Mathematical Ideas in Ancient Indian Poetry

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Abstract

Modern mathematics owes a big debt to India's contributions to the subject. Of particular importance is the decimal, place value number system that appeared in India during the Vedic period or soon after, circa 1300 BC to 300 AD, and made its way to Europe during the Middle Ages. That period of time in India also produced a heady mixture of poetic works: poems, songs, grand epics, biographies and books of instruction in verse covering millions of pages. Mathematical ideas are interwoven into the metaphysical, religious and aesthetic fabric of many of these works. This article brings a selection of poems from that time period that provides a taste of ancient India's mathematical preoccupations in their cultural and esthetic context. They also highlight India's mathematical accomplishments of the period, and uncover instances where seeds of future mathematical concepts made their first appearance. The concluding remarks touch lightly on current Indian-inspired uses of mathematical poetry as a pedagogical tool.

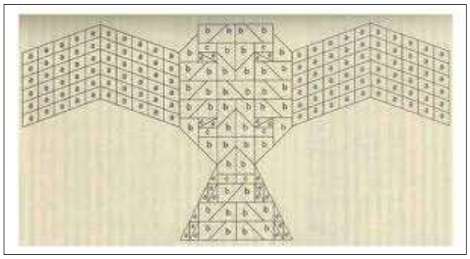
The Vedas and Supplementary Texts: Calendrical Calculations and Geometry

The most ancient Indian literary works are the four *Vedas*. Controversially dated around 1300 BC, the oldest of the four, the *Rig Veda*, is a collection of 1028 hymns written in classical Sanskrit. Mathematical ideas involving astronomy and time reckoning appear in a number of the hymns, particularly those describing the creation of the world. Contrary to other cultures, in Indian creation myths the world has no absolute beginning—a new world emerges from an already existing one. The creation is a sacrificial act in which the old gods sacrifice a primordial man, Purusha. The world, the seasons, the celestial bodies which start time reckoning of this universe's time cycle, and the new gods themselves, are made out of various parts of Purusha's body. Below is a fragment from a creation hymn from *Rig Veda* [21]. Note the precise mathematical measurements, including a very early mention of fractions:

From: **Rig Veda (Book 10, Hymn 90): Purusha**

A thousand heads hath Puruhsa, a thousand eyes, a thousand feet.
On every side pervading earth he fills a space ten fingers wide.
…………..
All creatures are one-fourth of him, three-fourth eternal life in heaven.
With three-fourths Purusha went up: one fourth of him again was here.
……………..
Forth from his navel came mid-air; the sky was fashioned from his head;
Earth from his feet, and from his ear the regions. Thus they formed the worlds.

An important task of ancient mathematics was to develop accurate ways of counting time. The ancient Indian calendar was a lunisolar calendar with intercalated months in five year cycles. In addition, each region implemented a number of changes based on local astronomical and religious customs making calendrical calculations very complicated. Reckoning of sacred time, the time that returns unchanged year after year at each religious holiday, was interwoven into the construction of sacrificial altars. In addition to the four *Vedas*, the Vedic literature includes a large number of supplementary texts. One of these texts, the *Satapatha Brahmana* (c. 600-500 BC) [22], contains verses of instruction for the construction of sacrificial altars to the fire god, Agni. Mathematically, two interesting features stand out: the interweaving of the notion of sacred time reckoning into altar construction and the precise and extensive plane geometry necessary for the constructions of the altars themselves.



A public fire altar was built in the shape of a falcon consisting of bricks of various geometric shapes. The fire altar was identified with a fixed unit of time—the year. Elements in the construction of the altar corresponded to calendrical elements of the year. Mircia Eliade explained the process as follows: "…with the building of each fire altar… the year is built too,…time is regenerated by being created anew" [4]. In the poem fragment below, the altar is the year with 360 enclosing bricks corresponding to nights, 360 Yagushmatî bricks identified with days, and 36 bricks that are "over" corresponding to the intercalary month. "Space-filler" refers to ruler of the world.

