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Geometric Visual Instruments Based on Object Rolling

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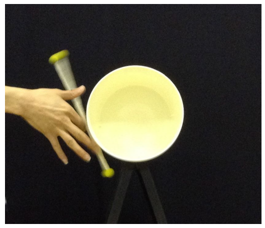
Abstract

If some object can be manipulated using a part of a body, especially, using hands, to create a sequence of aesthetically or geometrically attractive visual patterns, we call it the *visual instrument*. In this paper, we present visual instruments based on rolling objects on geometric surfaces. First, we present visual instruments that use line segments (or physically, batons) and geometric surfaces such as a cylinder, a globe, and a sequence of truncated cylindrical surfaces on which to roll the batons. We also present the visual instrument called the "gourds", which are made of some circular arcs. We made simulation for visualizing tempo-spatial patterns and roulettes the instruments create. Through construction of the visual instruments and their manipulation, we have confirmed that they have a high potential for use in manipulative visual performance and play.

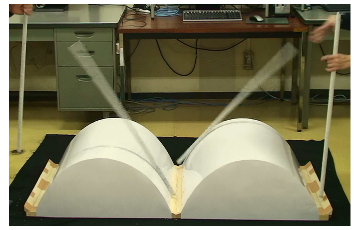
1 Introduction

Study on geometric curves on a plane has a long history. Among them there is a line of research on curves rolling along a fixed curve without slipping. One of the most studied cases is a circle rolling on a straight line. In this case, the locus of a point on the circle, that is, the roulette of the point, is the cycloid. In the 17th century, the cycloid was extensively studied by the great scientists such as Galileo, Pascal, Huygens, and Bernoulli. Exploration on the cycloid has been done not only for mathematical purposes but also for applications in various fields such as mechanics [1], astronomy [2], and even in the design of musical instruments [3]. The other important combination of curves is a line (segment) rolling tangent to some fixed curve. The roulette in this case is an involute, which was first considered by Huygens in designing accurate clocks [4]. Involutes have also many applications. For example, the circle involute, which appears when a line segment is rolled along a circle, has been commonly used for the profile of the teeth of gears. Another recent application of involutes is the vehicle with square wheels rolling on a sequence of truncated catenaries, which was first considered by G. B. Robinson in 1960 and was constructed by Stan Wagon in 1997 [5]. Success of these applications should rely on the dynamics and functionality of the rolling curves that matches practical requirements.

In this paper, we are interested in realizing visual patterns of rolling curves created through physically manipulating actual objects. Accordingly, we name that if some object is manipulated using a part of a body, especially, using hands, to create a sequence of aesthetically or geometrically attractive visual patterns, we call it the *visual instrument*, just like an instrument designed to make musical expression through manipulation is called a musical instrument. With this definition, visual instruments should include classical props and instruments used in performing arts and rhythmic gymnastics such as balls, clubs, rings, batons, and ribbons. One of the authors has also proposed some visual instruments [6-8]. In [6], balls are put inside a large clear globe and are rolled on the inner surface of the globe to create a variety of spherical patterns. In [7,8], balls are put inside a large cylinder and are again rolled inside to create somewhat anti-gravitational motion of balls, moving up and down inside the cylinder.



(a)



(b)

Figure 1: *Rolling of batons* (a) *around a cylinder; and* (b) *on a sequence of truncated surfaces.*

In this paper, we present new visual instruments based on rolling curves. The first instrument is based on a line segment rolling on a circle. It is physically implemented using a straight baton rolled on the outer surface of a cylinder or a globe. We made simulation for visualizing the roulettes the moving endpoint of a line segment generates. Then, we materialized the instrument and experimented the rolling of a baton around a cylinder as shown in Figure 1(a).

