Tiles and Patterns of a Field: From Byzantine Churches to User Interface Design

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Abstract

Analysis of geometrical tile patterns from the 5th century (Petra, Jordan) and 14th century (Natanz, Iran) shows how coherent perceptual units are "picked up" by a viewer. For every pattern we identify the "basic" or smallest polygon that can generate the pattern, a polygon grid that can "float" over the pattern, as well as the smallest rectangular tile ("practical tile,") that can be used as a template to physically construct patterns. We then make the link between patterns and interface design, showing how patterns can be used to "house" data and information. A proposed helicopter engine display, inspired by a 15th century tile pattern (from Gazarghah, Afghanistan), is used to illustrate the approach.

Introduction

The city of Petra (Greek for "rock") was the celebrated capital of the Nabatean kingdom, a commercially-oriented monarchy established in 168 BC. Currently located in modern-day Jordan, east of Wadi Arabah, it was the convergence of the silk and spice roads, making it an extremely wealthy trading center. The city of Petra was well-known for its richness and its grandiose sandstone façade buildings, many of which survive to this day. In 106 AD, Rabbel II Soter, the last ruler of the kingdom, died, and the Romans used this event to take over the Kingdom, renaming it Arabia Petraea. In the fourth century AD, Petra, like all the other localities in the Roman Empire, embraced Christianity. From that period onwards, Petra was a Byzantine town with Byzantine inhabitants, culture and architecture. In this paper we analyze several tile patterns in a church known as the Petra church, which was built between the 4th and 5th centuries. Recent excavations, conducted mostly in the 1990s, uncovered three churches: Petra, Ridge, and Blue. During the excavations, a set of papyri in the Petra church was unearthed, in which the name of the church is mentioned along with the fact that it was dedicated to the "blessed and all holy lady, the most glorious mother of God, and ever virgin Mary." When it was originally built, the Petra church had a single apse at the end of a broad nave, and two aisles [3]. During the 6th century, it was renovated. The altar under the apse was raised and an elaborate opus sectile pattern of marble and stone was installed in the altar area (Figure 1, left).

Opus sectile, Latin for "cut work," is an inlaid design made of pieces of marble shaped geometrically to create a pattern. It is considered more luxurious than mosaic—the pieces are larger, more fragile, and the stone is usually more precious than in mosaics. Opus sectile originated in Asia Minor (in the Biblical book of Esther, 1:6, there is a reference to inlaid tiles in the temple of Ahasuerus—King Xerxes I of Persia). The Romans used it primarily for floor and wall decoration in houses and public buildings, it has been found in several early Christian Byzantine churches. The opus sectile tilework in the Petra church consists of six rectangular areas that have four different geometric patterns (labeled A, B, C and D in Figure 1, right).

The objective of the paper is to understand how a visual space is organized, both in terms of its impact on the viewer but also how the pattern was physically constructed. This is not only valuable from an aesthetic and architectural point of view, but can also have implications for modern user interface design—primarily in the way information can be embedded within patterns.