From: **Satapatha Brahmana (X Kanda, 5 Adhyaya, 4 Brahmana)**

But, indeed, that Fire-altar also is the Year,—
the nights are its enclosing-stones, and there are
three hundred and sixty of these, because there
are three hundred and sixty nights in the year; and
the days are its Yagushmatî bricks, for there are
three hundred and sixty of these, and three hundred
and sixty days in the year; and those thirty-six
bricks which are over are the thirteenth month,
the body of the year and the altar, the half-months
and months,— there being twenty-four half-months,
and twelve months. And what there is between
day and night that is the Sûdadohas; and what
food there is in the days and nights is the earth-
fillings, the oblations, and the fire-logs; and what
is called "days and nights" that constitutes the
space-filling brick:—thus this comes to make up
the whole Agni, and the whole Agni comes to be
the space-filler and, verily, whosoever knows this,
thus comes to be that whole Agni who is the
space-filler.

Satapatha Brahmana's instructions for the construction of fire altars referred not only to the number of bricks that went into each construction, but also to the proportions among various parts of the falcon shaped altar. To achieve specified area proportions, it was necessary to convert some geometric shapes into others without changing the areas involved. This was done by a cut-and-paste plane geometry technique that involved precise measurements and sophisticated geometric considerations. For example, particular cases of the result known to us as *Pythagoras Theorem* were employed. The second stanza of the fragment below explains how to crop two pieces from the top of both sides of a rectangle and "glue" them to the bottom of the sides in order to get a trapeze of the same area. This constructs the falcon's tail and the first step in the construction of each wing. In the next step, not reproduced here, each wing was given a "bent," that is, each wing changed shape from a trapeze into two "glued" parallelograms.

From: **Satapatha Brahmana (X Kanda, 2 Adhyaya, 1 Brahmana)**

Pragâpati was desirous of going up to the
world of heaven; but Pragâpati, indeed, is all the
sacrificial animals— man, horse, bull, ram, and
he-goat:—by means of these forms he could not
do so. He saw this bird-like body, the fire-altar,
and constructed it. He attempted to fly up, with-
out contracting and expanding the wings, but could
not do so. By contracting and expanding the
wings he did fly up: whence even to this day birds
can only fly up when they contract their wings and
spread their feathers.
………………………………..
He contracts the right wing inside on both
sides by just four finger-breadths, and expands it
outside on both sides by four finger-breadths; he
thus expands it by just as much as he contracts it;
and thus, indeed, he neither exceeds its proper
size nor does he make it too small. In the same
way in regard to the tail, and in the same way in
regard to the left wing.

The Grand Epics: Mathematical Games, Probability and Statistics

The 5th to 4th century BC saw the beginning of the writing of the two major Sanskrit epics of ancient India, the *Mahâbhârata* and the *Rámáyana.* Written in verse, both books include narratives of grand proportions along with philosophical and devotional material, and offer tantalizing glimpses of mathematical interests in lush and exotic settings. The *Mahâbhârata* in its longest version consists of 200,000 lines of verse. Among the principal narratives of the *Mahâbhârata* is the story of King Nala and his wife Damayanti.

The story begins when King Nala looses everything, including his kingdom, in a dice game. He abandons his faithful and loving wife, Damayanti, for her own good. After a series of misadventures, Damayanti is reunited with her parents and her two children. Meanwhile Nala, under the assumed name Vâhuka, obtains a position as cook and charioteer of the King of the Forest, Rituparna. Damayanti devises an ingenious plan for bringing Rituparna, and therefore his driver, Nala, to her father's castle. A "mathematical incident" occurs on the way, after which Nala arrives free of the passion for gambling and they live happily ever after. Below are three verse fragments, with commentary, describing the "mathematical incident" as rendered in English by Sir Edwin Arnold (1832 - 1904)[20]:

From: **Mahâbhârata: Nala and Damayanti**

A little onward Rituparna saw
Within the wood a tall Myrobolan
Heavy with fruit; hereat, eager he cried:—
"Now, Vâhuka, my skill thou mayest behold
In the Arithmetic. All arts no man knows;
Each hath his wisdom, but in one man's wit
Is perfect gift of one thing, and not more.
From yonder tree how many leaves and fruits,
Think'st thou, lie fall'n there upon the earth?
Just one above a thousand of the leaves,
And one above a hundred of the fruits;
And on those two limbs hang, of dancing leaves,
Five crores exact; and shouldst thou pluck yon boughs,
Together with their shoots, on those twain boughs
Swing twice a thousand nuts and ninety-five!"