The second visual instrument is based on a line segment rolling on a sequence of truncated curves, or physically, on a sequence of truncated cylindrical surfaces. The idea is similar to the aforementioned square-wheeled vehicle, but for actual manipulation and for creating a variety of tempo-spatial patterns, we use batons to roll instead of squares. We designed and constructed two types of instruments of this kind: one is for a single player and the other is for a pair of players who stand on opposite sides and pass the batons by rolling them on the surfaces as shown in Figure 1(b). Through actual manipulation, we confirmed that it is possible to manipulate multiple batons on the surfaces to create a variety of tempo-spatial patterns.

The final visual instrument consists of several circular arcs and looks like a gourd with two heads; thus, we call it the "gourd". Two gourds are used for manipulation where one is rolled along the other. We show simulation results for the roulettes they make and also show demonstration of manipulation.

The remaining of the paper is written as follows. In Section 2, we summarize some basics of rolling curves. In Section 3, we present our idea of visual instruments and show results on software simulation. In Section 4, we report on the actual construction of the instruments and show some demonstration of manipulation. Finally, concluding remarks are given in Section 5.

2 Rolling Curves and Roulettes

We summarize some basic concepts and examples concerning rolling curves on fixed curves. The general setting is shown in Figure 2. A *roulette* is the locus of a point on a rolling curve that is generated while the curve rolls without slipping along a given fixed curve.

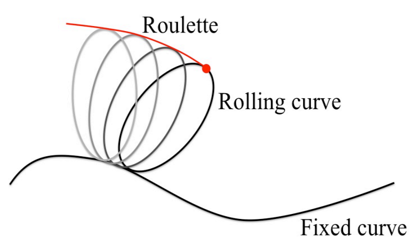


Figure 2: *A curve rolling along a fixed curve*.

There are a variety of combination for the pairs of curves as the rolling and the fixed curves. One of the most frequently considered combinations is a rolling circle along some fixed curve. When a fixed curve is a line, the roulette of a point on the circle forms a cycloid as shown in Figure 3. When the fixed curve is also a circle, and the first circle rolls internally or externally along the fixed circle, then the roulette forms a hypocycloid or an epicycloid, respectively. Figure 4 shows the case of the radii of both circles to be identical, generating the epicycloid as the roulette.

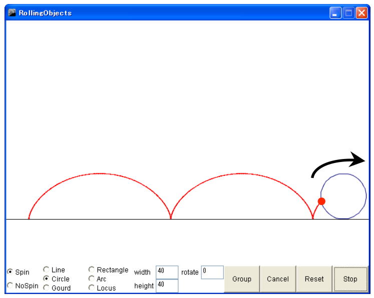


Figure 3: *A circle rolling on a line.*

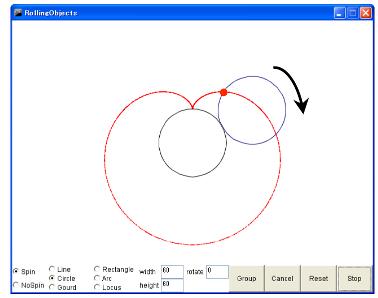


Figure 4: *A circle rolling on a circle of same size.*

Another frequently considered case is a line or a line segment rolling along some curve. The roulette in this case is generally called an *involute*. It is interpreted as the curve obtained by attaching a taut string to the fixed curve and tracing its free end as it is wound onto the fixed curve. When the fixed curve is a circle, cycloid, or catenary, the roulette is a circle involute, a cycloid, or a tractrix, respectively. One of the amusing applications of involutes is the vehicle having wheels of a regular polygon, say, a square. Figure 5 illustrates a square rolling on the "road" composed of a sequence of truncated catenaries. The crucial property of this construction is that the centroid of the square traces a horizontal line; therefore, the saddle of the vehicle does not change its height. While the examples of a taut string and a rolling square are physically implementable and they generate visual patterns in the real world, they are not sufficient as visual instruments in that both of them have left little space for a performer to manipulate multiple objects in various ways to create a variety of tempo-spatial patterns. We resolve this by simply rolling a line segment, or a baton, on a given curve.

