A Comparative Geometric Analysis of the Patterns Found on the Pavement Mosaics of the Chedworth Roman Villa

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Abstract

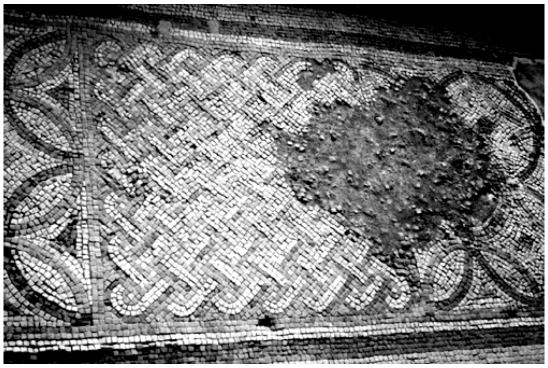
The Roman Villa found in Chedworth, England comprises several pavement mosaics that contain a variety of geometric patterns. The most notable of these patterns is the intricate swastika meander found in the dining room, recently subjected to an explicit geometrical analysis. However, the remaining mosaics in the villa, characterized by their uniqueness and apparent variety, have not received the attention they deserve. In this paper, the remaining pavements of this ancient villa are analyzed from a transformation-geometry point of view, thus filling this lacuna. Although the patterns exhibit considerable perceptual differences, the analysis reveals that they are closely related, and may be easily morphed from each other by means of a few simple geometric transformations. It is also shown that they are closely related to patterns found in Basque and Islamic decorative arts, as well as to families of curves well studied in the mathematics literature. Finally, it is shown how these patterns may be incorporated into a modular design process that yields many new and interesting pattern designs.

Introduction

The dining room was the principal and most splendidly decorated room in typical ancient Roman villas, a tradition that the Romans probably inherited from the Greeks [15], [28]. Evidently, this is also the case for the Chedworth Roman Villa in the United Kingdom. In fact, since at least 1903, the main large and intricate swastika meander pattern of this dining room has received much attention [8], and more recently, in spite of its claimed complexity as a testimonial to the genius of its creators, a simple geometric algorithm has been discovered for its generation [18]. The remaining mosaics in the villa, characterized by their uniqueness and variety, have not received any attention, and it is the purpose of this paper to fill this lacuna. In the past one hundred years such patterns have been analyzed and compared predominantly by means of the symmetry properties they possess [6], [14], [27]. However, there exist other possible approaches to pattern analysis. Indeed, according to Radovic and Jablan [21], "the theory of symmetry, literally taken from mathematical crystallography, is probably not the only way, and maybe not the best explanation for the construction of ancient antisymmetric patterns." Indeed, algorithmic, computational, and modular approaches such as those in [18], [19] and [21] have been recently explored as useful tools that offer explanations for the construction of patterns, and for measuring their resemblance to each other. Accordingly, this paper does not focus on the symmetries present in the patterns of the Chedworth Roman Villa. Rather, the remaining unexplored pavements of the villa are analyzed using a geometric transformation approach. Although at first glance these patterns exhibit considerable perceptual differences, the analysis reveals that they are closely related and may be easily morphed into each other by means of a few simple geometric transformations, thus suggesting ways in which the patterns may have been originally discovered and constructed. It is also shown that they are closely structurally related to other patterns used in the geographically distant Basque and Islamic decorative arts, as well as to families of curves well studied in the mathematics literature. Finally, it is shown how these patterns may be incorporated into a modular design process that yields many new and interesting pattern designs.

The Patterns in the Corridor of the Villa

The long corridor leading off to the side of the dining room is filled with patterns in various states of preservation. In this section, these patterns are described and analyzed in turn.



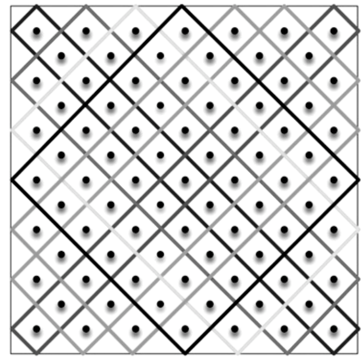


Figure 1: *A 7 × 7 guilloche pattern with 7 strands (left). The mirror-curve representation of the 7-strand guilloche pattern (right).*

A photograph of a noteworthy pattern in this main corridor is shown in Figure 1 (left). In spite of the fact that a significant portion of this mosaic has been destroyed, it is possible to reconstruct the structure of the complete geometric over-under guilloche pattern present, using the algorithm described in [18]. The resulting reconstruction yields the pattern shown in Figure 1 (right). This pattern is identical to the *peanut tillage* pattern used in the sand drawing tradition of the Chokwe people of Angola, except that the latter consists of six rather than the seven curves evident here [12]. These patterns fall under the domain of *mirror curves*, and detailed studies of mirror curves as well as algorithms for their construction may be found in [11]-[13], [16]-[17].

