Everything is Number – Mathematics Meets Arts

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Abstract

The project "*Everything is Number"* started in the year 2008 with a calendar illustrated by the Swiss artist Eugen Jost. All paintings deal with mathematical topics. The interplay of arts and mathematics provides fascinating insights into mathematics especially for students at school. Sponsored by the German employers' association *Gesamtmetall* the calendar paintings became part of an exhibition at more than 150 schools all over Germany. Students were invited to discover new access to mathematical problems and tasks. The feedback of the public towards the calendar and the exhibition was overwhelming. In the meantime the nonprofit project grew and we could publish books and a second calendar in 2010. In special workshops for teachers we show how to work with Jost's paintings in the classroom to motivate students for mathematics. Since October 2009 the project hits another peak with a temporary exhibition at the Museum of Technology in Berlin.

Year of Mathematics 2008

In 2008 the German government proclaimed the "Year of Mathematics" to stress the diversity and importance of mathematics in society and everyday life. Especially children and young people were to be inspired by an interesting and exciting science. A lot of national and local projects were started. Exhibitions, contests, festivals and other activities pointed out the beauty, the importance and the fascination of mathematics as a science and a subject at school.

For the year of mathematics the chair of mathematics and mathematics education at the University of Bayreuth contributed a big and colorful wall calendar. Only on this occasion the Swiss artist Eugen Jost created thirteen paintings. On the backside of each calendar leaf the mathematical topic of the corresponding painting was explained. These texts are not interpretations of the paintings, they give some mathematical and historical background information, and they want to stimulate the reader's curiosity.

The artist Eugen Jost lives and works in Thun in Switzerland. On the one hand his paintings are inspired by numbers and patterns. On the other hand he is dealing with language. The longer he has worked in those two fields – numbers and language – the less he has seen clear border lines between them. His paintings tell stories; they ask questions and they invite the viewer to do mathematics. Numbers, patterns and language are three important fields which characterize mathematics education.

Arts in Mathematics Education

This calendar was the initial point for our widespread project called *"Alles ist Zahl"* respectively *"Everything is Number"*. The feedback to the calendar was extraordinary. We got a lot of letters from enthusiastic people and many teachers told us, that they use the calendar paintings in their lessons to motivate several mathematical topics.