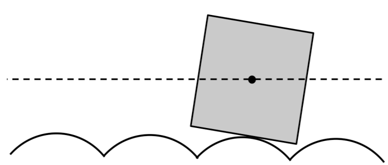
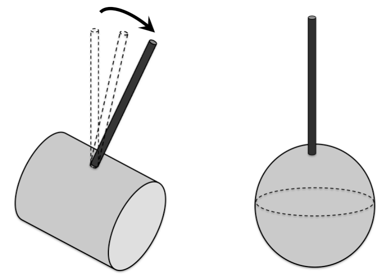


Figure 5: *A square rolling on a series of catenaries.*



(a) (b)

Figure 6: *Rolling a baton* (a) *on a cylinder; and* (b) *on a globe.*

3 Three Types of Visual Instruments Based on Object Rolling

Rolling Batons on a Circle. As geometric forms, we first consider a line segment rolling on a circle. If the line segment is tangent to the circle, this is exactly the case of generating a circle involute stated in Section 2. If we consider its physical implementation, there is also another phase of the line segment rolling on the circle with its endpoint fixed as the center of rotation. Figure 6 shows two kinds of physical implementation of this rolling where a baton is rolled (a) on a laid cylinder and (b) on a globe. We made simulation for visualizing the movement of the rolling in the plane. Figure 7 shows the situation of a line segment rolling on a circle. Figure 8 shows a part of the roulette of an endpoint of the line segment. It is observed that the roulette consists of two parts: one is the circular arc with radius equal to the length of the line segment, which is generated while the line segment rolls on the circle with its endpoint fixed, and the other is the circle involutes generated while the line segment rolls tangent to the circle.

Since the roulette of an endpoint of a line segment varies according to the ratio x of the length of the line segment to the circumference of the circle, we visualized the movement of the line segment and observed the roulettes for a number of values of x. Figures 9 and 10 show the cases of x = 1, 3/2, 2, and x = 1/2, 1/3, 1/5, respectively. When x = 1, that is, when the length of the line segment is equal to that of the circumference of the fixed circle, an endpoint of the line segment winds around the circle twice as shown

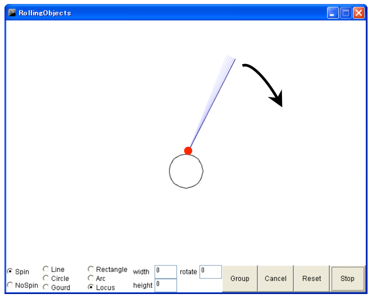


Figure 7: *A line segment rolling on a circle.*

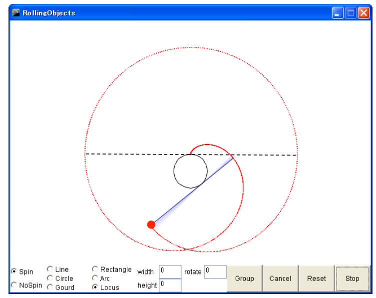


Figure 8: *A locus of an endpoint of the line segment.*

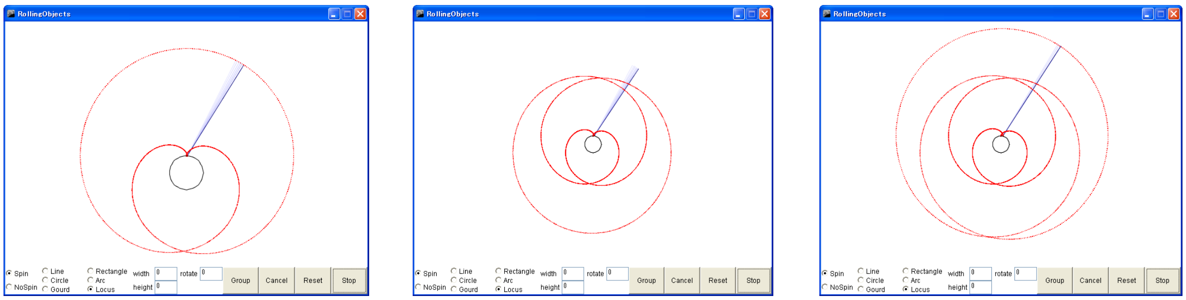


Figure 9: *Roulettes of an endpoint of the line segment when x =*1, 3/2, *and* 2.

