Intersecting Helices

Paul Gailiunas
25 Hedley Terrace, Gosforth
Newcastle, NE3 1DP, England
email: paulgailiunas@yahoo.co.uk

Abstract

In two dimensions a sequence of equally spaced parallel lines will intersect with another such sequence set at some angle to form a regular tiling of rhombi. If the lines are replaced by sine curves there are many more possibilities, depending on the relative phases of the curves. In three dimensions sets of parallel lines will intersect only in particular cases, since usually lines from different sets will be skew. A helix is a natural three-dimensional analogue of a sine curve, and again arrays of helices will intersect only in particular cases. Such configurations are so intricate visually that even small pieces of the infinite structure provide interesting sculptural forms.

Rows of Helices

The projection of a helix perpendicular to its axis onto a plane is a sine curve (of course the projection along its axis is a circle). Rotating the helix about its axis changes the phase of the sine curve. A row of equally spaced parallel sine curves will look quite different depending on their relative phases, and overlaying two such arrangements can produce a wide range of visual effects. A few of the examples in *Tilings and Patterns* [2], for example figures (*a*) and (*c*) on p.34, show some simpler examples, and Koert Feenstra [1] has explored the moiré effects that can be produced.

Comparable arrangements of helices (figure 1) are not so common but Merklen Brothers used interlaced helices in their furniture, where it was known as "Moorish Fretwork" [3]. In this case adjacent are 180° apart to avoid intersections, with the relative phases of the overlay in the most symmetrical arrangement.

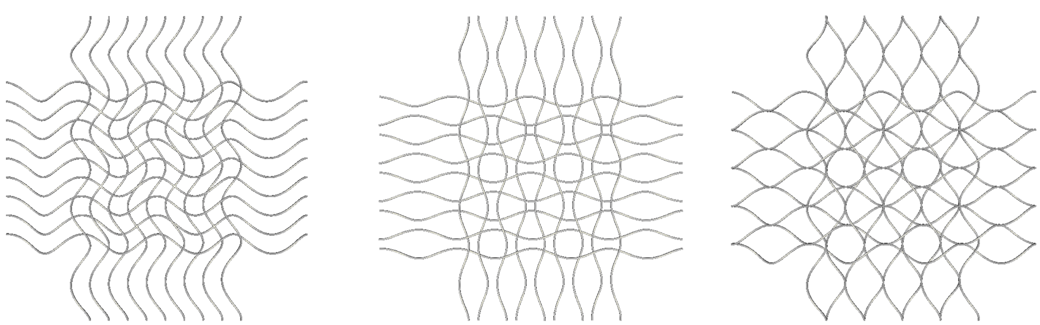

Figure 1: *Helices project as sine curves, and overlaying can produce many different patterns.*

Space Frames

A fully three-dimensional structure is produced when such two-dimensional arrays are stacked in parallel planes, and one (or more) further set(s) of parallel helices added at some angle(s) to the planes. Only the simplest case of three mutually perpendicular sets of helices will be considered here. Avoiding intersections is not difficult in three dimensions, and the most regular structures are those where the helices do intersect. An interlace version can usually be constructed by changing the radius of the helices slightly, but in general non-intersecting arrangements tend to be visually very confusing.

Adjacent helices in the "Moorish Fretwork" arrangement extended to three dimensions are still separate, and only intersect when their radii are increased to match their separation. The radius must be increased even further before non-parallel helices intersect, and the resulting configuration is probably too confusing to form the basis of a sculpture.

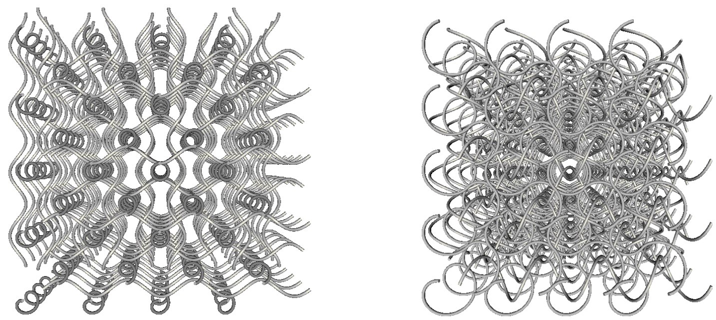

Figure 2: *In the "Moorish Fretwork" arrangement helices are well separated, and their radius must be increased to at least the repeat distance before any intersection is possible.*

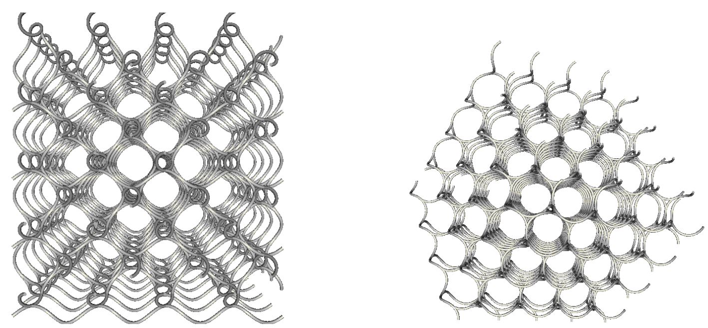

Figure 3: *Two views of the (10,3)-a configuration (down the axes of symmetry).*

(10,3)-a

Changing the relative positions of the three sets of parallel helices allows more promising possibilities. The most natural leads to a configuration corresponding to (10,3)-a, a very regular trivalent network first described in 1956 [4] (figure 3). The symbol "(10,3)" indicates that the shortest circuit in the network consists of 10 edges/vertices, and there are 3 edges at each vertex. The "a" seems to originate from the label of the diagram in Wells's plate [4]. There has been considerable interest in this structure, and George Hart has produced some very nice models of it [5]. As his images demonstrate it is actually possible to produce this structure by intersecting helices in four different ways: two using three mutually perpendicular sets (figures 3 and 4) and two using four sets at tetrahedral angles (not considered here).

