Toward a Tesseract Theater

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Abstract

Euripides' theatrical device *deus ex machina* and two landmark examples of its modern legacy are contextualized in spatial 4D. Stacked space, tesseract geometry and graph-theoretical trees are considered in the proposed design and use of a theater situated in the surface-volume of a tesseract.

Ancient Intuition

Spatial 4D is a product of nineteenth century mathematics, but ancient Greek tragedy, namely the plays of Euripides evince 4D spatial intuition in a controversial theatrical device called dues ex machina (DEM). To be clear, spatial 4D is defined mathematically as a Euclidean space with four orthogonal directions, where the fourth direction $4D^{+}$, dubbed ana, or $4D^{-}$, dubbed kata, is a degree of freedom beyond the familiar three, and DEM, characterized by divine intervention that typically saves a tragic hero from certain death, literally translates to "god from the machine". There are several liberties a 3-space entity could theoretically take while making a 4-space transit. Namely, one could disappear and reappear in the same or a different location, as Euripides intuited, and having perceived, communicated or reached across 2-space boundaries (i.e., seen through walls, voice inside someone's head, surgery without incision), unrealized by Euripides; most exotically however, one could disappear, rotate in a fourth direction and reappear orientation reversed (i.e., righty turned southpaw) [1]. It is hard not to imagine what theatrical marvels the Greeks could have conceived had DEM been an explicit result of higher dimensional space.

Consider three examples of Euripides' DEM hypothetically, albeit conservatively, contextualized in 4-space. He introduces it in his *Alcestis* (438 BC) to retrieve Admetus' self-sacrificing wife from Hades [2]: Heracles, a demigod, would transit kata to the Underworld, where he wrests her from Hades, and both would return ana to Earth. Later, *Medea* (431 BC) escapes Jason's wrath and the aftermath of her infanticide, when Helios' golden chariot alights kata through the Vault of Heaven to deliver her from Corinth to Athens [3]. Lastly, Euripides uses DEM twice in his *Iphigenia at Tauris* (414 BC), once toward then end of the plot, when her brother Orestes absolves his matricide by retrieving a wooden sculpture of Artemis that had fallen kata from Olympus, and again to save Iphigenia from sacrifice in Aulis; Artemis, a god, plucks Iphigenia, a mortal, from her imperiled position in 3-space, hypothetically granting her a fourth liberty of movement, whereafter she reappears safely in Tauris [4]. Now the ancient Greek model for Olympus, Earth and Hades reads more like a conventional 2-space map with all locations accessible by land or sea [5], but temporal thresholds that can only be crossed by death or ascension might better be described as extra-dimensional. Perhaps, the civilization that is heralded equally for its contributions to mathematics and theatre unwittingly experienced an interdisciplinary prescience.

A Precedent in Modern Drama

Although Euripides' legacy can be felt widely in all theatrical media today with historic milestone appearances of DEM on stage in Shakespeare's *Hamlet, Prince of Denmark* (1599-1601) and Artaud's "Jet de Sang" (Spurt of Blood, 1925), theatrical evidence of its spatial 4D companion (unlike other 4D models like time or emotion [6]) has but one precedent in modern drama—Ionesco's "La Contatrice Chauve" (The Bald Prima Donna, 1950). In this inaugural absurdist play, Ionesco develops a scene between two amnestic Brits Donald & Elizabeth Martin who discover over the course of their conversation that they are married, but this logically deduced conclusion makes an important but false assumption about the orientation of their daughter's one red eye and one white eye. We are later told by an omniscient maid that "they are both grievously deceived…whereas, it's the right eye of Donald's child that's red and the left eye that's white, it's the left eye of Elizabeth's child that's red and the right eye that is white" [7]. Certainly, Ionesco is out for postwar laughs at the expense of the English middle-class, but, to the extent that orientation reversal is the punch line, his existential referendum on logic, memory, marriage and, perhaps, 3-space gains momentum with this reference to a fourth spatial dimension.