Figure 1.1 for example shows the painting "Hardy's Taxi" – named after an anecdote by the English mathematician G. H. Hardy (1877 – 1947). Among others you can discover several number sequences in the picture. Some of them are obvious like the first nine natural numbers. Some of them are slightly hidden: 1, 1, 2, 3, 5, 8, 13, 21, 34, …, 377 – the Fibonacci numbers. Others are presented as symbols, e.g. the triangular numbers (figure 1.2).
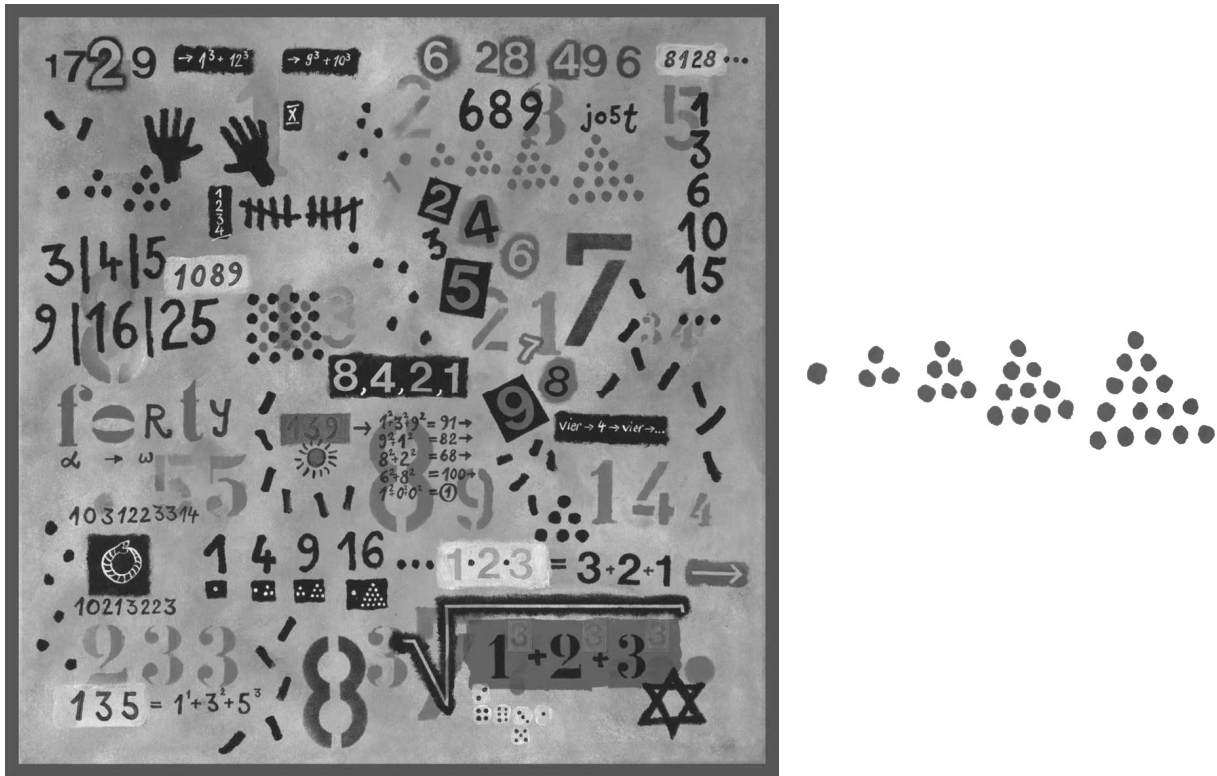

Figure 1: *"Hardy's Taxi" (1), Eugen Jost, project "Everything is Number", Triangular Numbers (2)*

At school students are asked to explore *"Hardy's Taxi"*. They will find a lot of numbers, signs and symbols. On a closer look they will discover patterns and structures. Finding number sequences, discovering rules and common properties of each sequence is one possibility to explore the painting. Finally students will be able to continue the number sequences, they start doing mathematics on their own.

The Exhibition on Tour

The calendar 2008 was not a singular event. The feedback from teachers shows how wonderful Eugen Jost's paintings fit in mathematics education. Therefore we have created an exhibition with 13 digital prints plus explanatory texts for schools. Figure 2 gives an impression of these so-called roll-ups (size 2m x 1m each). The texts to the pictures are to encourage reflection on mathematics. The exhibition package additionally contains a roll-up with information on the project. Due to the financial support of the German employers' association of the metal and electrical (M+E) industry *Gesamtmetall* we had the ability to produce seven sets of this exhibition (six German sets and one English set). The sponsor even paid the costs of transportation. So we had an attractive offer for schools.
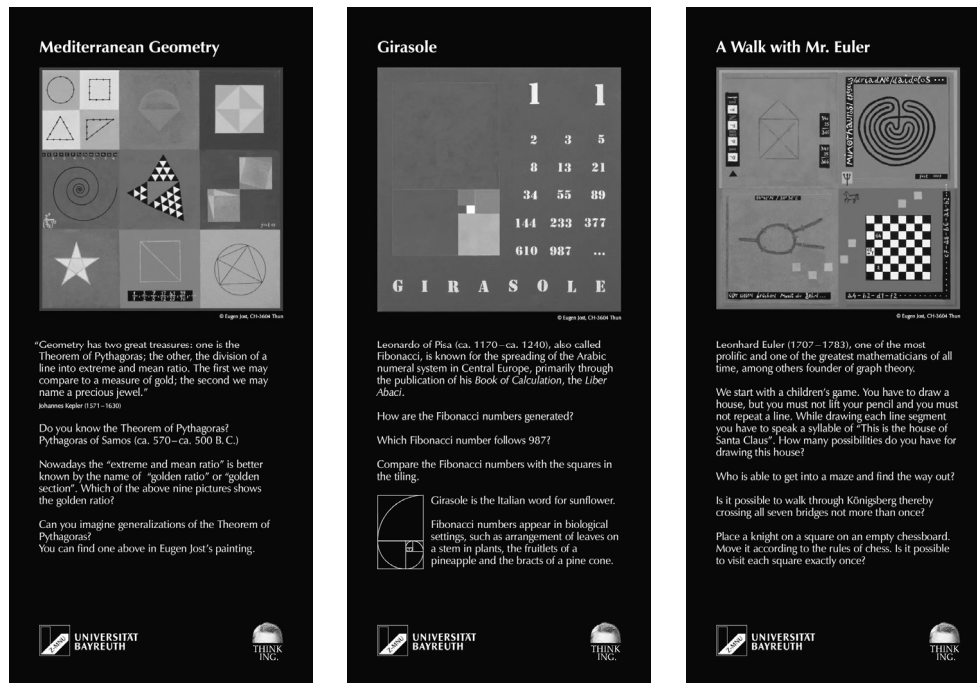
Figure 2: *English Roll-ups, project "Everything is Number"*