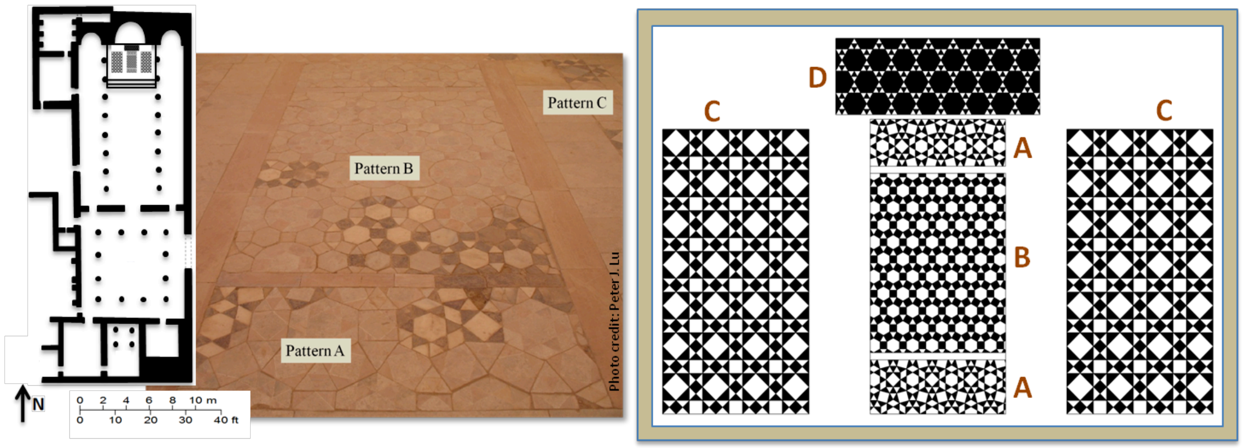

Figure 1. *Architectural layout of the Petra church and the location of the patterns in relation to the nave and apse (left). The middle picture shows the layout of the opus sectile in the church; note that some elements have been renovated (photo credit: Peter J. Lu). Illustration of the opus sectile, showing the locations of patterns A, B, C and D (right).*

The reminder of this paper is organized as follows: we begin with an analysis of patterns in the Petra church, define the concept of layers, and identify the basic units (building blocks), of the pattern. We then analyze and illustrate subtle relations among the different patterns within the church. Using a complex 14^{th} century Islamic tile pattern from a Sufi sanctuary in Natanz, Iran, we demonstrate the generality of the analytical approach and the kind of insights it provides. In the last section of this paper, we make the link between patterns and user interface design and present an example of a tile-based interface.

Analysis of Patterns

To analyze the different patterns, we define a "layer" (or a layer unit) of a pattern as a subset of the pattern which appears to the human eye as a coherent unit that does not appear to belong to a larger unit. At times these coherent units overlap, resulting in emergent features and shapes that form sub-layers. For example, in Pattern A, the circle-like shape (yellow) seen in Figure 2 is a layer unit. It immediately attracts the eye of the observer. These circles overlap throughout the pattern, resulting in oval-like shapes of different types (highlighted in red, turquoise and blue). Each of the oval shapes can be combined with ovals of the same type to create coherent units such as flowers and stars.

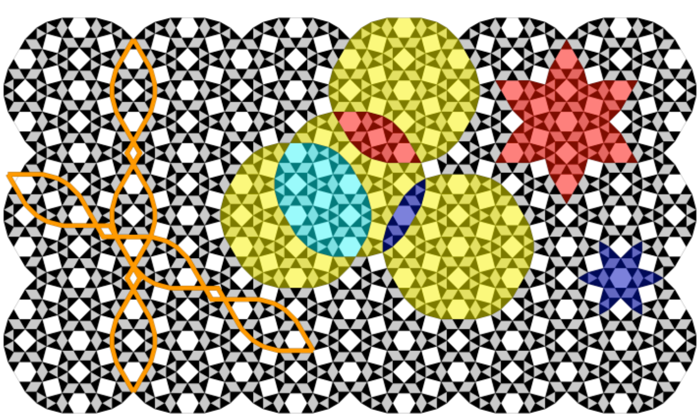

Figure 2. *The circle-like layer unit A-1 (in yellow), and its sub-units (red, turquoise, and blue).*

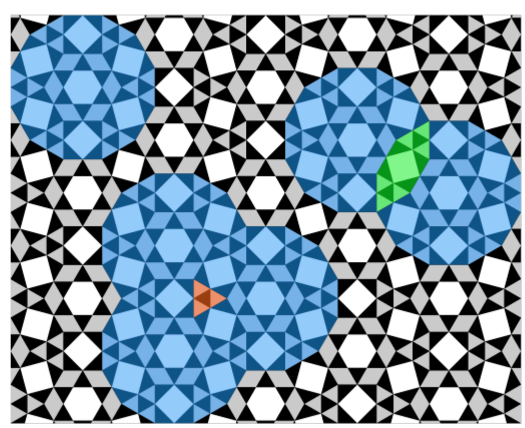

Figure 3. *The dodecagon layer unit A-2, (cyan) and its sub-units (green, brown).*