Other Precedents

Other media have made further strides with spatial 4D, and contemporary theatre might find its inspiration there. Namely, in the literature genre of short science fiction, Bond's "Monster from Nowhere" (1939) and Heinlein's "—And He Built a Crooked House—" (1940) explore 3-space entity interplay with other 4-space liberties [8]. Bond's tragic hero Burch manages to pin a 3-space cross-section of the 4-space monster he hunts so that its cross-section has lost the fourth degree of freedom, and, while pinned, his monster utters, from 4-space, a "tonic wave of supernatural heights" that "tortured the eardrums" [9]. On the other hand, Heinlein's hero Mr. Teal is the indomitable architect of a house situated in the surface-volume of a tesseract: "They climb a fourth flight of stairs, but when the cover at the top lifted to let them reach the level above, they found themselves, not on the roof, but standing in the ground floor room where they had entered the house" [10].

This is the spirit of contextualizing DEM in spatial 4D from which contemporary theatre can model its classical departure. Artaud's bold rhetorical challenge—"[T]heatre has been created to teach us that…the sky can still fall on our heads." [11]—corroborates his radical twist on DEM in "Jet de Sang" in which his tragic heroine rejects divine intervention by biting the hand of God [12]. Had Artaud informed this coup de theatre with spatial 4D, we might have seen something more akin to Bond's cross-sectional pinning. Certainly, creative authority granted contemporary stage directors may still have the final word on Artaud and else.

Stacked Stages

If we push the envelope even further, the conventional black box that describes theatrical playing space may develop into the surface-volume of a tesseract, composed of eight cubes. Certainly, this notion deserves a rationale which can be found in Hinton's popular representation of 4-space events as occurring over stacked 3-spaces [13], say three cubes. See Figure 1a. Consider this fundamental strategy for representing a 4-space event against 261 ways to unfold a tesseract [14]. Which unfoldings optimize sets of three stacked cubes whose interiors are at least partially visible from most points of view (audience) embedded in a single cube? Surprisingly, the answer requires little, if any, subjectivity and can be best explained citing a catalog of 23 8-node trees [15], where a tree is a connected graph with n nodes and n-1 edges [16], and, in particular, those 8-node trees that enumerate distinct pairings of non-adjacent nodes.

Since there is a one-to-one mapping between unfoldings of a tesseract and paired 8-node trees [14], of the 22 8-node trees that enumerate pairings, F. Harary & G. Prins' 11th & 22nd trees (Figures 1c, 2b) correspond to surface-volume unfoldings (Figures 1b, 2a) that optimize visible sets of three stacked cubes. To demonstrate a counter-example, their 1st tree may be a row of eight cubes, but one might argue the need for so many cubes to represent a 4-space event or, for that matter, the clarity of doing so from any embedded POV. On the other hand, Tree 11's only pairing corresponds to a surface-volume that resembles a proscenium stage theater (Figure 1b). Tree 22's first pairing (of two) represents a variant of the proscenium stage theater (Figure 2a). Whereas the unfolded tesseract theater structure is informed by its tree and pairing, what we think of as non-adjacent cubes in 4-space is informed by distinct pairings and would be of particular interest to theatre practitioners (i.e., playwrights, directors, choreographers) whose stage directions and blocking should properly locate within the folded tesseract the 4-space path of an object or an actor. See arcs connecting shared faces of adjacent cubes in Figures 1b, 2a.

Take *Hamlet*, for instance, staged in Tree 11's unfolding, say, Stage 11. There are two, arguably more, 4-space events that would benefit by context in a tesseract theater. First, in Ii and Iv to warn of foul play in his death, recently departed King Hamlet appears as a ghost before Prince Hamlet [17]. Adhering to Elizabethan notions of afterlife [18], the upper body of unresolved King Hamlet could be suspended in the fly, wrestled in 4-space transition perhaps by two winged angels, and simultaneously his legs suspended in the trap (seen through a transparent floor), held by winged demons, in a tug-of-war over his tortured soul, but the face of King Hamlet passes through Prince Hamlet's 3-space on the mainstage in order to deliver his haunting admonition. Second, in Vii, to support Shakespeare's criticized pirate intervention (DEM) in absentia to deliver Prince Hamlet from execution in transit to England [19], tortured King Hamlet might appear again with arms loosed by the angels to reach into the air in Prince Hamlet's 3-space above the ship deck, clasping his son's arms and hefting him in a DEM transport to the pirate ship. Perhaps this gesture ultimately serves King Hamlet's blood lust, and the angels consequently acquiesce his soul to the demons. These two 4-space events in *Hamlet* require three stacked cubes {fly, mainstage, trap}, but general usage of Stage 11 may also explore {house, mainstage, backstage} or {off-stage left, mainstage, off-stage right} or even something counter-intuitive like {lobby, fly, backstage, off stage left} since these four cubes are adjacent when the surface-volume folds back into a tesseract [20].