In the ancient Roman mosaic pavements one often finds that the borders of a pattern are surrounded by straight-line meanders called *key patterns* [8], [10]. One of the mosaics in the Chedworth Villa is surrounded by the rather unusual step key pattern illustrated in Figure 2.

Figure 2: *An unusual step key pattern.*

A geometrical analysis of this key pattern reveals elementary transformations that convert it to two of the most common meander patterns found in ancient Greek and Roman mosaics [9], [19], [23], the very simple square snake key pattern of Figure 3 (a), and the simplest known swastika meander pattern of Figure 3 (e). The relations between these three patterns are explicated visually in Figure 3 by means of simple geometric transformations. The step key pattern of the Chedworth Villa is shown in (c). One possible intermediate step to transform (a) into (c) is shown in (b). Finally, an intermediate step transforming the Chedworth pattern to the swastika meander is shown in (d). These transformations suggest that the swastika meander may have evolved in this manner from the square snake key pattern.

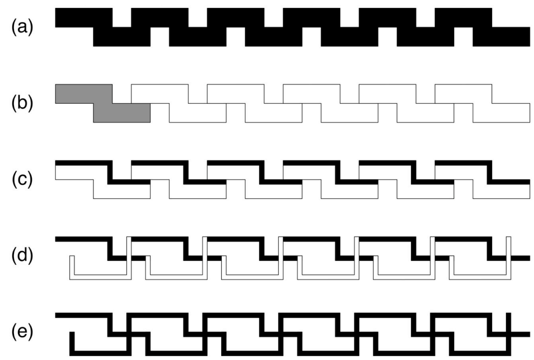


Figure 3: *Transforming the step key frieze pattern to a swastika meander.*

The straight line is considered the simplest element in pattern design, and following the straight line, the simplest of all figures is arguably the circle. Since antiquity, the circle has played an integral part in the pattern design of many cultures. Lewis Day devotes Chapter 5 of his pattern design book [8] to the circle, starting the chapter with the sentence: "A most important element in geometric pattern is the circle: with it curvilinear design begins at once to flow more freely." Eric Broug [5] expresses a more categorical sentiment; his first sentence in Chapter 1 reads: "The starting point for every geometric pattern is a perfect circle." Once a single circle is drawn (which he calls the *primary* circle), the next most important decision in its application is the selection of the number and location of a group of *secondary* circles. Broug demonstrates with numerous examples that a large number of geometric patterns used in Islamic art emanate from the addition of four secondary circles that go through the center of the primary circle, and that are equally spaced apart at 90 degrees from each other, as illustrated in Figure 4 (left). If the primary circle is deleted from this pattern the four remaining circles form a unit that fits in a square that may be repeated, with or without overlap, to tile the floor plan, as in Figure 4 (center). The four-circle unit may also be rotated at 45 degrees to form another square region that may tile the floor, shown in Figure 4 (right).

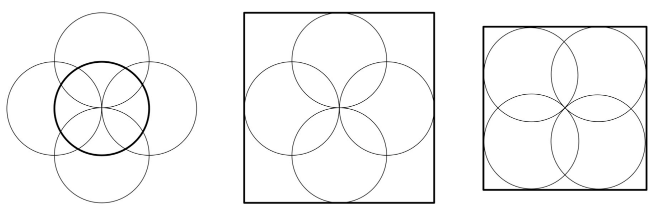


Figure 4: *Primary circle (in bold) with four secondary circles as a basic pattern design unit.*

The two basic units of Figure 4 are employed in several mosaics found in the Chedworth Roman Villa. Interestingly, although both patterns are the same in the sense that one is a mere rotation of the other, the resulting changes in orientation significantly alter the perceptions of the viewer. In the center diagram of Figure 4, the *lunes* (intersections of two circles) lie in diagonal orientations, whereas in the diagram on the right the lunes are oriented vertically and horizontally.

A black-white version of a photograph of one of the pavement mosaics in the Chedworth Villa that uses the unit in Figure 4 (center) is shown in Figure 5, and an idealized geometric version of it is pictured in Figure 6. The vertically aligned circles in Figure 4 (center) correspond to the dark circles in Figures 5 and 6, whereas the horizontally aligned ones correspond to the light grey circles. Note that the overall patterns in these pavement mosaics may be viewed as the superimposition of one array of circles (dark) over a diagonal translation of itself (light grey). In addition, the centers of each circle are marked with a diamond shaped tiling with shades (colors) matching those of their containing circles.