For example, the six red petal-like shapes form a lotus flower unit. Beyond individual ovals, note how the exterior lines of the ovals form “chains” (orange) that run throughout the pattern. The second layer unit in Pattern A is the dodecagon shape (cyan) seen in Figure 3, defined as A-2 for the purpose of this analysis. In comparison to A-1, this layer unit only intersects in one way, resulting in a hexagon whose sides are equal in length (colored light green). The intersection of three A-2 layer units yields a small triangle (brown). Sub-layers can also be formed when two different layers come together: For example, if we subtract six instances of A-1 from A-2, we get a roundish six pointed star (of David), which can be seen in Figure 4, right side, in blue). If we subtract three instances of A-2 from the central instance of A-1, we get a rounded three-axel shape (upper left side, yellow). If we subtract all six surrounding dodecagons, we are left with the central hexagon (lower left side, yellow).

We find that it is possible, with *intersection*, *subtraction*, and *union* of layer units A-1 and A-2, to capture every shape in the pattern. This is because the contours of layers A-1 and A-2 generate the entire pattern as can be seen in Figure 5. To show this, it is sufficient to prove that every atomic shape (i.e. one which is not intersected by any other line) in the pattern can be formed by employing A-1 and A-2 and using the three operations (*intersection*, *subtraction*, and *union*); this is because any other larger shape in the pattern is a union of several atomic shapes. The general idea is that we (1) look at the different edges that make up the atomic shape we wish to form. Next, (2) we find a group of layer units whose contours “touch” this atomic shape. We then (3) divide these units into two groups. The first group incorporates all layer units that include the atomic shape, and the second group is its complement. We now (4) take the intersection of all layer units of the first group and subtract all layer units of the second group. The end result is the atomic shape.

So far we have looked at the pattern from a perceptual perspective; that is, its visual and holistic impact on the viewer. We can also consider a pattern from a mathematical and building construction point of view [12]. Here we propose three techniques for defining and generating the pattern: the basic polygon, floating polygon, and practical polygon: a *basic* polygon is a shape whose edges do not necessarily lie on the edges of the pattern, which can cover the entire field by only using isometrics—*translations*, *rotations* and *reflections*. Naturally we try to find the smallest polygon with this property (c.f. *fundamental domain*). To identify the basic polygon we consider all centers of rotational symmetry in the pattern. In pattern A there are three distinct centers of rotation (the center of the white hexagon, the center of the white square, and the center of the small black triangle). By connecting the vertices of these three centers we get a 30°-60°-90° triangle (upper left in Figure 6, brown) -- this is the basic polygon. This triangle, when rotated and reflected can generate the entire pattern, as is progressively shown in the Figure.

