Learning Mathematics Through Dance

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Abstract

Mathematics educators and cognitive scientists are exploring the importance of embodied, multisensory, arts-infused experiences as a central component of mathematics learning, countering pedagogic traditions that valued only abstract, disembodied representations. Dance has great potential for sensory, fully-embodied mathematical engagement -- but how can abstract mathematical concepts be danced? I introduce the work of acclaimed Canadian modern dancer Sarah Chase and her explorations involving number theory, connecting her work to a Bridges 2009 paper on multisensory representations in number theory and exploring attentional aspects of her cross-patterning movement work and simultaneous verbal storytelling. This paper will be accompanied by an original short film of Sarah Chase's dance-based teaching.

Introduction: Embodied, Kinesthetic Mathematics Learning

Mathematical relationships are most familiarly represented abstractly, and the experience of many mathematics learners (and more often an alienating experience than otherwise) is of a subject that excludes most sensory, bodily and human-scale experience in favour of Platonic abstraction [5]. For many years, the students that were rewarded in most mathematics courses were those who could sit quietly and silently, doing mental calculations and scratching a few symbols down with pencil and paper to arrive at a correct answer. Movement, colour, sound, touch and other physical modalities for the exploration of the world of mathematical relationships were scorned in this system as primitive, coarse, noisy and not sufficiently elevated and abstract. This disembodied approach to mathematics education was encouraged, despite the documented fact that professional research mathematicians actually *do* make extensive use of sensory representations (including visual, verbal and sonic imagery and kinesthetic gesture and movement) and sensory models (drawings, physical models and computer models), both in their own research work and in their communication of their findings to colleagues in formal and informal settings [7], [27]. These bodily experiences ground the abstractions of language and mathematical symbolism [19].

In education and mathematics education in particular, there has recently been increased attention given to multisensory, embodied ways of learning (see, for example, [2], [17], [21], [22], [24], [25]). This interest coincides with a move away from the Platonic/Cartesian notion of a mind-body split and toward a recognition of the integration of physical, emotional and intellectual experiences that allow learners to engage in learning as whole people situated in body, place and culture. Cognitive neuroscientists have also recently established that knowledge cannot be separated from perceptual experiences [3], [11]. That is to say that knowing something implies some sort of cognitive reenactment of the multiple sensory modalities by which it was experienced. This perspective stresses the multisensory nature of the brain activity associated with our interpretations of our interactions with our world. Drawing from Merleau-Ponty [20], we might say that the forms of mathematics education have a phenomenological character: they are meaning-endowed structures with strong connections with the body, the senses and the world, with the complex relation of subject-mathematical object underpinned by visual, tactile, auditory, motor, spatial and temporal fields of perception.

Of course (and as I have written elsewhere [15]), necessary embodied experiences are not *sufficient* for the learning of mathematical concepts. Bodily movement, sensation and patterning do not 'speak for themselves' to mathematics learners, any more than graphs, diagrams and equations 'speak for themselves', without further explanation, to the uninitiated. Rather, an integrated approach, one that moves among multisensory, graphical, algebraic, numeric and verbal inquiries of mathematical phenomena, offers an ideally balanced program for teaching and learning. Learners' ability to bring together abstract intellectual understanding, algorithmic operational fluency and a visceral feel for mathematical concepts is the basis for robust, generative mathematical understanding where any one of these alone would not be.

With a new focus on embodied, multisensory facets of mathematics learning, educators can begin to ask new kinds of questions about curriculum and pedagogy. When learners are approaching a new mathematical topic, would it be helpful for them to *hear* a mathematical relationship, to *touch* it, or to know it through *movement*? Can learners learn the equivalence of a variety of representations that include, but go beyond, those usually invoked in mathematics classes? How might multisensory, bodily experiences of mathematical patterns and relationships serve as semiotic resources as students develop the ability to work with generalization and abstraction in mathematics? What resonances may be invoked when focused multimodal experiences and abstract mathematical symbolism are both present in learners' attention in the math classroom? A focus on embodied mathematics learning, and the growth of work on mathematics and art through organizations like Bridges, have also led to considerations of teaching mathematics through arts-infused pedagogies, since the arts are involved in the creation of sensory (visual, tangible, audible, kinesthetic), performative, embodied products that may be based upon mathematical patterns and structures.

Teaching Mathematics Through Dance: Possibilities and Limitations

Our human bodies are the means by which we experience dance and embodied mathematics, and our bodies, generally and particularly, have certain affordances and limitations. Elbows, knees, hips, wrists and ankles only bend in certain directions and through certain angular arcs. Limbs, feet, fingers are of a particular size that cannot be made bigger or smaller. Our strength, weight, flexibility and energy can vary with training, but there are limitations on how fast a person can move, how high one can jump and how far one can bend. Particularities in terms of special abilities, injuries or disabilities, age, height and so on create a range of human movement possibilities, but this range is bounded even for the athletes and acrobats amongst us.