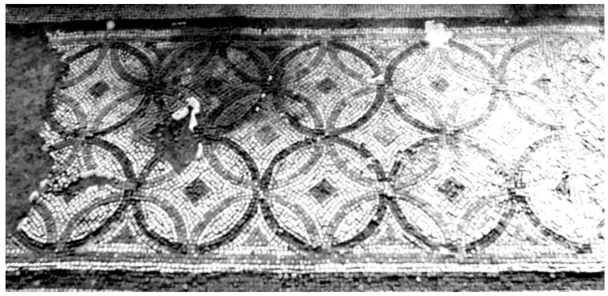


Figure 5: *Chedworth Villa mosaic that uses the unit of Figure 4.*

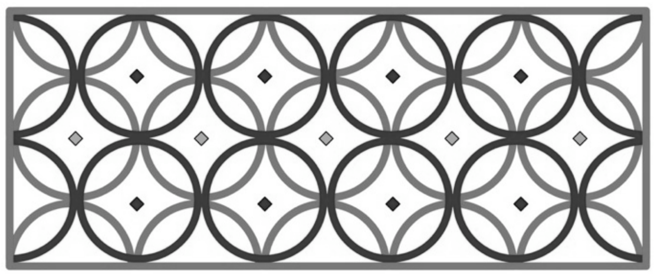
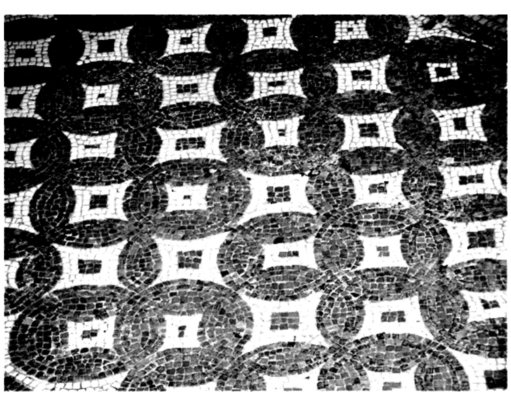


Figure 6: *Idealized geometric structure of the mosaic of Figure 5.*

Figure 7 (left) shows a black-white version of a photograph of another pavement mosaic found in the corridor of the Chedworth Villa, along with its idealized geometric structure on the right. Note that although at first glance the mosaics of Figures 5 and 7 look very different, a more careful comparison shows that the underlying geometric structures are identical, except for the fact that one is rotated by 45 degrees. The apparent significant difference in perception of the two patterns is caused by the rotation, as well as the different coloring schemes used.



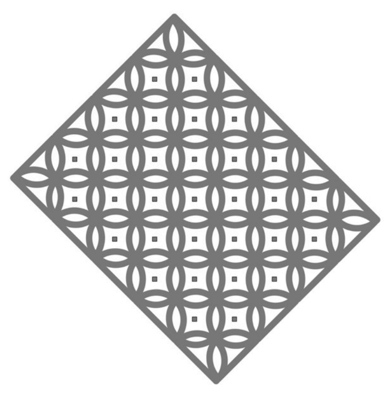


Figure 7: *A pattern (left) with the same idealized geometric structure as the mosaic in Figure 5, but with a very different apparent look. The idealized structure of the mosaic rotated by 45 degrees (right).*

One of the richest geometric mosaics in the Chedworth Villa is shown in Figure 8. The main and most interesting components in this design consist of two types of units. The first is made up of two smaller circles contained in the larger circle. The second is obtained by cutting the first unit in half, bisecting the smaller circles, resulting in two concave semi-circles contained in a larger semi-circle.