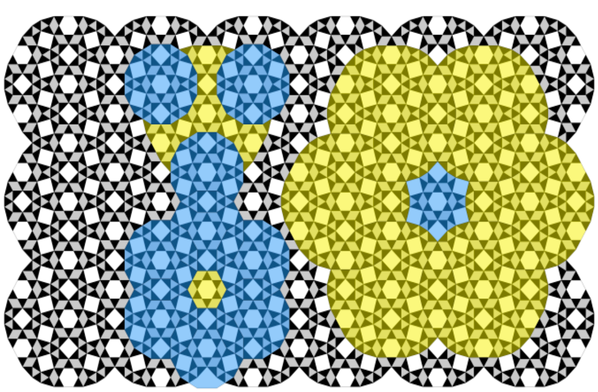

Figure 4. *Relation (subtraction) between layers A-1 and A-2.*

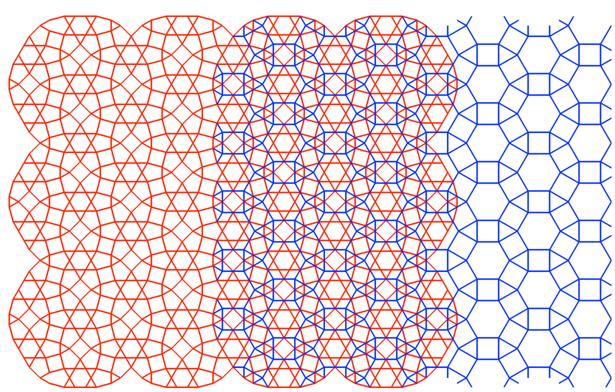

Figure 5. *Contours of units, layers A-1(red) and A-2 (blue).*

A *floating* polygon is a shape that can generate the whole pattern using only translations. We can find a floating polygon, or more precisely a floating parallelogram, by considering the directions in which the pattern can be moved. An interesting observation about such a polygon is that it is not a subset of the pattern, and therefore can float in every direction. In Figure 6 we see two such floating polygons – the large rectangle and the smaller parallelogram (note also the vector lines for each floating polygon). The third polygon type is a *practical* polygon. It is the smallest rectangle that can be used to generate the pattern using isometrics. The only difference between it and the basic polygon is that a practical polygon is rectangular, which makes it possible to be used as a template for construction (as will be discussed later). It is worth noting that while every repeating geometrical pattern has a basic polygon, it does not necessarily have a practical one. We can find a practical polygon by first considering the pattern's rectangular floating polygon; we place that polygon such that its edges pass through the pattern's centers of rotation. That way we can use the pattern's rotational and reflectional symmetries in order to find a smaller rectangle (if one exists). The practical polygon of pattern A and the sequence of isometrics used to create the pattern are presented in Figure 6 (in green).

Up to now we have described two ways to analyze and describe patterns. The first focused on its visual impact on the viewer; the layers and the various sub-layers are perceived by the human eye as a coherent and stable shape. The second viewpoint, inspired by the work of [10], focused on the basic elements for generating a pattern. In the next section we discuss how two patterns come together to create a visual field.

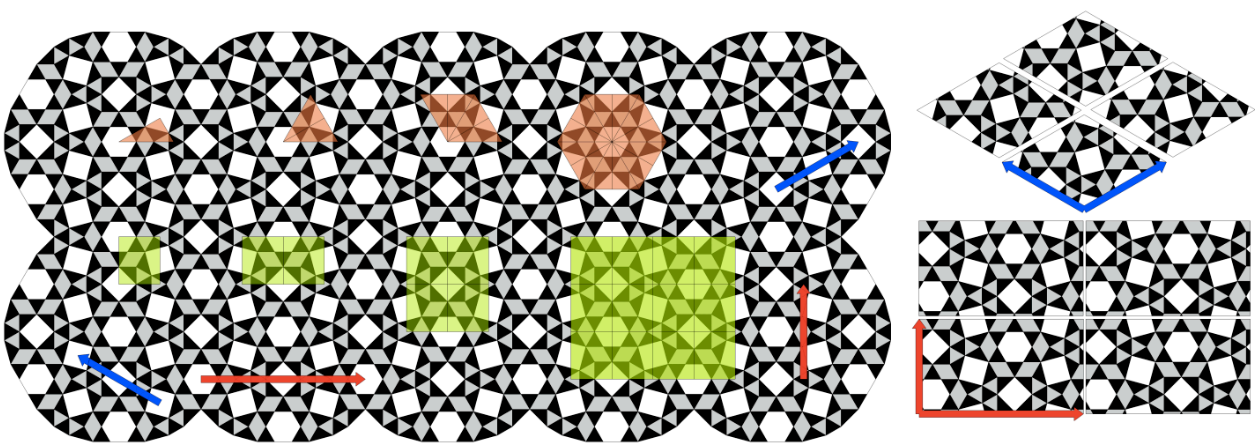

Figure 6. *Basic (brown) and Practical (green) polygons. On the right are two floating polygons. Note that the edges of the polygons agree with the vectors on the pattern only in length and direction (there is no requirement that they agree in location).*