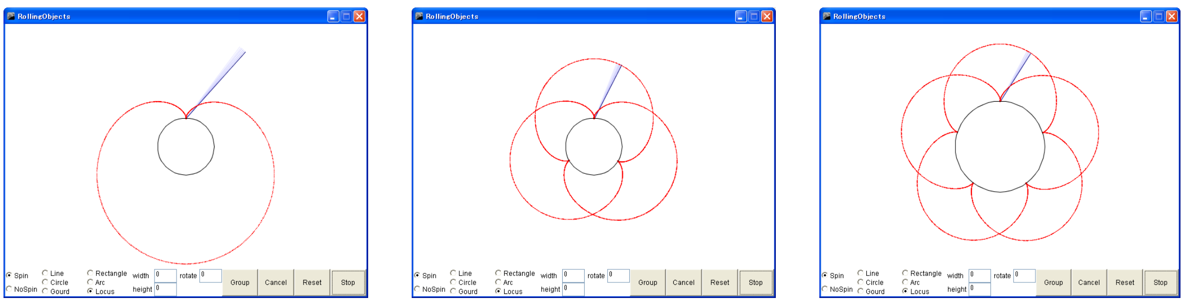


Figure 10: *Roulettes of an endpoint of the line segment when x =*1/2, 1/3, *and* 1/5.

in the left-hand side of Figure 9, and the roulette consists of the half circular arc and the two truncated circle involutes. When $x = 1/2$, the roulette shown in the left-hand side of Figure 10 is similar to the epicycloid shown in Figure 3, but their shapes are different. We note that for any positive value of x, the roulette is a curve that consists of circular arcs and circle involutes.

Rolling Batons on a Sequence of Truncated Curves. Similarly to the case of the square-wheeled vehicle, we use a sequence of truncated curves as a fixed curve. However, to make the visual instrument as simple as possible for a performer to manipulate it skillfully easily to create a variety of tempo-spatial patterns, we use a line segment again instead of a square as an object to roll. This results in allowing any smooth convex curves as truncated curves as far as the following two conditions are satisfied. First, the lengths of the truncated curves should be equal to the length of the line segment. Second, the upper angle between two neighboring truncated curves must be more than or equal to 90 degrees to avoid the line segment getting stuck to the cusp between the curves. Figure 11 shows line segments rolling on truncated circular arcs with upper angles of 90 and 120 degrees, respectively. In both cases, the roulette consists of a circle involute and a circle arc whose radius is equal to the length of the line segment.

As an actual visual instrument, line segments are implemented by batons and the fixed truncated curves are implemented by cylindrical or conical surfaces. In its manipulation, a performer rolls several batons on the surfaces simultaneously. Furthermore, when two performers stand face to face at each side of the sequence of surfaces, rolling and passing of batons can be done between them.

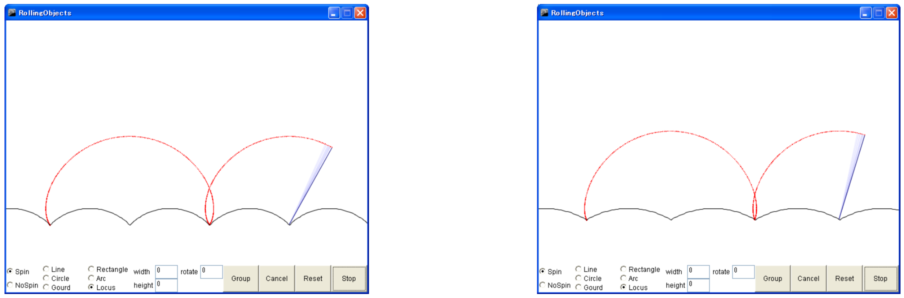
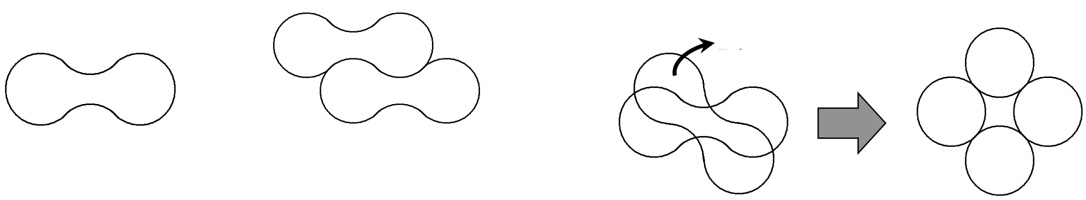