Spatial 4D may have deep roots in theatrical origins but, despite little development through modern drama, offers several exciting, even revolutionary, applications that serve as rare mathematical contributions to the theatre, with significant potential to forever alter theatrical vocabulary of space.

References

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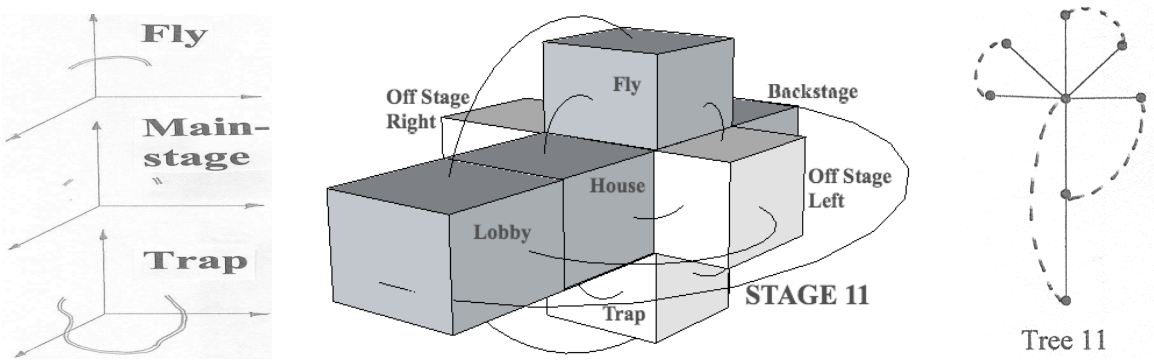
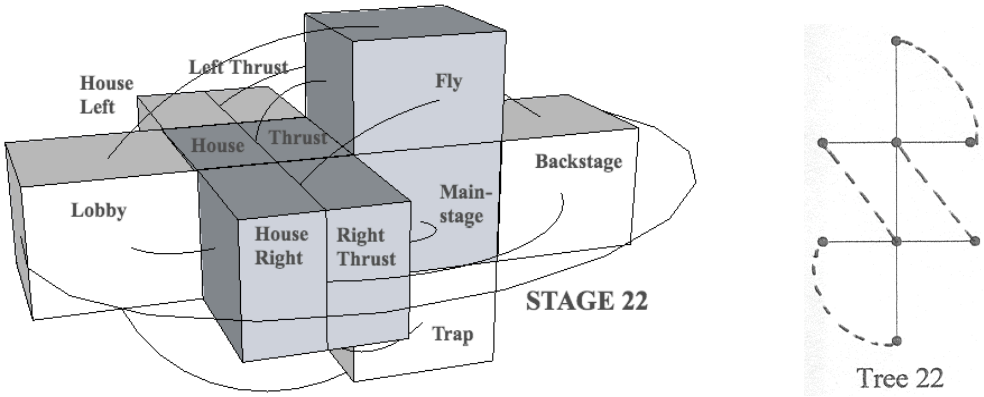
[19] W. Shakespeare, op. cit., pp. 1116-1117 (Vii: 1-81)
[20] E. Burger, M. Starbird, op. cit., pp. 225-227

Illustrations



Figures 1a (left), 1b (middle), 1c (right): *Hinton's stacked 3-space model demonstrates the lifting of a closed loop into 4-space (left) which could be presented in Stage 11 with arcs mapping folds (middle), modeled on Turney's tesseract unfolding corresponding to Harary & Prins' 11th 8-node tree omitting Turney's only pairing (right) and useful as actor or object paths in 4-space.*



Figures 2a (left), 2b (right): *Stage 22 (left) modeled on Turney's tesseract unfolding corresponding to Harary & Prins' 22nd 8-node tree with arcs mapping folds omitting Turney's first (of two) pairing (right)*