Rotate, Reflect, Recycle

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Abstract

I describe the details of a sort of algorithmic art where the input is a sketch with some radial symmetry. The algorithm arose during my process of creating a recent work (Three Strange Dreams, 2010).

Step One: The Input

My project started with a doodle I sketched during a mathematics department meeting in 2010. It is pictured in Figure 1. The fact that the center of the sketch is empty turns out to be necessary in the final stage of the algorithm.

For the purposes of my description, I will render the input generically, as in Figure 2 below. The letters ABCD are near the center of the figure to indicate that the doodle is at the center of the input and surrounded by blank space.

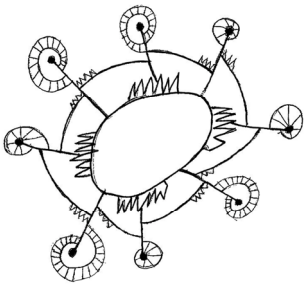

Figure 1: *The Doodle*

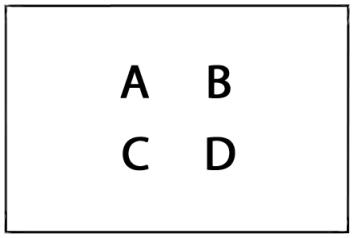

Figure 2: *The Input*

Step Two: Rings and Monoids

With the input settled, we proceed to the second step, which can be described in more than one way. I prefer the following one, which evokes the concept of topological covering: *Take four copies of the input and rotate each of them 180 degrees. Then, after translating them all so that each one has a quadrant covering the original image, crop the result at the same dimensions as the original*. Using our input from Figure 2, we can illustrate the result as follows:

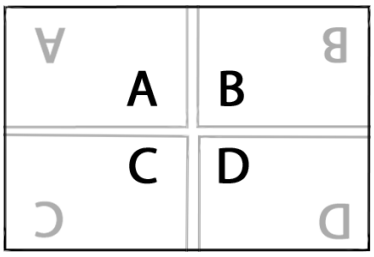

Figure 3: *Step Two*

I implemented this algorithm manually (using Adobe Photoshop Elements, once the input sketch was scanned), and I always allowed myself artistic license. In Step Two, for example, it made sense to adjust how I positioned the covering to achieve a visually pleasing result. I also took some liberties with colors, layers, and opacity. My digital print "Rings and Monoids" (2010) arose at this stage. I displayed it at the La Crosse Area exhibit of the Wisconsin Regional Artists Program (WRAP, 2010).

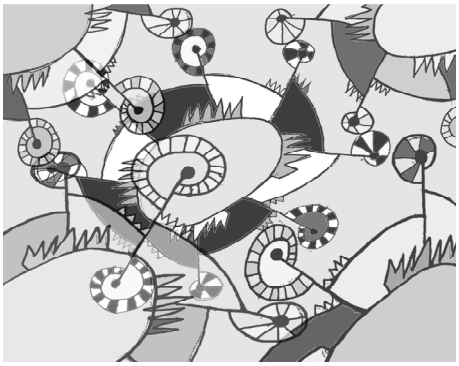

Figure 4: *Rings and Monoids (2010)*

Step Three: Tiling

Not feeling that the project was done, I implemented the next step, which is a tiling: *Reflect the image successively across its right edge, then across the bottom edge of the result, then the left edge, then the top edge*. This result is partially rendered in Figure 5 below.