Figure 11: *Roulettes of a baton rolling on a sequence of circle arcs with angles 90 and 120 degrees.*

Gourd: Shape That Consists of Circular Arcs. Here we show one more shape we use as a rolling-based visual instrument. Figure 12(a) shows the basic shape. It consists of two circular arcs of 270 degrees in both sides and two circular arcs of 90 degrees in the center part, all of which have the same radius. Due to it shape, we call it the "gourd" (with two heads). It has an interesting geometric property that one gourd is piled tightly onto another as shown in Figure 12(b). The upper gourd can be also piled tightly onto the narrow part of the lower gourd. Furthermore, when one of two gourds is rotated as shown in Figure 12(c), then two gourds perpendicular to each other generate four illusional circles. We further investigate what kinds of roulettes are generated by the two gourds piled tightly. Figure 13 shows four snapshots of the piled upper gourd rolling around the lower fixed one. After the rotation, it is observed that the upper gourd is again attached tightly to the fixed one. Roulettes of two typical points on the upper gourd while it rolls are shown in Figure 14. Figure 15 shows roulettes of two typical points on the upper gourd that is first piled on the narrow part of the lower fixed gourd.



(a) (b) (c)

Figure 12: (a) *The gourd;* (b) *piled gourds;* (c) *four illusional circles after rotating a gourd.*