A "crore" is equal to 10, 000,000. In this stanza Rituparna tells Nala the number of fruits (nuts) and leaves on the ground, the number of leaves on "two limbs," and the number of nuts on "twain boughs." All this, without apparently doing any counting. Statistics is the modern mathematical discipline that investigates procedures of inference of the sum total by examining small samples. This is the first known mention of statistical inference in literature. Intrigued, Nala stops the carriage to check the prediction by counting fruits and leaves. The numbers match and the following dialog occurs:

To Rituparna spake: "Lo, as thou saidst
So many fruits there be upon this bough!
Exceeding marvelous is this thy gift,
I burn to know such learning, how it comes."
Answered the Raja, for his journey fain:—
"My mind is quick with numbers, skilled to count;
I have the science."
"Give it me, dear Lord!"
Vâhuka cried: "teach me, I pray, this lore,
And take from me my skill in horse-taming."
Quoth Rituparna— impatient to proceed
Yet of such skill desirous: "Be it so!
As thou hast prayed, receive my secret art,
Exchanging with me here thy mastery
Of horses."

As soon as Nala learned the "secret art" he became cured of his addiction to gambling. Thus is the power of mathematics!

Thereupon did he impart
His rule of numbers, taking Nala's too.
But wonderful! So soon as Nala knew
………………
From the afflicted Prince
That bitter plague of Kali passed away.

In addition to statistics, the story of Nala and Damayanti anticipates the development of other modern branches of mathematics. In the first part of the story there is a description of the game of chance in which Nala lost his kingdom:

That hour there sat with Nala, Pushkara
His brother; and the evil spirit hissed
Into the ear of Pushkara: "Ehi!
Arise, and challenge Nala at the dice.
Throw with the Prince! it may be thou shalt win
(Luck helping thee, and I) Nishadha's throne,
Town, treasures, palace— thou mayst gain them all."

And Pushkara, hearing Kali's evil voice,
Made near to Nala, with the dice in hand
(A great piece for the "Bull," and little ones
For "Cows," and Kali hiding in the Bull).
So Pushkara came to Nala's side and said:—
"Play with me, brother, at the "Cows and Bull;"

It seems that the game was played with a number of dice or pieces, a large one "bull" and a few small ones "cows." The game is not identifiable, but dice games gave rise to the need of calculating odds of winning and thus set the stage for the development of the modern mathematical area of Probability. This game is of mathematical interest in another way as well, since it appears to be a board game played with a number of pieces, and as such has some similarity, and perhaps is a precursor, to chess. It is generally believed that the game of chess originated in India. The Indian Sanskrit name for chess was "shatranga," meaning four "anga" (detachments) following the arrangement of the troops in the Battle of Kurukshetra described in the *Mahâbhârata.* The game was originally played with four armies and a dice [6]. Chess is a particularly mathematical game, but all games involving strategy, contributed to the development of the modern branch of mathematics called Game Theory, which is concerned, among other things, with determining winning strategies. It is rare to find references to games in ancient manuscripts. Perhaps the earliest such mention is a reference to "draughts" (checkers) in *The Papyrus of Ani, The Egyptian Book of the Dead* (c. 1400 BC) [5], where the title of Plate VII reads: "…the forms of existence which pleased the deceased, of playing at draughts and sitting in the Seh Hall….."

Two men playing chess, 16th century Persian (see [13])

In another grand epic, the *Sháhnáma (Epic of the Kings),* the Persian poet, Abu'l Qasim Firdausí (932-1025 AD), tells the story of the introduction of chess into Persia around 550 AD, and the legend of the game's origin: Gav and Talhand, two sons of an Indian queen, quarreled about the succession to her throne. A battle ensued and Talhand perished. To clear his name and to console his mother, Gav asked the sages of the region to invent a game which will show every move of the decisive battle and in this way prove him innocent of his brother's death. A small fragment from *Sháhnáma* is reproduced below [7]. Firdausí claims that the original game of chess was played with two armies on a chess board of 100 squares, which decreased to 64 by the time chess reached Persia.

