

The Immersive Bridge Between Math and Art

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Abstract

A metaphoric bridge between math and art suggests that math is on one side, and art is on the other. This doesn't mean that mathematicians and artists are on different sides, but that there are separate mindsets or modalities. The introduction of the multi-touch user interface offers a new connection between math and art. To illustrate, we show how a new application helped discover new geometric forms and has posed interesting questions.

From Paper to Silicon

Mathematicians have constructed geometric patterns on paper for centuries. These constructs have been the basis for many works of art in the east and west, with the artist using fields of color and varying line widths to evolve their patterns.

Over the last two centuries, various mechanical amusements such as the harmonograph[1] and the Spirograph[1] have produced pleasing patterns proportional to the operator's skill and esthetic aptitude. The optical kaleidoscope[1] required only simple rotation to perturb its colorful and symmetric pattern. These devices were simple bridges between math and art, and have modern digital counterparts.

Over the last few decades, computer-generated fractals and tessellations have inspired artists. The patterns produced reflect the taste of the artist and/or the creator of the program. The artist and programmer might be the same person, but the work typically proceeds iteratively with the program taking input values from the artist who views the result. The artist then modifies the parameters or the programmer modifies the program, bridging math and art in their collaboration.

In the last half decade, multi-touch technology has allowed a more direct and intimate interaction between the “canvas” and mathematical models. Older user interfaces were “interactive” because the user interacted with a program on a step-by-step transactional basis. Multi-touch gestures replaced pointing devices and reduced the need for on-screen controls, fostering a feeling of real-time “immersion” with the subject matter. The user-artist sees the model change immediately in response to gestures made directly on the display. Such “Apps” can be highly visual, kinetic, and aural in their interpretation of a mathematical, physical, or musical theory.

An Immersive Application for Exploring Geometric Trees

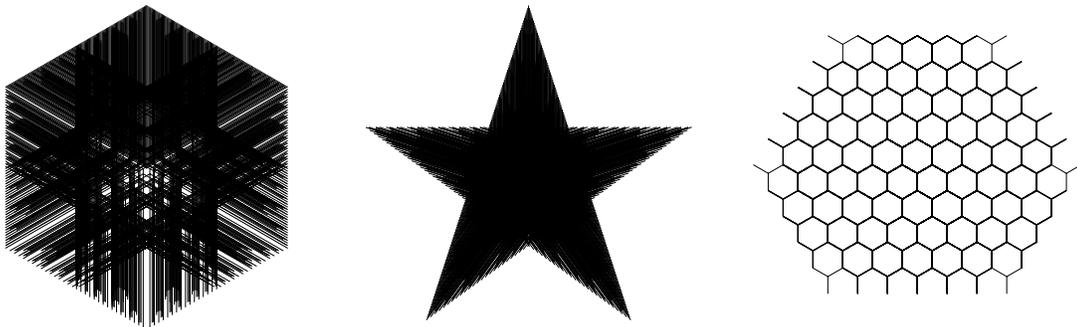
In 1980, I wrote a program to draw geometric trees. Trees were perfect for teaching recursion, and the output could be displayed on the Tekscopes in the lab. My computer science students explored combinations of branching angles and the common ratio between one level and the next in the tree. We discovered that for a given angle, there was a “perfect” common ratio that allowed the tree's branches to

be as long as possible without overlapping. Using that observation, I made an animated sequence of a “self-contacting” tree opening up from 0° to 180° – but that was the end of my exploration. I thought I’d seen everything that was interesting (and so did others exploring the tree form at the time, evidently).

When the iPad came out, I resolved to couple its multi-touch interface with my tree drawing algorithm. I wanted people to be able to easily vary a tree’s parameters and see the results in real time, so I decided on an initial set of gestures and refined them as I saw how the program behaved.

Changing the angle or common ratio changes the way the tree fills space, so before drawing anything in response to a gesture, I generated all the nodes of the tree and then scaled them to fit the display. This allowed the parameters to be freely varied without the tree growing “wild” (going off the display). Once I got the app running, I found the self-contacting trees very familiar because of my 1980 animation. It was none the less a joy to see the tree change in response to my finger moving on the display.

Changing the common ratio was less familiar to me than changing the angle. Stunted trees were familiar, but uninteresting. With the scaling taken care of, I could test common ratios that I had always assumed would lead to a scribbled mess. As the developer, scientist, and artist, I went exploring and soon discovered previously unseen polygons, stars, grids, pinwheels, and many other beautiful patterns!



As one might expect, tree branches form geometric patterns that come into focus at special angles. One might think that these crystalline grids are the “Art”, but more interesting complex meta-patterns appear in the transition between the special angles. Changing the common ratio ever so slightly can also yield an artistic result. The immersive nature of the interface facilitates this exploration. The mathematics of these meta-patterns need not be understood to appreciate them, but their existence begs explanation. The above trees have parameters $(n=3, \text{angle}=60^\circ, \text{ratio}=1/2)$, $(3, 144^\circ, 1/2)$, $(2, 120^\circ, 1/1)$. [2]

Conclusion

The kaleidoscope, Spirograph, and harmonograph demonstrate that art can be generated by physical laws. Likewise, geometric trees can be drawn by algorithms wired to multi-touch gestures. The resulting immersive application can provide an esthetic or visceral experience in itself, exposing new territory on *both* sides of the bridge between math and art.

References

- [1] See Harmonograph, Spirograph, and Kaleidoscope on Wikipedia
- [2] <http://www.geom-e-tree.com/explore.html>