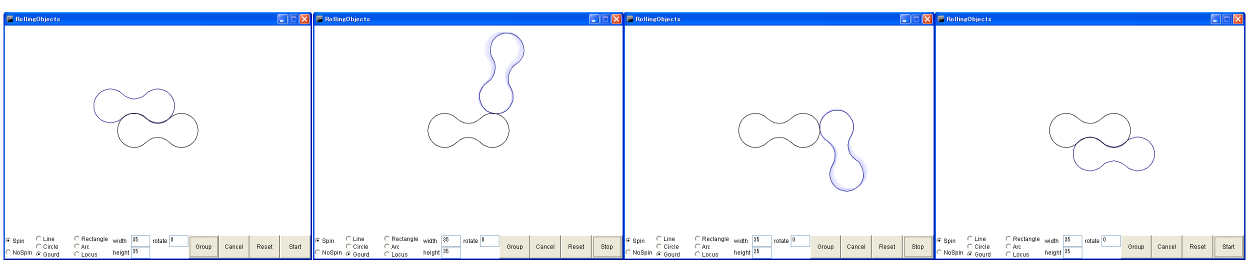


Figure 13: *Transition of the upper gourd rolling around the lower one.*

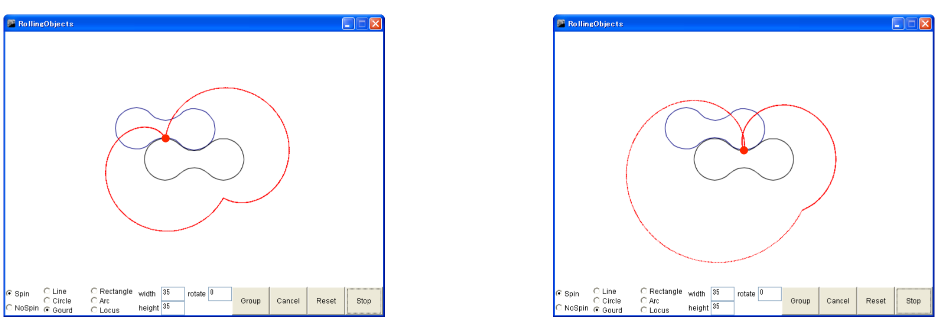


Figure 14: *Roulettes of points on the upper gourd (1).*

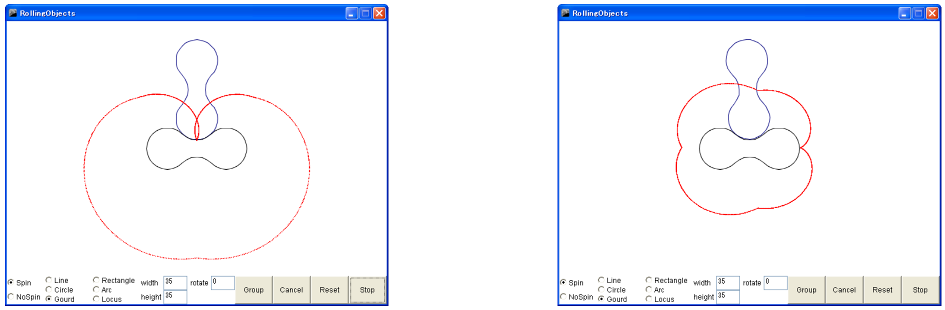


Figure 15: *Roulettes of points on the upper gourd (2).*

4 Construction of Visual Instruments and Their Manipulation

We constructed the visual instruments stated in Section 3 and investigated methods for the actual manipulation. We first consider a baton rolling on a cylinder. The length of the used baton is 30 cm and the radius is 1 cm. It is coated with rubber to avoid slipping on the cylinder. The diameter of the cylinder is about 20 cm. The set of the parameters is close to the case of $x = 1/2$ in Figure 10. Figure 16 shows the snapshots of the baton rolling around the cylinder in half revolution. It is further rolled to make one revolution. We note that even for this movement, some skill using fingertips is needed to manipulate the baton smoothly, especially in the transition from the second image to the fourth in Figure 16.

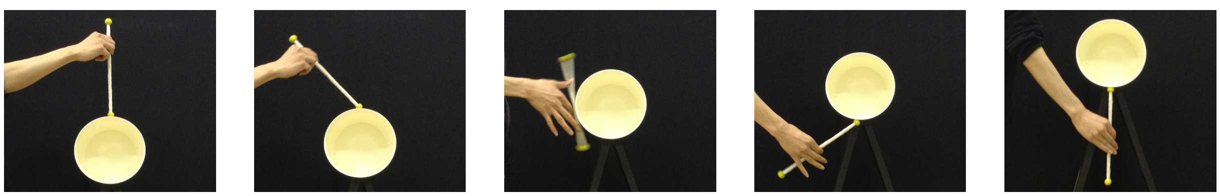
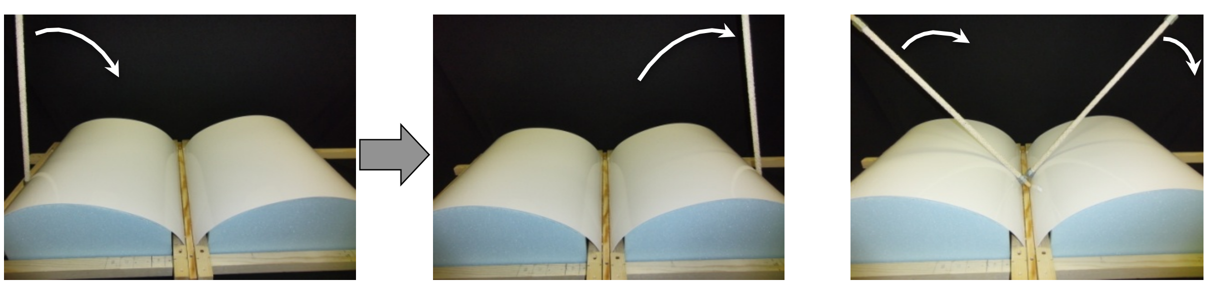