In the years 2008 and 2009 the exhibition sets were shown in more than 150 schools and institutions. Approximately 140.000 students, teachers and interested individuals visited the exhibitions. We evaluated and documented these local events to get information how the exhibition was used to improve mathematics education at school (figures 3 and 4).

Figure 3: *Students from primary and secondary schools are working with the exhibition*

Figure 4: *Prof. Dr. Peter Baptist (University of Bayreuth) presents the exhibition*

The Exhibition at School

Teachers and students worked in many different ways with the exhibition. Some schools organized workshops for their students. Therefore they put together roll-ups and hands-on experiments (figures 3.2, 3.3 and 9.1). For example the students made *"A Walk with Mr. Euler"* (figure 2.3) to find a way over the *Seven Bridges of Königsberg* realized with adhesive tape on the floor.

Other teachers created quizzes, exercise sheets and questionnaires for their students to get them in contact with the mathematics motivated by Eugen Jost's paintings. Depending on the students' age they had to solve questions on different mathematical levels.

Often teachers used the roll-ups to motivate a new topic. *"Mediterranean Geometry"* is shown in figure 2.1. This painting contains nine small pictures arranged in a matrix, all of them dealing with the Pythagorean Theorem. In the middle of the last column we see a visual proof of the theorem. Students are asked to reconstruct the theorem (figure 5.1).
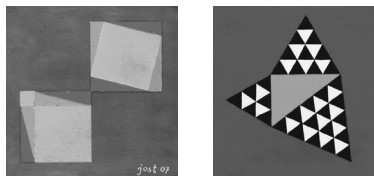

Figure 5: *Details of "Mediterranean Geometry" (1), Generalization of Pythagorean Theorem (2)*

In the center of *"Mediterranean Geometry"* Eugen Jost placed a generalization of the Pythagorean Theorem. We don't have squares over the three sides of the right triangle but equilateral triangles. Each of them has a tessellation of smaller congruent equilateral triangles in colors black and white. Counting the small triangles that tessellate the leg triangles and the ones of the hypotenuse triangle leads to a conjecture of a generalization of the Pythagorean Theorem. Consider also other similar figures over the sides of a right triangle.

Our project title *"Everything is Number"* (figure 6) goes back to the Pythagoreans too. Pythagoras of Samos and his devotees recognized that numbers are hidden in everything, from the harmonies of music to the orbits of the planets, and this led them proclaim "everything is number". By exploring the meaning of mathematics, the Pythagoreans were developing the language that would enable them and others to describe the nature of the universe.

Figure 6: *"Everything is Number", Eugen Jost*

In addition to the exhibition Peter Baptist, full professor at the university of Bayreuth, wrote an interesting and enjoyable book about the calendar paintings called *"Alles ist Zahl"* [1]. He explains the mathematical and historical background, he stimulates mathematical activities and he gives teachers advice how to work with the exhibition.

