

The Evolution of An Idea Inspired by 70

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Abstract

The paper presents an artist's recounting of the design steps behind a series of artworks based on combinatorial mathematics. Several design and media contexts are explored, including color print on paper, hand-colored origami units assembled into a quilt-like model, and a tessellation made with the non-folded origami quilt units.

Introduction. In this paper I describe a tour through some familiar, but nonetheless inspiring, mathematics, which I explore as an inspiration for visually rendered art. Along the way, I describe a dialogue between the mathematics and the art. My thinking is inspired by the conceptual art tradition, especially by Sol LeWitt, in which mathematics was sometimes used as an idea and way of setting conditions within which to work. In our 2011 Bridges paper [2] we started with a piece by LeWitt (Morrow & Morrow, 2011) and deconstructed it to discover a possible mathematical basis for its creation, then used that discovery as the basis for a related piece of origami art. In this exploration I start with a mathematical idea, use it to create two works of art in different media (a print and folded paper), and then use the unfolded pieces of paper that I designed for the origami piece to create a tiling.

Social Inspiration. I often create origami birthday gifts that have the number of pieces of paper matching the age of the birthday celebrant. Thus I am always looking for interesting ways to configure the paper, which involves not only finding appropriate origami units that can be interconnected, but also choices for the layout of interconnected units. Mathematics is an excellent source for inspiration, and I almost always turn there for ideas. Factors are an obvious aspect of integers, and sometimes the factors of a number can inspire a pleasing visual display. For instance I find that factors are a rich source of ideas for a number like 60 or 80 — birthdays that I have encountered frequently among my friends and mentors lately. Here is an origami “quilt” made with 60 squares of paper and arranged into a pleasing array of 15 squares, each made with 4 pieces of paper (Figure 1).



Figure 1: 60 Unit Origami Quilt

Now that some of us are progressing toward 70, I have been challenged to think about ages with not-so-obviously pleasing factors. The factors of 70 (which are 1, 2, 5, 7, 10, 14, 35, 70) or the prime factorization of 70 (which is $2 \times 5 \times 7$) did not inspire any obvious designs adaptable to an origami model. While a 7×10 layout is a nice shape, you cannot subdivide it further in a nice way. One could make 14 pentagons out of 5 pieces of paper each and then display them in 2 rows with 7 in each row, but it is difficult to get a nice visual array in just 2 rows. One could array the pentagons in 3 or 4 rows with an unequal number across rows. But the visual appeal of these ideas was not extremely high, and models for pentagons made with 5 pieces of paper are not very appealing either. Thus I went on a quest to find a different aesthetic and mathematical visual link to 70. Finding an idea that would render well with origami would be an added bonus.

Mathematical Inspiration. I turned to the idea of combinations, and, luckily for me, 70 is a binomial coefficient, specifically $70 = \binom{8}{4}$. Note that this notation means the number of unique combinations (disregarding order) of 8 things taken 4 at a time is 70. It is also true that $\binom{7}{3} = 35$ and also $\binom{7}{4} = 35$, thus $\binom{7}{3} + \binom{7}{4} = 70$. So now I had two different combinatorial ideas that could give me an array of 70 objects.

I began making squares divided into fourths. This allowed me to place each of the 70 combinations of 8 colors taken 4 at a time into one square and end up with 70 uniquely colored squares (Figure 2). Here I am considering only combinations of 4 colors, not permutations, which are discussed below. (Note that the colors appear as shades of gray or fill patterns, hereafter referred to as “colors,” in the print version of this article so as to make the figures understandable. In the CD version, the figures are in color. The mathematical ideas are identical in both versions.)

I was struck particularly by the enormous number of different arrays that could be made from this very limited set of elements. Of course the elements could be varied endlessly by choosing different shapes in which to display the required subset of colors, but even just keeping to a square, the number of options was huge. Artists are very familiar with the idea of producing stunningly interesting pieces by placing strict limit on building blocks. Here I are just a few ways to think about variations:

- Given the 70 4-colored squares, any one of them can be placed in the first spot in the layout, any one of the remaining 69 squares can be placed next, etc, giving 70! choices for the final layout.
- Given the 4 colors assigned to a particular square, there are 4! (or 24) ways to place the colors on each of the squares, giving another layer of choices with a huge number of possibilities.
- The color assigned to each area within the square can be varied almost endlessly.
- The areas can be equal or not.