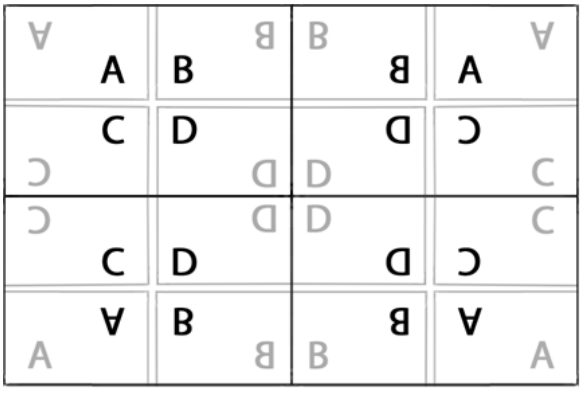

Figure 5: *Step Three*

At the center of Figure 5, we have a convergence of four copies of one of the quadrants (D) from the input sketch (Figure 1). Moreover, since the input sketch had an empty region at its center, so does the center of Figure 5. Obviously, there are three other locations in the tiling where this sort of empty region arises, corresponding to the A, B, and C quadrants of the original. All four are visible in Figure 6.

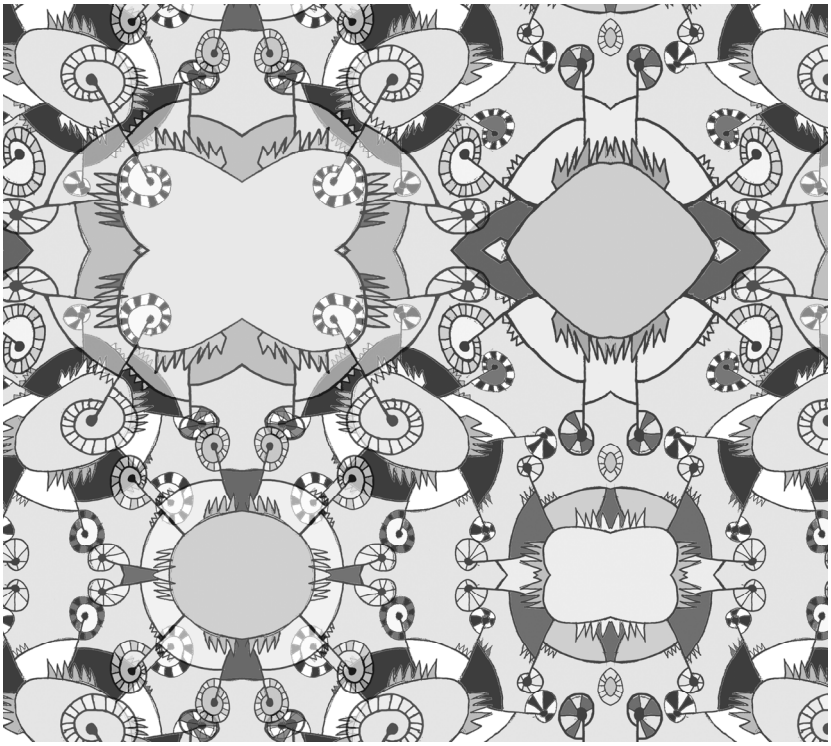

Figure 6: *The Tiling*

Step Four and Output: The Strange Loop

After I applied the tiling process (Step 3) to "Rings and Monoids," I obtained Figure 6, with its four empty regions. Recalling the notion of a "Strange Loop," which Hofstadter [1] defines as occurring "whenever, by moving…through the levels of some hierarchical system, we unexpectedly find ourselves right back where we started," I imagined passing through a gateway, ending up on the other side with another gateway before me, passing through it, and always finding another gateway on the other side. I imagined eventually ending up in front of the first gateway again. To render this Strange Loop visually, I used the empty regions of Figure 6 as the gateways. I chose three of them, reserved a separate image for each, and made the next gateway in the sequence visible in each image. In terms or our algorithm, we have this final step: *Form three images by suitably cropping the gateways corresponding to the* D*,* A*, and* C *quadrants. Then, embed a second gateway into each image,* A *into* D*,* C *into* A*, and* D *into* C*.*

My digital print "Three Strange Dreams" (2010) was the output from Step 4 and appeared in the Exhibit of Mathematical Art at the Joint Meetings in New Orleans (2011) and in the WRAP State Exhibit in Madison (2011).