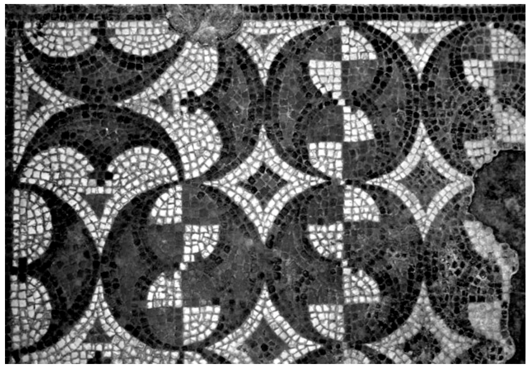


Figure 8: *An intricate pattern obtained by introducing two small circles inside a larger one.*

The types of plane curves employed in the mosaic of Figure 8 are called PC (*piecewise circular*) curves in mathematics [3], and have been applied to geometric modeling in the manufacturing industry [22]. As the name suggests, a PC curve consists of pieces (arcs) of circles connected together. The arcs may come from a single circle, or they may be taken from circles of different radii. The first type of PC pattern used (the left-most pattern in Figure 9) is closely related (by some simple geometric mutation transformations) to several other patterns that have a long history in the decorative arts. If the right semi-circle of the upper small circle, and the left semi-circle of the lower small circle are erased, the second diagram (a pair of embracing commas) is obtained, which becomes the well-known PC curve known as the Yin-Yang symbol in ancient Chinese philosophy when one of the commas is shaded, as in the third diagram [2], [24]. Finally, if the second diagram consisting of two PC curves is superimposed on a rotated version of itself, then the resulting four PC curves become the curved swastika in the right-most diagram. From the geometrical transformation point of view then, the Yin-Yang pattern is clearly related to the swastika, suggesting that one may have been obtained from the other in this simple manner. Indeed, Rudolf Arnheim (1961, p. 392), supports this view, writing that the Yin-Yang symbol "is likely to be a derivative of the ancient spiral, whorl-circle, and swastika patterns." [1].

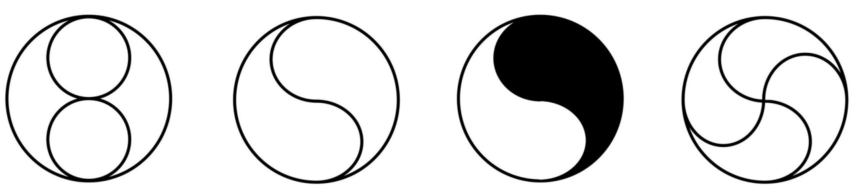


Figure 9: *The fundamental unit found in the mosaic of Figure 8, consisting of a circle containing two smaller circles, and some of its most common elaborations.*

Very little use has been made of the Yin-Yang symbol in pattern design. Amor Fenn (1993), in his compilation of hundreds of border designs lists only two (labelled N^o 204 and N^o 204^A) that use this symbol [10]. These are illustrated in Figure 10. Historically, the bright component is located at the top [1], but Fenn has chosen to disregard this convention in his designs.

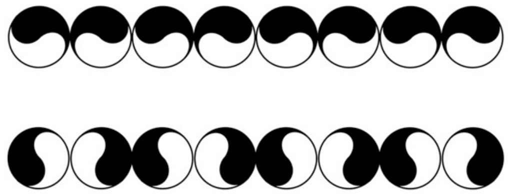


Figure 10: *The two border designs that use the Yin-Yang symbol in Amor Fenn's catalogue [10].*

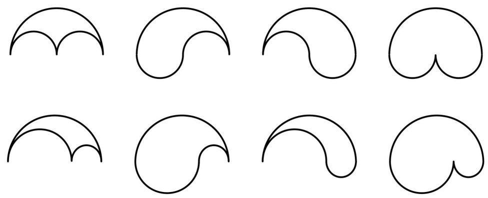


Figure 11: *The symmetric arbelos curve (top leftmost), its three siblings (top), and their asymmetric counterparts (bottom).*

The second type of unit used in the mosaic of Figure 8 is a special case of a well-studied PC curve in mathematics known as the *arbelos* [4], [20]. In the general arbelos curve, the two smaller semi-circles need not be the same size. In Figure 11 (top left-most), and the decorative arts exposited here they are the same size, and the resulting curve is therefore called the *symmetric* arbelos. The arbelos has three siblings determined by whether the smaller semi-circles are concave as in the traditional arbelos, or not. One can make only the first one convex, only the second one convex, or both, as illustrated in Figure 11. The symmetric convex-convex arbelos curve in Figure 11 (top right) is called a *cardioid*, and has the interesting property, in common with a circle, that any line that goes through the cusp (*center of length*) cuts the curve into two pieces of equal length [3]. In a circle, its center is also the center of length.