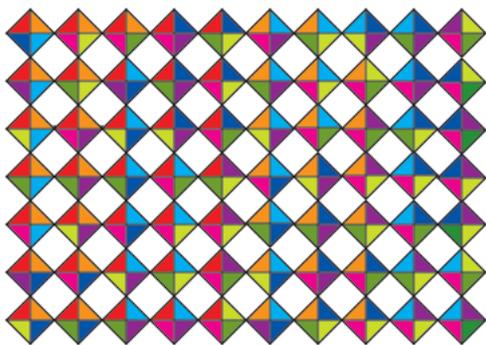


Figure 2: Systematic $70 = \binom{8}{4}$ Coloring

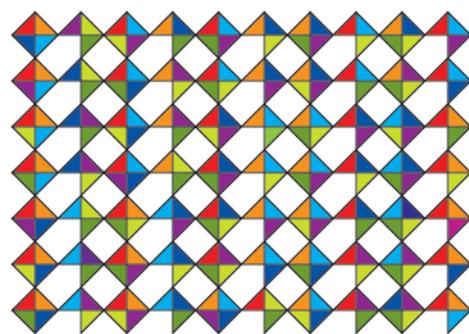


Figure 3: “Paired Coloring for $\binom{7}{3} + \binom{7}{4} = 70$ ”

I then thought about the $\binom{7}{3} + \binom{7}{4}$ option. I puzzled over how to divide a square into thirds to 3-color it as well as fourths to 4-color it. I didn’t come up with an idea that I really liked until I thought of simply leaving one of the fourths blank (Figure 3). I next thought it would be interesting to embed a pattern of each unique combination of 4 colors paired with its non-overlapping, unique set of 3 colors, thus coming up with 35 paired squares. Each pair encompasses all 7 colors, and each of the 2 components of the pair are unique. At this point I invented a new design within the square consisting of 3 L-shaped elements plus a small square (Figure 4). In this design, the colors will not have equal areas within a square, but this idea seemed more visually appealing and suggests more variability.

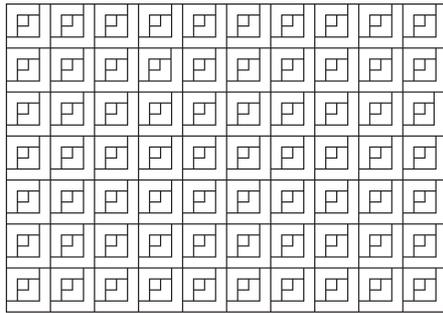


Figure 4: "Layers" Design Template

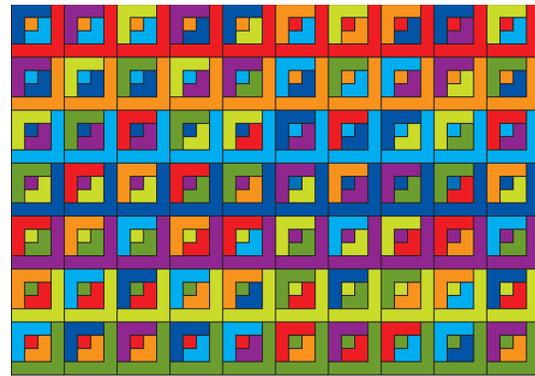


Figure 5: "Layers" Coloring for $70 = \binom{8}{4}$

While I had a very methodical way of coloring the squares so as to keep track of the combinations --- like the tree diagram method for listing combinations and permutations, but using a visual rather than numeric vocabulary --- displaying the squares in the order created in this way produced large blocks of the beginning colors and very small amounts of the last colors in the list (see Figure 2 or Figure 5, for example), which was as aesthetically appealing. At this point it is good to remember, as mentioned above, that there are a huge number of choices for making a layout. One could certainly mix up the squares randomly, or seemingly randomly, but I found it more satisfying to find a subtle pattern for the layout.

I present here three different pieces that can be made using the above ideas with quite different-looking results. Figure 5 and Figure 6 (which will appear, in color, at the Bridges Art Exhibit) are based on a $\binom{7}{3} + \binom{7}{4} = 70$ coloring. Figure 5 has the $\binom{7}{4} = 35$ in the left half and the $\binom{7}{3} = 35$ in the right half. Figure 6 encodes into the design black dots to show the related pairs of a 4-colored square and a 3 colored square with non-overlapping colors, thus together representing all 7 colors. Non-related pairs have squares between them across the rows and slanted lines between down the columns. There are many other choices for deploying other media from fabric to buttons to found objects.