Mathematical concepts are not bounded in the same ways. The most immediate and obvious example is any concept to do with infinity – and as Caleb Gattegno has written, mathematics is "shot through with infinity" [12]. Any mathematical entity that deals with infinitely large or infinitely small quantities or sizes, or any idea that involves extreme changes of scale is very hard to feel and illustrate with our bodies – ask anyone who has attempted to dance the principles of infinitesimal distances and areas that underlie calculus, or Zeno's paradox, asymptotes or fractals. If mathematics learners have an imaginative sense of how these infinite or infinitesimal concepts work, based on a teacher's skillful narrative or a computer simulation that allows for zooming, then a bodily experience of these concepts can work as a reminder or a simulation that recalls the imagined concepts. (For example, when Tim Chartier [8] mimes an infinitely long rope, the audience builds upon his verbal explanation and their prior knowledge to 'see' a finite motion as representing the infinite.) The dance acts as a kind of bookmark or memory jogger in these cases, rather than as the primary experience upon which concepts are built.

There are similar problems when we try to represent the graphs of functions in a static way with our individual bodies, since our bones, limbs and joints limit the curves and angles we can represent. One of the currently most popular ideas for math dance among North American teachers is the idea of 'function calisthenics' (see Figure 1). These 'beautiful dance moves' look fine when drawn with a squiggly stick figure, but real-life human arms cannot form the smooth rounded curves of the stick figure's arms. What is more, the positioning of our arms on our bodies means that important pieces of the curve can't be shown in this way (for example, the vertex of the quadratic or absolute value graph), and the asymptotic nature of the reciprocal function is not convincingly shown. That is not to say that 'function calisthenics' are not useful in math class, but they are helpful as memory jogs for concepts learned in other way, rather than as a primary way to understand the graphs of functions.

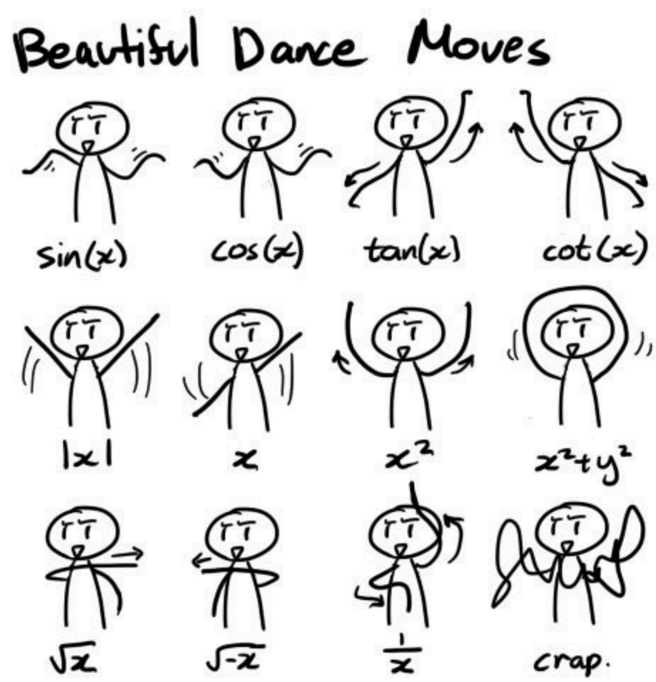

Figure 1: *Math Dances! Human arms are not quite as bendable as the ones drawn here. Photo by Dylan231, used under Creative Commons license.*

Another factor in designing dances to teach mathematical concepts has to do with the possibilities and limitations of dance as a form and a cultural practice. Dance teachers are often provided a curricular chart like the following in Figure 2, which although simplified, gives a sense of the elements that could be combined in the making of a modern dance. I will refer to the 'time' and 'space' elements of dance sketched in this chart to discuss some of the affordances and constraints of dance as a medium for the embodied teaching of mathematical topics.

Particular mathematical topics often map most easily onto particular elements of the dance, while others are very difficult to model through dance. I have already discussed the difficulty of representing anything to do with infinity through dance. Similarly, it is difficult to dance areas of mathematics that concentrate on symbolic manipulation; how to dance the rules of algebra or of symbolic logic?