Pattern B and its Relation to Pattern A

Pattern B has only one layer unit (B-1). This is the dodecagon (light brown) visible in Figure 7. The intersection of two B-1s creates an equilateral hexagon (purple). These hexagons form a sub-layer that looks like chains (green) running throughout the pattern. The overlap of three B-1 layer units creates a triangle (light blue). While pattern B is rather simple, there seems to be some kind of relation with pattern A as both patterns have regular hexagons whose sides lie on the edges of squares. A closer analysis shows that pattern B is in fact a reduction of pattern A: Pattern B's basic and practical polygons are reductions of A's (as can be seen in the rightmost side of Figure 7). Layer unit B-1 is a reduction of A-2, and since B-1 contours create the whole pattern, A-2's contours (blue in Figure 5) create pattern B.

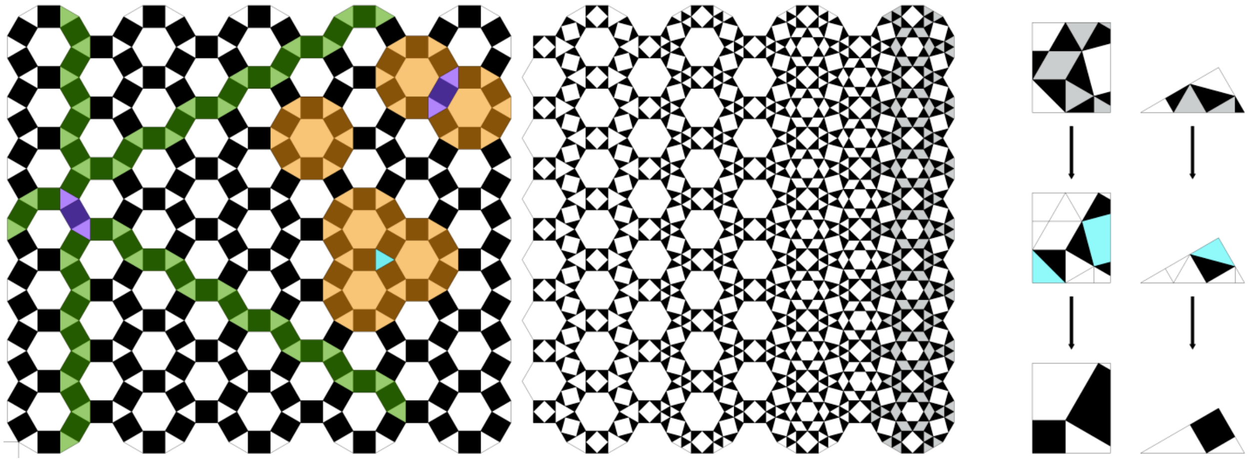

Figure 7. *Layer of pattern B and the transform from pattern A to pattern B. On the far right are reductions in the basic and practical polygons of A (creating the basic and practical polygons B).*

We now proceed to consider the relation between the physical locations of patterns A and B in the church. To do so, we draw on the notion of "field" from Christopher Alexander's work on pattern language and his "theory of centers" [1] to describe an area that constrains a given pattern and may include a juxtaposition of more than one pattern. In particular we look for a coherency within the field; for example, meaningful relations between the patterns in the field. And indeed, patterns A and B are related. First of all, their practical polygons adhere to the same grid (Figure 8). Second, pattern B is scaled such that its regular hexagon is the same size as the regular hexagon in pattern A. Finally, there is a prolongation of these hexagons from one pattern to the other (shaded in red in the Figure). Interestingly enough, this prolongation only exists on the right side of the field, not on the left.

In addition to patterns A and B, two other patterns exist on the raised platform inside the church. These are patterns C and D (see Figure 1). Pattern C is made up primarily of squares and D combines equilateral triangles (similar to those seen in pattern A) as well as regular hexagons (not the same size as in A and B, though). C's layer unit is a square with a border and there is an overlap that creates a small white square. D's layer unit is a 6-pointed star (c.f. kagomé lattice) and its triangles are identical to the triangles in A (but somewhat smaller). Generally speaking, patterns C and D are much simpler than A (because there is only one layer), and we were unable to identify any physical and/or layout relations between these two patterns.