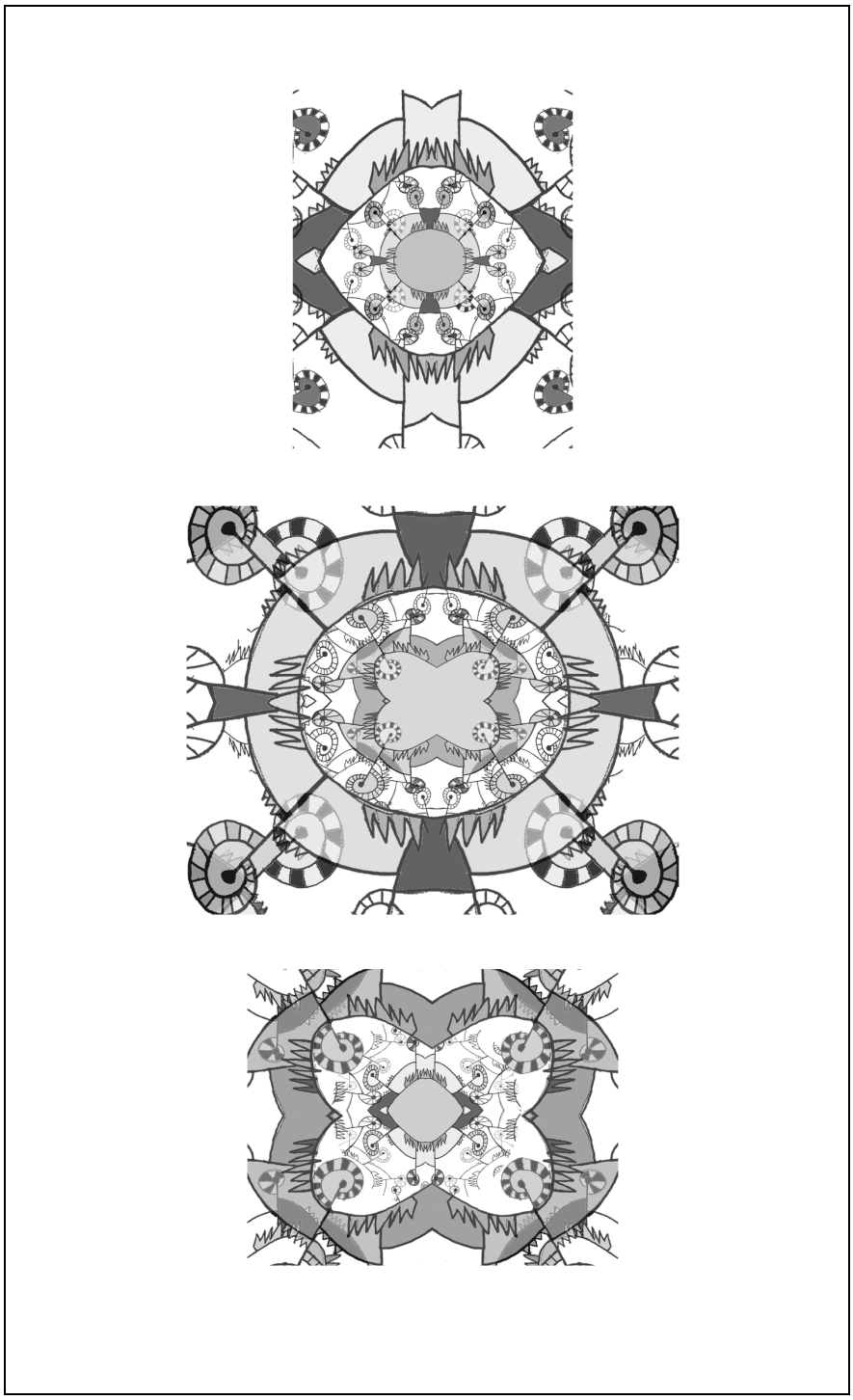

Figure 7: *Three Strange Dreams (2010)*

References

[1] Hofstadter, Douglas R. *Gödel, Escher, Bach,* New York: Basic Books, 1979.