In contrast to these, the broad area of geometry (including topology and even graph theory) maps easily onto body shapes, spatial relationships among dancers and spatial pathways, and invites the choreographer or teacher to use props like string or ribbon to create lines marking out spatial pathways. Number theory, in contrast, maps most comfortably onto metered time or onto regular, clocklike spatial models of metered time. (There are some subtleties in the choice of how to represent relative times when dancing number theory, and I will discuss these later in this paper.) Kant noted that geometry is about space and arithmetic is about time; anyone who has tried to teach embodied mathematics through dance discovers this contrast immediately.

The Elements of Dance

The Elements of Dance can most easily be remembered by using the acronym "D.R. B.E.S.T." (Dance is Relationships, Body, Energy, Space and Time). The chart below lists some way to describe each of the elements – there are many more possibilities for each element. Can you think of other descriptive words? Add your own ideas and words.

	Relationship	Body	Energy	Space	Time
Dance is . . .	***Personal Relationships:*** Friends, siblings, parents, romance, etc. and all of the emotions that accompany those relationships (happiness, envy, hatred, love, etc.). ***Spatial Relationships:*** In front, beside, behind, over, under, alone/connected, near/far, formations ***Timing Relationships:*** Before, after, unison, canon, sooner than, faster than	***Parts of the Body:*** Head, eyes, torso, shoulders, fingers, legs, feet . . . ***Initiation:*** Core, distal, mid-limb, body parts ***Body Shapes:*** Symmetrical, asymmetrical, rounded, twisted, angular, arabesque . . . ***Body Systems:*** Muscles, bones, organs, breath, balance, reflexes ***Inner Self:*** Senses, perceptions, emotions, thoughts, intention, imagination ***Non-locomotor (axial):*** Stretch, bend, twist, turn, rise, fall, swing, rock, tip, shake, suspend ***Locomotor (traveling):*** Slide, walk, hop, somersault, run, skip, jump, do-si-do, leap, roll, crawl, gallop, chainé turns	***Attack:*** Sharp/smooth Sudden/sustained ***Weight:*** *Strength:* push, horizontal, impacted *Lightness:* resist the down, initiate the up *Resiliency:* rebound, even up and down, action/reaction ***Flow:*** Free, bound, balanced, neutral ***Quality:*** Flowing, tight, loose, sharp, swinging, swaying, suspended, collapsed, smooth	***Size:*** Large, small, narrow, wide ***Level:*** High, medium, low ***Place:*** On the spot (personal space), through the space (general space), upstage/downstage, stage right/left (specific space) ***Direction:*** Forward, backward, sideways, diagonal, right, left ***Orientation:*** Facing front, back, side, etc. ***Pathway:*** Curved, straight, zig-zag, random ***Spatial Relationships:*** See Relationship column	***Metered:*** Pulse, tempo (fast/slow), accent, rhythmic pattern ***Free Rhythm:*** Breath, open score, sensed time, improvisation, cued ***Clock Time:*** Seconds, minutes, hours ***Timing Relationships:*** See Relationship column

Figure 2: *Elements of Dance, from Peel Schools Elements of Dance, Ontario, Canada. Retrieved from <http://myclass.peelschools.org/ele/8/14689/Lessons/Dance/Introduction%20to%20Dance%20Unit/The%20Elements%20of%20Dance.pdf>*

Teaching Mathematics Through Dance: Some Examples

A number of dancers and mathematics educators have worked with embodying mathematical patterns and teaching them through dance. Many of these dancer/educators have presented at Bridges conferences.

One prominent example is the California dance company 'Dr. Schaffer and Mr. Stern' who have been producing and teaching mathematical dances since 1987. This group's pioneering work includes dances, classes and a book, *Math Dance* [26] dealing with mathematical concepts including combinatorics and probability, symmetry, the geometry of polygons and polyhedra, and number theory ideas. New work from this group includes multimedia dance representations of the history of mathematics in *The Daughters of Hypatia: Circles of Mathematical Women*. Sarah-marie belcastro and Karl Schaffer, collaborating mathematician-dancers [4] have co-written an article that considers 'artistic dance' and mathematics in some generality, considering issues of symmetries (both of the individual dancer's body and moves and those of a group of dancers), rhythms and their relation to number theory ideas, the use of dance to teach geometry, choreography informed by mathematics, and Laban's polyhedral modeling of dimension of dance movement.

A dance representing "symbolic dynamics that arise from cutting sequences on Veech surfaces and

Bouw-Möller surfaces" (a topic in geometric topology) won Brown University's Diana Davis a prize in the 2012 *Dance your PhD* contest sponsored by *Science* magazine. The dance can be seen at [6] and offers an understanding of the spaces using mirrors, color coding, trajectory lines representing cuts, and dancers in colored t-shirts and hats representing sequences of sides of polygons in the space. Clearly, mathematics at both elementary and advanced levels can potentially be represented via dance.