Figure 16: *Rolling of a baton around a cylinder.*

We next constructed visual instruments of two sequences of truncated cylindrical surfaces: one for a single player and the other for two players. Frames of the instruments are made of wood and the truncated surfaces are made of acrylics. The curves of the surfaces are naturally formed by the tension and gravity, so should be closed to catenaries. The lengths of the cylinders are 45 cm for a single player and 60 cm for two players. The lengths of the batons, which are the same with those of the curves, are 60 cm for a single player and 90 cm for two players. The batons are coated with rubber to avoid slipping on the surfaces.

Figure 17(a) shows rolling of a baton by a single player where the baton rolls twice on the surfaces. Figure 17(b) shows two batons rolled one by one to the right. Once we confirm such basic patterns, there are some systematic ways to create a variety of tempo-spatial patterns. For example, several parallel paths on the surfaces can be used to roll the batons. Furthermore, we can use mathematical notation called *siteswap* [9], which was originally developed for juggling to manipulate multiple objects at a time, to roll multiple batons on the surfaces without collisions.



(a) (b)

Figure 17: *Rolling of batons on the sequence of two cylindrical surfaces.*

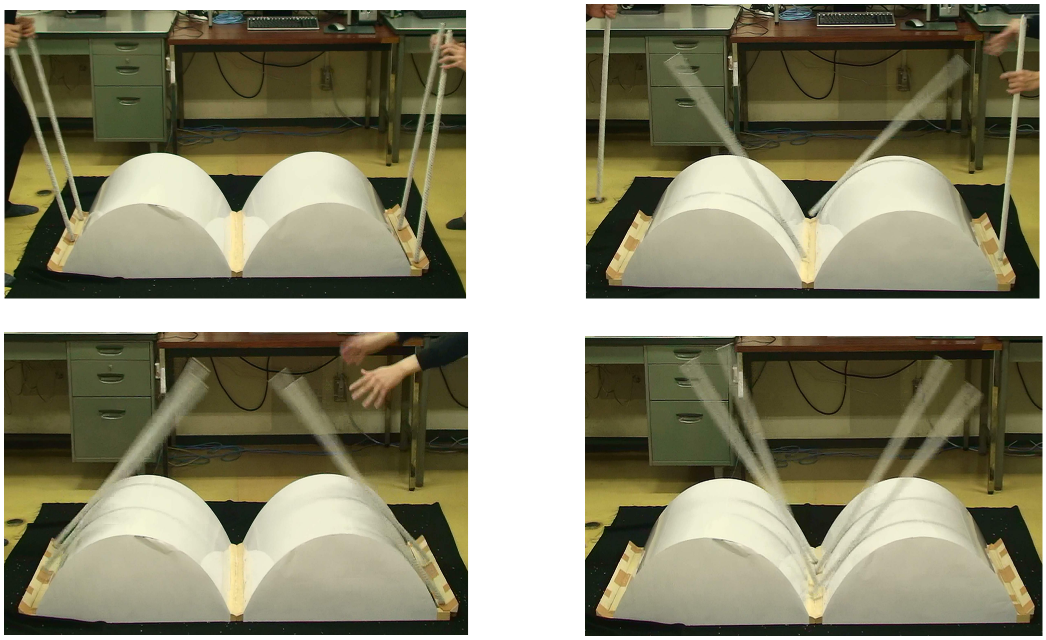
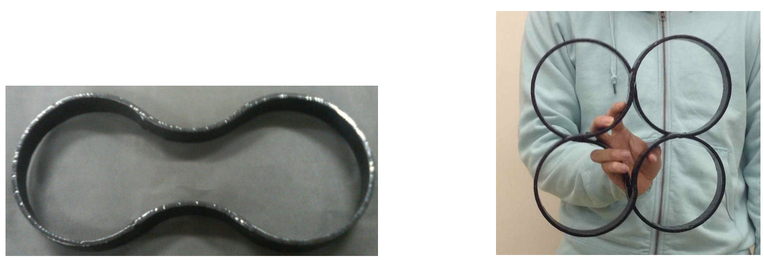