From: **Sháhnáma: The Story of Gav and Talhand and the Invention of Chess**
by Abu'l Qasim Firdausi

Those men of wisdom called for ebony,
And two of them—ingenious councilors—
Constructed of that wood a board foursquare
To represent the trench and battlefield,
And with both armies drawn up face to face.
A hundred squares were traced upon the board,
So that the kings and soldiers might manoeuver.
Two hosts were carved of teak and ivory,
And two proud kings with crowns and Grace divine.
Both horse and foot were represented there,
And drawn up in two ranks in war-array,

The steeds, the elephants, the ministers,
And warriors charging at the enemy,
All combating as is the use in war,
One in offence, another in defense.
…………
King Gav, the great and good, affected much
The game of chess suggested through Talhand;
His mother studied it. Her heart was filled
With anguish for that prince. Both night and day
She sat possessed by passion and by pain,
With both her eyes intent upon the game.

Buddhism and Jainism: The Number System and the Transfinite

Around 500 BC India went through a political and religious upheaval. Indian states were established, and two new religions, Buddhism and Jainism, rebelling against Vedic values and caste system, came into prominence. Buddhism was founded by Prince Siddhartha Gautama, called Buddha (620-543 BC). Jainism, believed to be of ancient origin, venerated the last enlightened teacher, Prince Vardhamana, called Mahavira (599-527 BC). The two religions have many aspects in common, among them the incorporation of scientific and mathematical ideas into their philosophical and metaphysical systems. Both leaders were believed to be scholars, well versed in sciences and the other learned subjects of the day. In *Kalpa Sutra* [19], the story of Mahavira's life, it is said that he was "… versed in the philosophy of the sixty categories, and well-grounded in arithmetic, in phonetics, ceremonial, grammar, metre, etymology, and astronomy." A charming story appearing in *Lalitavistara* [17], the story of Buddha's life written around 100 AD, shows Buddha's early prowess with mathematics. The verse fragment below, from Edwin Arnold's *Light of Asia* [1], is based on verses from *Lalitavistara*:

From: **The Light of Asia (Book the First): The Education of Buddha**
by Edwin Arnold

And Viswamitra said, "It is enough,
Let us to numbers.

After me repeat
Your numeration till we reach the Lakh,
One, two, three, four, to ten, and then by tens
To hundreds, thousands." After him the child
Named digits, decades, centuries; nor paused,
The round lakh reached, but softly murmured on
"Then comes the kôti, nahut, ninnahut,
Khamba, viskhamba, abab, attata,
To kumuds, gundhikas, and utpalas,
By pundarîkas unto padumas,
Which last is how you count the utmost grains
Of Hastagiri ground to finest dust;
But beyond that a numeration is,
The Kâtha, used to count the stars of night;
The Kôti-Kâtha, for the ocean drops;
Ingga, the calculus of circulars;
Sarvanikchepa, by the which you deal
With all the sands of Gunga, till we come
To Antah-Kalpas, where the unit is
The sands of ten crore Gungas. If one seeks
More comprehensive scale, th' arithmetic mounts
By the Asankya, which is the tale
Of all the drops that in ten thousand years
Would fall on all the worlds by daily rain;
Thence unto Maha Kalpas, by the which
The Gods compute their future and their past.
"'Tis good," the Sage rejoined, "Most noble Prince,
If these thou know'st, needs it that I should teach
The mensuration of the lineal?"
Humbly the boy replied, "Acharya!"
"Be pleased to hear me. Paramânus ten
A parasukshma make; ten of those build
The trasarene, and seven trasarenes
One mote's-length floating in the beam, seven motes
The whisker-point of mouse, and ten of these
One likhya; likhyas ten a yuka, ten
Yukas a heart of barley, which is held
Seven times a wasp-waist; so unto the grain
Of mung and mustard and the barley-corn,
Whereof ten give the finger-joint, twelve joints
The span, wherefrom we reach the cubit, staff,
Bow-length, lance-length; while twenty lengths of lance
Mete what is named a 'breath,' which is to say
Such space as man may stride with lungs once filled,
Whereof a gow is forty, four times that
A yôjana; and, Master! if it please,
I shall recite how many sun-motes lie
From end to end within a yôjana."
Thereat, with instant skill, the little Prince
Pronounced the total of the atoms true.