Examples of elementary and secondary school classes dancing mathematical topics can be found in a few places on the internet – for example, [18] covering a variety of elementary school mathematics topics, and [30] for a particularly thought-provoking dance on dimensionality, with points of light, shadows on a flat screen, and finally movement in 3 space representing the progression from zero to three dimensions. A number of dance/movement educational ways of teaching mathematics are part of the growing collection of digital mathematics performances at the 'mathfest.ca' festival site.

In the realm of folk music and dance, several mathematicians have analyzed regularities in contradance moves – for example, Peterson [23] uses matrices and group theory to list all the states and locomotion moves of the contradance set. Peterson refers to Copes' 2003 talk to the Mathematical Association of America (MAA) and demonstration with the National Council of Teachers of Mathematics (NCTM) on the use of group theory to choreograph contradance sequences (available at [10]). Although it might seem, from a practical choreographer's point of view, a huge apparatus used to achieve a fairly simple result, and although the original attempts required some tweaking to make them danceable, it is interesting mathematically as an exercise in modeling all possible locomotion moves and end states.

My own work through Bridges has touched on a related tradition, English longsword and rapper sword dance [14], and the beginnings of an analysis of the geometry of longsword locks through the physical algorithms used to create them as part of the dance tradition.

How Sarah Chase Dances (and Teaches) Aspects of Number Theory

Those of us who have taken on the challenge of teaching aspects of number theory (divisibility, factoring, least common multiples, relatively prime numbers) in embodied ways often run into problems of representing two contrasting, syncopated rhythms in a fixed time in a way that is difficult for all but the most experienced percussionists to achieve. To offer a few examples of this:

• I have tried to show the relationship between the numbers 2 and 3 by walking side by side with a dancer friend, trying to walk two steps to her every three, and instructing learners to watch for where our steps coincide. In actual practice, this is very difficult to achieve; steps cannot be slowed down too much or the walkers will lose their balance, and it is hard to time 2 steps against 3 in the same fixed time, even with a third person keeping time with an audible series of handclaps or other pulses.

• In a 2009 Bridges paper [13], four of us used clapped, syncopated musical rhythms over a fixed time to teach these number theory notions in an embodied way. We also used Godfried Toussaint's circular representations of musical rhythms [29] and Spirograph patterns using the familiar geared drawing toy to make visual, polygonal or star-shaped representations of the relationships between numbers that are either relatively prime or which share factors. Once again, we ran into difficulties in clapping syncopated patterns in a fixed time – for example, attempting to clap 7 against 3 is very difficult for all but an accomplished percussionist.

• Dr. Schaffer and Mr. Stern use a similar tactic of clapping syncopated rhythms against each other to embody these number theoretical concepts, although they use a colour-coded number line along with circular notation as a visual display of the results. Schaffer and Stern use the more workable system of having a time-keeper tap out every beat (rather than every bar), so that, for example, the person clapping a 3-rhythm would join in on every third beat, and the person clapping a 7–rhythm would join in on every seventh beat. Nonetheless, it is still difficult to keep track of the count and of the relationship between 3 and 7. Accomplished percussionists like Keith Terry, Evelyn Glennie and others also perform these virtuosic feats (and can be seen doing so in online videos), but for most mathematics learners, even beating a syncopated rhythm of ‘2 against 3’ with left and right hands is very difficult and takes practice.

Canadian dancer and choreographer Sarah Chase [1], [16], [28], working independently in dance, and outside the field of mathematics education, has come up with an ingenious solution to the pedagogical problem of teaching about divisibility and relatively prime numbers in an embodied way. The remainder of this paper will describe Chase’s Number Theory dances and the ways she has used combinatorics, cross-patterning and simultaneous narrative to stimulate both left and right sides of the body and brain, activating a mathematical awareness of number that is challenging, visceral and learnable.

Chase is an acclaimed modern dancer whose physicist grandfather led her to a lifelong interest in mathematical patterns. Her approach to divisibility and related number theoretic concepts is a combinatoric one, physically realized on the left and right sides of the body at once, a bit like patting your head and rubbing your stomach. She engages both number and emotive storytelling and imagery in the dances. Here is a description of a very simple version of her number theory dances presented at a workshop with young children on Hornby Island, BC, Canada this past summer:

Participants were instructed to use their left arm to cycle slowly through two positions: high (representing summer), then low (winter). At the same time, their right arm would cycle through three positions: high (representing ‘my mother’), horizontally extended (‘my father’), and low (‘me’). Slowly, participants cycled left and right arms through all the possible combinations, moving both arms simultaneously: both arms held high (‘my mother in summer’), left arm low, right arm horizontal (‘my father in winter’), left arm high, right low (‘me in summer’), left low, right high (‘my mother in winter’), left high, right horizontal (‘my father in summer’), and finally both arms low (‘me in winter’). Once both arms coincided in the low position, a full cycle had completed and would be repeated. Participants were asked to move slowly, letting an image develop for each of the family images associated with each move – ‘my mother in summer’; ‘ my father in winter’.