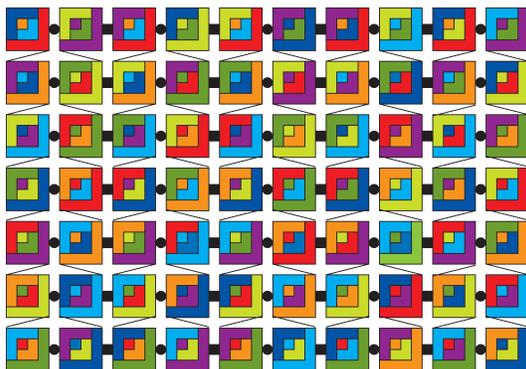


Figure 6: Coloring Using $\binom{7}{3} + \binom{7}{4} = 70$

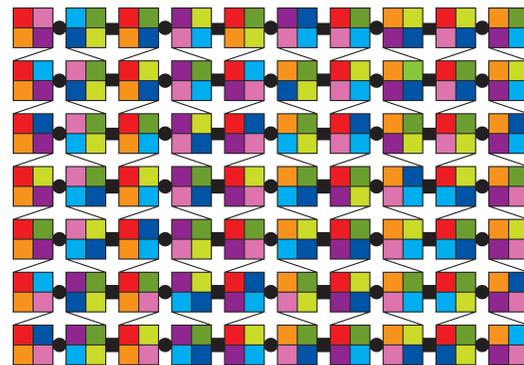


Figure 7: Coloring Using $70 = \binom{8}{4}$

Returning to Origami. My final challenge was to see whether I could translate a combinatorial design to an origami context. I would need pieces that interlock and that would look pleasing in both 3 and 4 colors. I could not use my usual square made from 4 units because 70 is not divisible by 4 and I did not want

partial squares. Because each piece would need to be a unique set of colors, I decided that each piece should be hand colored because this approach would make it possible to use origami paper. I chose triangular Fuse origami quilt units [1] and a $\binom{7}{3} + \binom{7}{4}$ pattern. Triangles can be nicely divided into either 3 or 4 equal sub-triangles, as can be seen in Figures 8 and 9. As a result of both the coloring pattern and folding process each piece is either 3-colored or 4-colored on both sides, but the coloring pattern is different on each side, as can be seen in Figures 8 and 9.

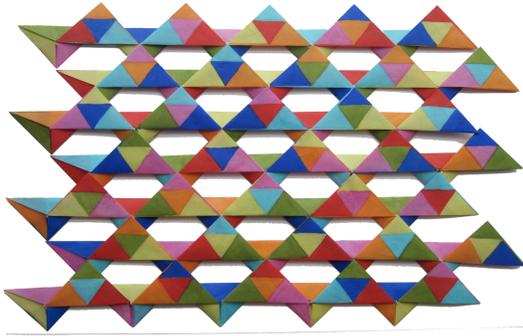


Figure 8: *Origami Quilt, Side 1*

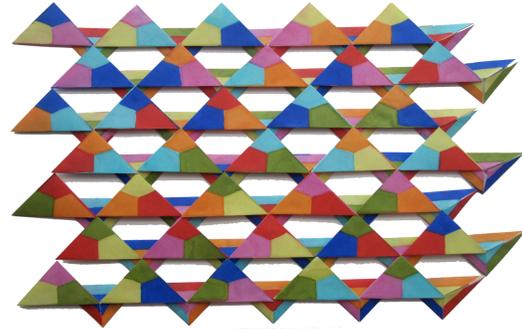


Figure 9: *Origami Quilt, Side 2*

Math/Art/Play. As a final exploration, I made a tessellation using the units that were 4-colored for Side 1 of the origami quilt. These pieces are non-regular pentagons (see Figure 10) and, surprisingly, tessellate as shown in Figure 11.



Figure 10: *Pentagonal Tessellation Unit*

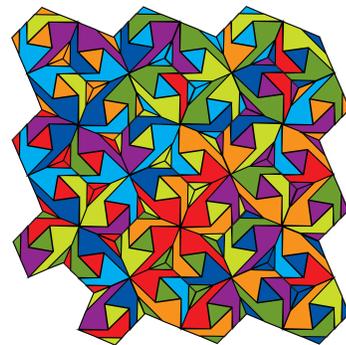


Figure 11: *Pentagonal Unit Tessellation*

Conclusion. This investigation was specific to $\binom{n}{k} = 70$ combinations. However, in this process I was led back to Pascal's Triangle to see the rich possibilities for other combinatorial problems that will provide many artistic inspirations for birthdays other than 70.

References

- [1] Fuse, Tomoko. *Unit Origami Quilts*. Tokyo: Japan Publications, 1990.
- [2] Morrow, Charlene and Morrow, James. A Group Theory Approach to (re)Constructing Sol LeWitt's Drawing Series IV, #413. *Proceedings of Bridges: Mathematical Connections in Art, Music, and Science*. University of Coimbra: Coimbra, Portugal, 2011.