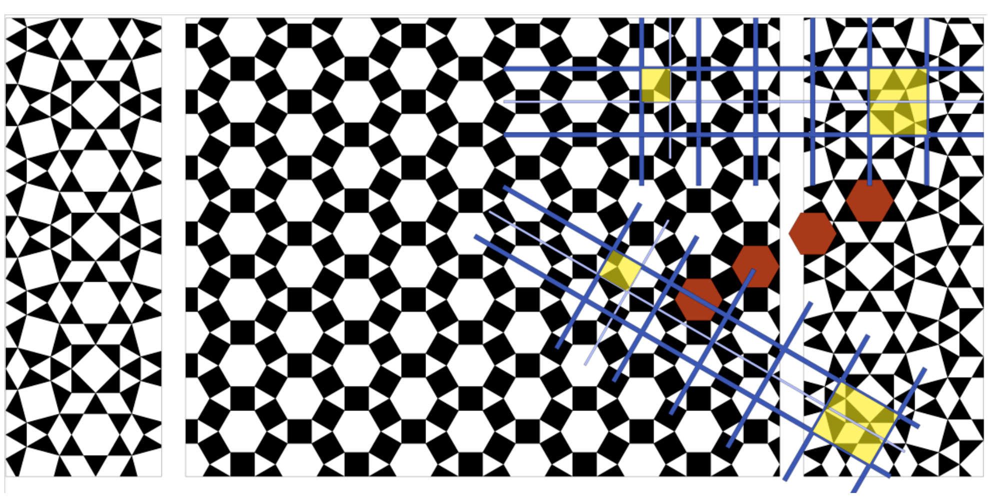

Figure 8. *The grid underlying pattern A and pattern B.*

Brief Analysis of a 14th Century Pattern

Figure 9 shows a lively and rich pattern constructed between 1304 and 1307, made from tiles, stucco, and paint [11]. It is located at the entrance to a Sufi sanctuary in the town of Natanz, Iran [4], and contains three layer units: the blue octagons, the zigzag lines, a painted hexagon-like shape and its eight small squares. As can be seen in Figure 9, many sub-layers are formed from the intersection, subtraction, and union of layer units. Three main symmetry centers can be identified: one is around the dark star, the second is at the intersection of the stucco pattern, and the third has three chevron-like forms. Connecting these three centers forms the smallest basic polygon (in brown, Figure 9). The practical polygon (colored in green in the figure) can be found using the procedure described above. Finally, what is interesting and revealing about the practical polygon of this pattern is that it may have been used to construct the pattern within the bounds of the field, as demonstrated in Figure 9.