With an adult group, Sarah might use this imagery to develop some therapeutic work around healing and family relationships. With the kids, she asked how many moves it took to get the 2 on the left and the 3 on the right back in sync. When the kids answered “Six!”, she said, “So now you know *why* two times three equals six”.

This very simple two-against-three two-handed exercise exemplifies the principles of Chase’s more complex number theory dances: using combinatorics to approach number theory; using bilateral and cross-body movement to stimulate new physical/ cognitive/ emotional patterning and associations; bringing elements of storytelling and words spoken over the movements to add layers of meaning to the physical/ combinatoric experience. Chase does *not* use temporal syncopation in her embodied representations of the numbers, but rather uses a kind of spatial syncopation – that is, the moves are done on both sides of the body simultaneously, without reference to a fixed beat or timekeeping, and may actually be done quite slowly, but each arm is at a different part of its cycle, so that they are *spatially* out of sync until the cycle is complete. I encourage readers to try out this simple two-against-three exercise: elementary as it is, it really does feel mind-bending as the left and right sides of the body move slowly and simultaneously through their patterns and finally come to rest at the low position together.

Chase’s dances are both artistic and pedagogical, engaging workshop participants or audiences in a discussion of the nature of calculation, relative primeness of number pairs or triples, and the applications of combinatorics to life situations. Chase recounts that her starting point for these dances came from a wish to make dances that were useful to people in practical ways [9]. In particular, at a time when she began using an ocean beach as a rehearsal space, she came up with the idea of using the body to calculate the tides, since the tides depend only upon the relative movements of moon, earth and sun. From the idea of a human tide calculator came her human Chinese Horoscope calculator and human calculator of the 1001 Nights of Scheherazade's legendary storytelling. *Chinese Horoscope* uses the left side of the body to symbolize (gesturally) each the five elements and the right side, the twelve animals that combine to make the sixty-year cycle of the Chinese Horoscope, combined as well with two foot positions to symbolize yang and yin qualities of each. In performance, she begins with the year 1936, and can 'physically calculate' (through dance) the animal and element for any birth year up till 1996. *1001 Nights* uses both arms and one leg to physically run through all the combinations of the prime factors of 1001 (7, 11 and 13) while simultaneously telling stories as Scheherazade did.

Chase's dance performances work with pairs and triples of numbers that are relatively prime, but in her workshops with children and adults, she encourages participants to experiment with number pairs of their own choosing. Learners will often choose pairs of numbers that share a common factor, and will reach 'aha!' moments when the embodied combinatoric patterns "repeat more quickly than they expected" [9]. Chase reports learners devising their own algorithms to calculate how much faster the patterns repeats; for example, in a '6 against 9' pattern, one might expect 6 X 9 = 54 combinations, but the common factor of 3 reduces this to 54/3 = 18 combinations before the pattern begins to repeat. This kind of mathematical exploration, based on embodied, sensory experiences, could easily be adapted for use in senior elementary or junior secondary school mathematics classes, and could integrate mathematics with dance-making and storytelling.

Chase says that the experience of dancing combinatorics has given her "a profound understanding that number relationships are not just something written down on paper, but that they have to do with phasing, time and a kind of harmony. Math can be about relationships in time [9]". Chase has always enjoyed working with mathematics, but her school experiences with factorization and divisibility always involved static spatial representations of area and volume, or simply calculations. Through dance, "harmony and relationship and things coming in and out of sync with each other makes the mathematics *alive* in a different way. It no longer feels static, but living and in motion." This sense of mathematical relationships coming alive through embodied movement in time has much to offer learners as a cognitive resource for exploration, problem-solving and abstraction around numerical relationships and properties.

We are completing a short film on Chase's dance and her teaching work on number theory and story with groups of adults and high school students, to be shown at the Bridges 2013 short film festival. Chase is one of just two artists invited to participate in an international neuroscience symposium this year, based on the potential for her work affecting brain plasticity. Sarah Chase has brought a new approach to problems of teaching number theory through embodied movement and dance. I look forward to her collaborations with mathematicians and mathematics educators, and the possibility of participation in an upcoming Bridges conference.

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