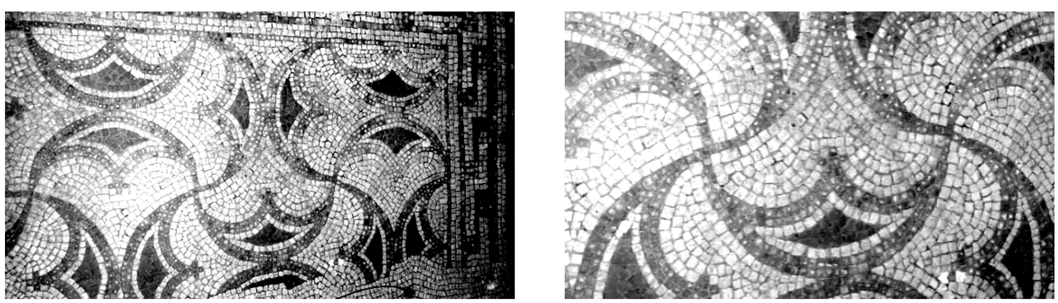


Figure 12: *This variant of large semi-circles that contain small semi-circles reveals curved swastikas (left). Detail of two curved swastikas (right).*

Another of the richest geometric mosaics found in the Chedworth Villa is shown in Figure 12 (left). It consists of a fascinating re-arrangement of arbelos curves that unwittingly generate simple small as well as larger stylized curved swastikas (see detail in Figure 12, right). The geometric structure of the emergent swastika patterns visible in the mosaic of Figure 12 may be described as the concatenation of four symmetric arbelos curves (with their interiors shaded) as shown in Figures 13 and 14 (left-most). Using the siblings of the arbelos curve (Figure 13), the arbelos swastikas may be transformed into their three siblings as illustrated in Figures 13 and 14. These arbelos curved swastikas may be described (in left to right order) as concave-concave, convex-concave, concave-convex, and convex-convex arbelos swastikas. It is natural to ask whether the three combinatorially generated "new" patterns in Figure 13, using a simple mathematical rule, are in fact new. It turns out that the second pattern from the left is a popular motif used in the Basque decorative arts [25], [26], but strangely enough seems not to appear in the traditional arts of other cultures. The third pattern appears frequently in the ancient Roman mosaics, a slight modification of it appears in the center of the dining room mosaic at Chedworth [18]. Yet, the fourth pattern on the far right is not found either in the Chedworth Villa or in other mosaic locations. Its childish and playful character is perhaps not elegant enough to make its appearance in decorative art.

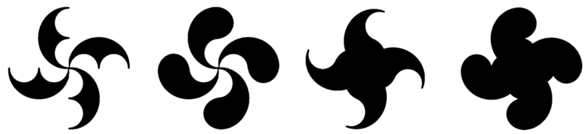


Figure 13: *The arbelos swastika in the Chedworth mosaic of Figure 12, and its three siblings.*

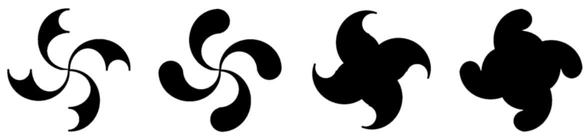


Figure 14: *The asymmetric arbelos swastika and its three siblings.*

Modular Pattern Design

The various patterns found on the pavements of the Chedworth Roman Villa can be incorporated into the framework of modular pattern design [7], [21] to create a vast number of new and interesting pattern designs that evolve from these ancient patterns. Space limitations do not permit an exposition of the possibilities. However, Figure 15 shows an example of what can be done by permuting the elements in the leftmost curved symmetric-arbelos swastika. A greater variety of interesting patterns may of course be obtained by incorporating the asymmetric arbelos unit, and alternating the colors of the various elements.

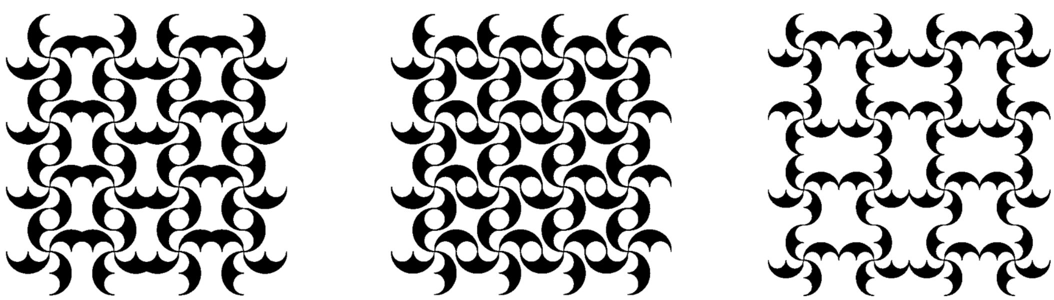


Figure 15: *Three pattern designs obtained with the leftmost curved swastika pattern of Figure 13.*

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