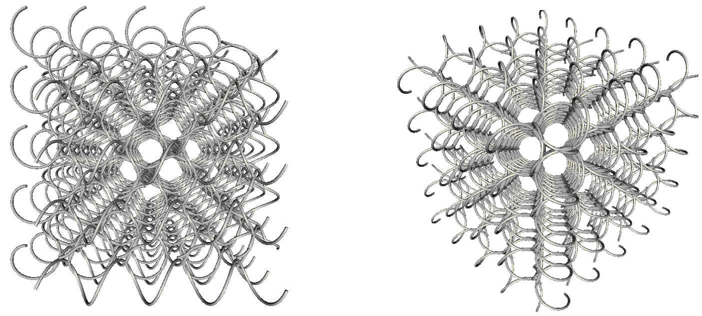

Figure 4: *A larger radius provides a different realisation of the (10,3)-a structure.*

A pleasing variant of this structure is produced if the radius of the helices is increased so that intersections occur at the vertices rather than helices osculating at the mid-point of the edges.

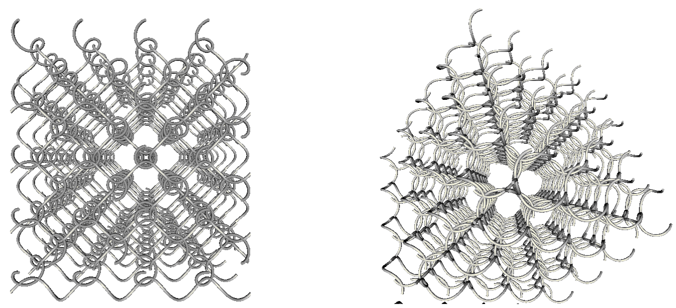

Figure 5: *Another form of (10,3)-a with helices intersecting at the vertices.*

A large section of the (10,3)-a network can be rather overwhelming even with a model and possibly smaller fragments would be more successful as sculpture. Towers are quite effective (figure 6), but even an apparently arbitrary piece seems to work (figure 7) aesthetically.

Figure 6: *Towers extracted from the configurations in figures 3, 4, and 5.*

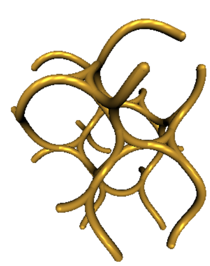

Figure 7: *A fragment of the configuration in figure 3.*

Different Configurations

Paradoxically, if they are all of the same phase, structures with intersecting helices are even more difficult to understand visually than those based on (10,3)-a, but again a small fragment makes an interesting sculptural form (figure 8).

Being constructed from helices these structures are all chiral. The introduction of mirror images provides several further possibilities. The classic (10,3)-a structure leaves a lot of space into which another framework can fit. Most famously it can accommodate its mirror image, which is also its dual (considering it to be a packing of trihedral with skew decagonal faces, so the dual has vertices at the centre of the trihedron and edges that correspond with the faces).

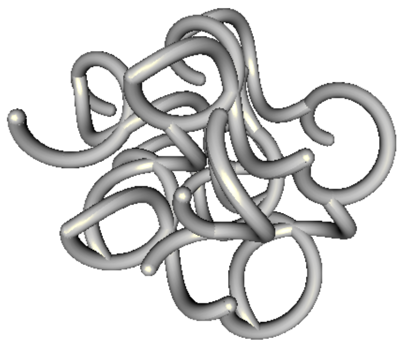

Figure 8: *A more complicated structure is produced if all helices have the same phase.*

Alternatively mirror images can be incorporated into a single structure, so that, for example, instead of adjacent helices having a phase shift of 180° they can be of the opposite chirality. The easiest way to understand these structures is to consider the two sets of helices separately. Each is half of one of the structures already considered, so that figure 9 can be seen as half of (10,3)-a (figure 3) merged with half of figure 2. Clearly there are many more possibilities.

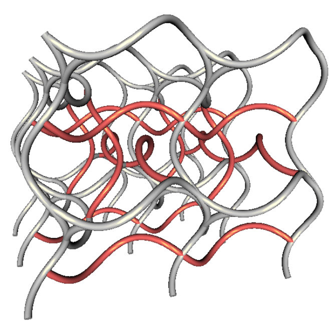

Figure 9: *A structure with helices of both senses.*

[1] Feenstra, K., Moiré, *Bridges Leeuwarden Proceedings 2008*, p.422.

[2] Grünbaum, B. and Shephard, G. C., *Tilings and Patterns*, Freeman & Co., New York, 1987.

[3] Tucker, P., "Moorish Fretwork", *Bridges Proceedings2004*, pp.181-188.

[4] Wells, A.F., *The Third Dimension in Chemistry*, Oxford, 1956.

[5] www.**georgehart**.com/rp/**10-3**.html (accessed Jan 2012)