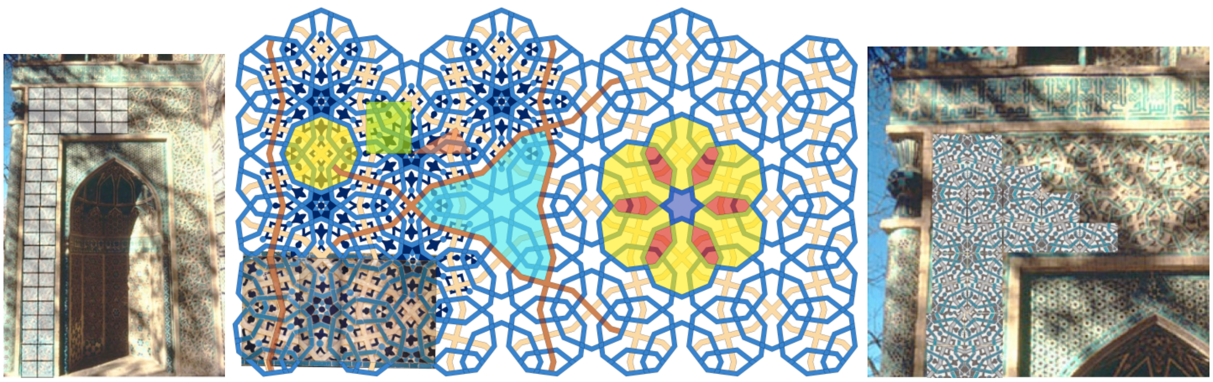

Figure 9. *Prayer niche at the entrance to the Khanqah (dervish sanctuary) and burial shrine for Shaykh 'Abd al-Samad in Natanz, Iran. The pattern surrounds the prayer niche. The pattern is composed of three layer units and the basic and practical polygons are in brown and green respectively. The practical polygon tiles the field completely as can be seen in the side pictures (photo credit: Sheila Blair and Jonathan Bloom; www.archnet.org, id=IMG11333).*

Implications for and Applications to Interface Design

When we talk about user interface design in the context of modern systems, we are really talking about organization of information so as to portray messages about a system's behavior in a coherent fashion. Thus, the problem of interface design is very much akin to the problem of how to best organize space. Both problems are about the creation of order within a space, or in our case, the field. As such, techniques and schemes for generating the underlying pattern on which one can "hang" information is a must [2], in particular when considering modern interfaces that are rich (if not glutted) with information. Consider for example the problem of interface design for the wealth of data coming from microprocessors in automobiles, aircraft, and spacecraft about the vehicle's health. This has given rise to a new field in AI and computer science dedicated to developing integrated vehicle health monitoring systems (IVHM) so as to analyze and provide this information to users.

In the following we briefly describe a research effort to use some of the concepts of tiling arrangements to organize vehicle health monitoring data on helicopter engines. The objective of the research is develop formal and systematic methods to visually organize multitudes of data -- in a way that helps the viewer quickly identify deviations from expected system behavior as well as obtain an holistic understanding of the system. The display design is based on a 15th century tilework from Gazarghah (near the town of Herat), Afghanistan (Figure 10, left). The pattern shows a wild and powerful composition, replete with stars and flowers that come together to create an integrated pattern [8]. The step-by-step procedure for generating a tile-based display is described in [7]. Here we illustrate two of the main elements in the display: the actual engine data and the composite information. The engine's parameters are organized in the middle of the main flower. Each one of the four engine parameters—Power Turbine Speed (Np), Gas Generator Speed (Ng), Torque, and Fuel Flow—is represented by a petal. Naturally, we place the parameters of the left engine on the left side of the flower and the parameters of the right engine on the right in order to achieve symmetry. Rotor speed, which is a product of both engines' outputs, is represented on top like a keystone supported by the two sets of parameters. Figure 10 shows the progression from the abstract flower formation of tilework to an arrangement for placing engine parameters. In addition to the engine parameters, we use the shapes created by the pattern's sub-layers to embed composite information. For example, it is possible to compute a value that is a composite of the pair "Rotor speed" and "Np" (yellow). This composite is sensitive to any deviations from expected (or normal) values [9]. Along the same lines, it is possible to create a composite made out of the triplet "Rotor speed," "Np," and "Ng" (light blue). These composites are computed using data mining and clustering techniques [5,6]. An interesting observation about such composites is that the larger the composite, the more sensitive it becomes to real, as well as unexpected, anomalies and resistant to nominal fluctuations and random noise in the data. The middle picture in Figure 10 is a snapshot from a simulation of the display running actual engine data (of a NASA UH-60 helicopter). The red arrow on the left points to a gray pentagon inside the triplet composites "Rotor speed," "Np," and "Ng," visible on the left side of the display, and indicates an anomaly. As can be seen in the figure, this anomaly cannot be observed by merely looking at the parameters (inside the flower), or for that matter by the information provided by the pair composites. This anomaly, which deserves serious attention by the pilots, only appears in the triplet.