Figure 18: *Rolling and passing of batons between two players.*

Figure 18 shows the sequence of cylindrical surfaces for two players where they stand on the opposite sides of the sequence and manipulate up to four batons to create various tempo-spatial patterns.

Finally, we state on the construction of the gourds and their preliminary usage. They are made of hard paper with thickness of 5 mm and depth of 3 cm. They are covered by vinyl tapes to avoid slipping. The diameters of the circles of the gourds are 14 cm. Figure 19 (a) shows the constructed gourd and (b) shows the two gourds held crossed, resulting in the four illusional circles. Figure 20 shows the transition of rolling the upper gourd smoothly along the lower fixed gourd.



(a) (b)

Figure 19: (a) *The gourd;* (b) *two gourds held crossed.*

5 Concluding Remarks

In this paper, we proposed the concept of visual instruments and presented three types of visual instruments based on rolling geometric objects. Through construction and manipulation of the instruments, we have confirmed that the instruments generate a variety of aesthetically and geometrically attractive visual patterns; thus, they have a high potential for manipulative performance and play. In the conference, we will show some demonstration and performance using the instruments. Future work on the presented instruments includes refinement of their sizes, materials, and the methods of manipulation. Somewhat ultimate future work is to clarify the whole landscape of the world of visual instruments. Also for this purpose, we should further develop visual instruments having unique visual, geometric, and manipulative attractiveness.

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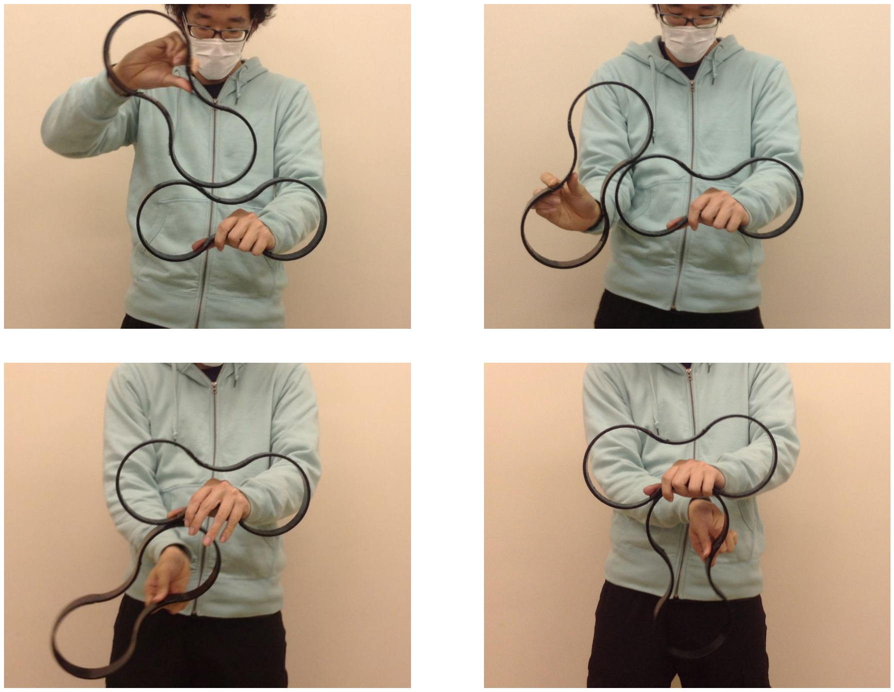

Figure 20: *Transition of rolling the upper gourd along the lower gourd.*

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