The above poem starts with young Buddha being asked to name all numbers up to "lakh," which means 100,000. But he continues beyond the lakh to "kôti," which equals 10^7, and through increasing powers of 10 to 10^{421}. Such large numbers had names, but the names were not standard, and it is difficult to say which is which in this poem. In the next stanza, young Buddha enumerates units of lengths up to a "yôjana," which is about 9 miles, and then apparently he also names the number of atoms in a yôjana. The number does not appear in Arnold's English translation/transformation of the poem, but *Lalitavistara* cites it as $384{,}000 \times 10^7$.

The cultural phenomena encountered here are deep respect for mathematical ability and a passion for large numbers. Large numbers came naturally to a culture where people were believed to live through multiple cycles of existence spanning seemingly never-ending periods of time. This may have been a motivating force behind the development of India's number system. India needed an adequate way to express and work with astronomically large numbers and therefore developed the decimal, place value numeral system and a notation for numbers that is conductive to arithmetic operations. In a place value system 5, for example, in the first place means 5, while 5 in 56 means 50, and 5 in 546 means 500. It was therefore inevitable that the development of such a numeral system introduced and used 0, the way we do today, as a number of 0 value. An earlier version of a place value system, but with base 60, appeared in Babylonian mathematics, and zero was used by the Babylonians to denote an empty space. The Chinese also claim priority for the invention of the decimal, place value system, using 0 as a number [16]. What is not controversial is that the Indian decimal, place value system (including zero) and notation for numbers, made its way to the Arab world, and through it, during the Middle Ages, traveled to Europe, where it interacted with the Greek method of mathematical deduction, to build the basis of present day mathematics [12, 14, 23]. Nobel Prize laureate Wislawa Szymborska's poem [24] reproduced below gives voice to the historical uncertainty of the origin of the number 0.

A Poem in Honor of
by Wislawa Szymborska

Once, upon a time, invented zero.
In an uncertain country. Under a star
which may be dark by now. Bounded by dates,
but no one would swear to them. Without a name,
not even a contentious one. Leaving behind
no golden words beneath his zero
about life being like. Nor any legends:
that one day he appended zero
to a picked rose and tied it up into a bouquet;
that when he was about to die, he rode off into the desert
on a hundred-humped camel; that he fell asleep
in the shadow of the palm of primacy; that he will awaken
when everything has been counted,
down to the last grain of sand. What a man.
Slipping into the fissure between fact and fiction,
he has escaped our notice. Resistant
to every fate. He sheds
every form I give him.
Silence has closed over him, his voice leaving no scar.
The absence has taken on the look of the horizon.
Zero writes itself.

Jainism's early texts exhibit the same passion for large numbers, but favor their expression as powers of 2 rather than powers of 10. Mathematically, an interesting addition is the development of an intuitive notion of numeral infinity, with several categories of infinity. We start with a fragment from the Jainist text *Tattvārtha Sutra*, *That Which Is* [25] describing what seems to be an infinite chain of concentric circles:

From: **Tattvārtha Sutra: The Lower and Middle Regions**
by Acharya Umāsvāti

There are islands and oceans that bear propitious names such as Jambu Island, Lavana Ocean and so on. The islands and oceans are concentric rings, the succeeding ring being double the preceding one in breadth. At the centre of these islands and oceans is the round island Jambu with a diameter of 100,000 yôjanas and Mount Meru at its navel.