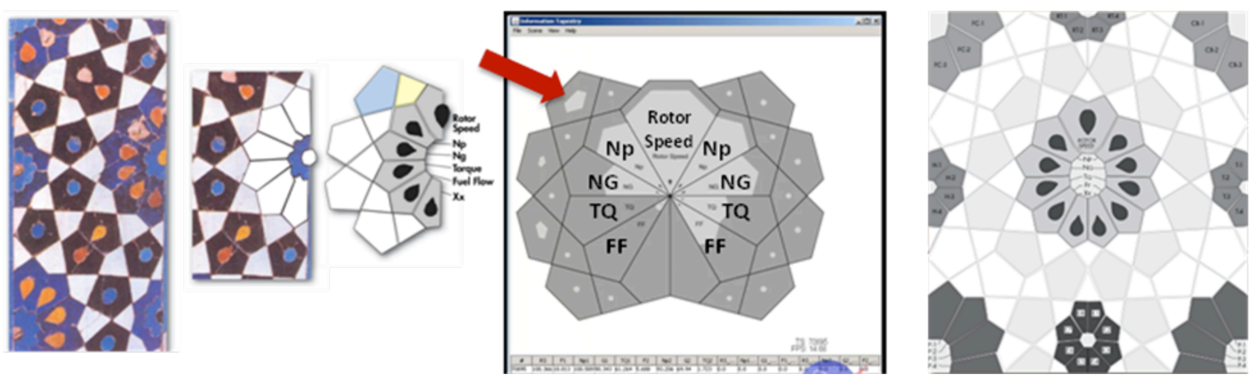

Figure 10. *From tilework to parameter arrangement. The picture on the far right shows an integrated display of engine parameters and eight other aircraft sub-systems. (Adopted with permission from [7])*

The idea behind such tile-based displays is to organize information in a coherent way. This involves a progressive move from simple shapes (such as a petal that houses one engine parameter) to increasingly more sophisticated sub-layers (of pairs, triplets, quadruples), and ending with an overall shape for a given system (e.g., the two engines of this helicopter). The underlying structure can then be further extended to include other sub-system information (e.g., electrical, hydraulic, pneumatic, flight control) as can be seen in rightmost picture in Figure 10. This concept of arranging the whole was explored by Gestalt researchers in the early 1930's and has many potential applications for interface design [13]: (1) we perceive information, initially represented as separate elements, as a unit because of the elements' "proximity" and "common fate;" (2) the unit fosters better understanding of the interrelations among parameters; (3) the unit and its resulting pattern(s) enable faster information processing; (4) any deviations from the structure of the pattern can be quickly detected; and (5) the implications of the deviation and impact on other parameters can be understood intuitively.

Summary and Conclusion

We began this paper with an analysis of an *opus sectile* tile pattern from the Byzantine church in Petra. We showed that behind seemingly simple and abstract geometric patterns there is an underlying order, both in the way elements of the pattern come together in an integrated way as well as how different patterns relate to one another and form a well organized and coherent field. We then made the link between the way tiles, patterns, and fields of patterns have been used for aesthetic and architectural purposes and their potential application in interface design and information organization. The notion of layers and sub-layers is important for user interface design as a method to describe and account for shapes and patterns on which information can be "hung." As for the polygons, just as an artisan can use the practical polygon as a template (or even a tile) to construct a complex pattern, it can also be used in display design to generate a complex pattern, and then easily "tweaked" when it becomes necessary to change and/or adapt the pattern to users' (changing) needs. Finally, the polygons help us to define and categorize the unit of a pattern and consider relations and similarities between patterns within a field. Generally speaking, a field, for the purpose of user interface design, can be a single display (e.g., for GPS navigation), a cluster of several displays (including their buttons and controls), all the way to the layout of an entire cockpit (of a car or aircraft). Nevertheless, for such a tile-based approach to gain wide acceptance among automotive, avionics, and display engineers, it must first be supported by a formal language of patterns. Future research should therefore focus on creating a generic language to describe patterns and developing methods and tools for their application.

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