There are seven continents on Jambu Island: Bharata, Haimavata, Hari, Videha, Ramyaka, Hairanyavata and Airavata. The six mountains that extend from east to west and divide the seven continents are Himavan, Mahahimavan, Nisadha, Nila, Rumkin and Sidharin. The mountains are, respectively, as golden as Chinese silk, as white as the Arjuna tree, as crimson as the rising sun, as blue as sapphire, as white as silver, as golden as Chinese silk.

Continuing the description of islands and oceans, *Tattvārtha Sutra* states that the number of islands and oceans is “innumerable.” Some historians argue that “innumerable” means “countable infinite.” But *Tattvārtha Sutra* also mentions that the concentric circles of islands and oceans stops at an ocean called Svayambhuramana, and thus the number of islands and oceans cannot be infinite. More likely, “innumerable” meant a very large number, perhaps large enough that it was not yet given a name. Of interest is also the following continuation of the treatment of infinity:

From: **Tattvārtha Sutra: Substances**
by Acharya Umāsvāti

There are innumerable soul units in a soul.
There are an infinite number of space units in space.
The number of units in clusters of matter may be numerable, innumerable or infinite.

The definitions of numerable, innumerable and infinite in *Tattvārtha Sutra* are obtained through a recursive process , via the distribution of mustard seeds among the concentric rings of islands and oceans described above. This is sophisticated mathematical thinking, but it is not mathematically precise in the mathematical sense of today. If we accept that “innumerable” is indeed countable infinity, then the recursive procedure leading to *Tattvārtha Sutra*’s “infinite” gives a cardinality of ${\aleph_0}^{\aleph_0}$, which is indeed infinite and uncountable. What we have here is an intuitive notion of two kinds of infinity. It took more than 1000 years longer for Cantor to create set theory and with it the precise mathematical tools required to describe various kinds of infinity.

Concluding Remarks

The Vedic period, the most ancient time considered in this article, was not the oldest civilization known to have inhabited the Indus Valley. Traces of the Harappan period, a five thousand years old civilization, were uncovered in several places in India. This civilization seems to have developed a certain amount of mathematics as seen in the mensuration and weighting devises found at excavation sites. They also developed a pictographic form of script, which unfortunately is still undeciphered [12, 14]. Perhaps poetic works older than *Rig Veda*, containing seeds of mathematical concepts, are still waiting to be decoded. Due to space restrictions, this article stops at the time the two great religions of India, Buddhism and Jainism, came into maturity—around 300 AD. But mathematics in poetic form has not ended its appearance in India at that time. In fact the golden age of Indian mathematics with its cultural tradition of recording mathematical results and problems in verse occurred during the Middle Ages. Some of the most charming mathematical poems come from this tradition. For example, Bhaskara (1114-1185 AD), the best known of medieval Indian mathematicians, wrote an algebra book intended for the education of his daughter, Lilavati. The book's title is also *Lilavati* (meaning "the beautiful"), and it was written entirely in verse [3, 12, 11].

Mathematics written in poetic form appears to this day, often in the service of mathematical pedagogy. Inspired by the mathematical poetry of medieval India, Barbara Jur [15] encouraged her algebra class to compose word-problems in poetry. The results have both mathematical and poetic merit. Jur's motivation was to enrich teaching by engagement, but in articles [2, 18] we find reports of such poetry writing experiments conducted in Pre-Calculus, Calculus, and Statistics classes that conclude that poetry writing in mathematics classes strengthens students' understanding and integration of the subject matter. The mathematical poetry of medieval India and the difficulties students have with word-problems in algebra feature in another article describing the use of poetry in a college algebra course [8]. Glaz and Liang [8] used poems from *Lilavati* and other historical sources to ease the difficulties students have with the transition between word-problems representing natural phenomena and the corresponding mathematical models—the equations representing the phenomena. The process yielded additional pedagogical benefits, such as the strengthening of students' number sense and mathematical intuition and the enhancement of retention and integration of the material [8]. A more extensive survey of the uses of poetry in mathematical pedagogy, as well as the poem *The Enigmatic Number e*, may be found in [9], while [10, 11] are sources for additional mathematical poetry. The connections between history, poetry, mathematics, and pedagogy unfold like the Indian myth of creation, with no absolute beginning and elements of surprise in the future.

Acknowledgements

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