Calendar 2010

Eugen Jost was fascinated by the success of the exhibition and the whole project. In early 2009 he had the idea of creating a new calendar. In the meantime he was very "productive" and we tackled a new calendar. But we didn't want to make a simple copy of the first calendar. So we developed a new layout for the back sides of the calendar leaves. We reduced the text length and we added more graphical

elements to raise the attractiveness (figure 7.2). The goal of the calendar remained the same: Creating an illustrative and popular access to mathematics with the help of Jost's paintings and stimulating texts. *Gesamtmetall,* the employers' association, supported the project once more.
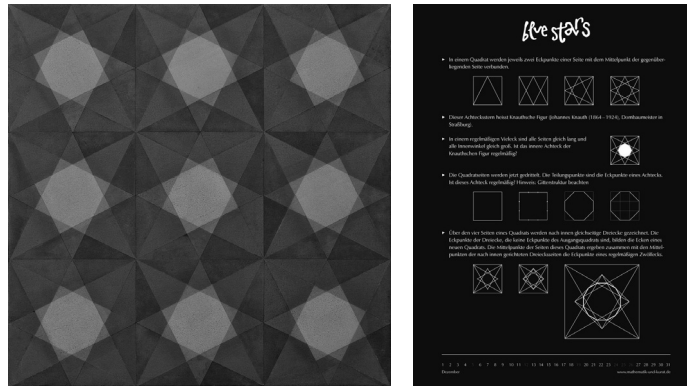

Figure 7: *"Blue Stars", Eugen Jost (1), backside – December, Calendar "Alles ist Zahl 2010" (2)*

"Blue Stars" (figure 7.1) is a good example to show how to work at school with the exhibition and the pictures of the calendar. At the beginning the students are asked to explore the painting. Therefore they will get a worksheet with some tasks:

- Find a basic pattern in the painting (see figure 8.4).
- Describe this basic pattern and make a construction in your exercise book.
- Find congruent figures and fill each of them with the same color.
- Mark all angles without measuring.

Primary school students will find several (right angled) triangles, quadrangles, different kites and octagons (figure 8.5). In secondary education they analyze the figures:

- How do we create such octagons?

The back side of the corresponding calendar leaf (figure 7.2 and figure 8.5) gives the answer. We can go further.

- Is this octagon regular? What criteria do we need to create a regular polygon (angle, side length)?
- Is it possible to tile the plane with regular octagons of the same size?
- Compare the area of the octagon to the area of the basic square (basic pattern)?

One simple basic pattern (figure 8.4) stimulates a lot of exciting questions. Variations of this basic pattern lead to interesting new problems.
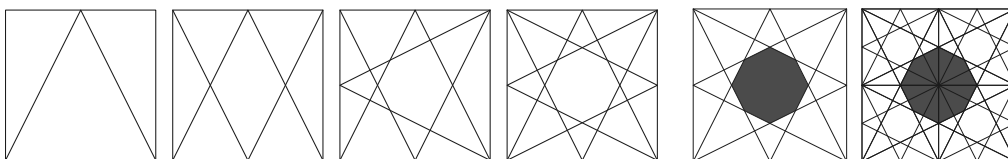

Figure 8: *Basic Pattern of "Blue Stars" (1) – (6)*

Another *"Blue Star"* topic is similarity. In the basic pattern we find the half regular octagon. This octagon is to be found several times in the painting. But there are more octagons inside. Not congruent but similar ones, as shown in figure 8.6: regarding four basic patterns arranged to a square we will find another octagon which is similar to the first one. Now we can ask: What happens by arranging nine, sixteen, twenty five, … basic patterns to a square? What size is the factor we have to multiply the length of sides of the original octagon to get the side-length of the similar ones? Can we find other similar figures?

In-service Teacher Training and Special Exhibition

For many teachers motivating mathematics via arts is a new way of teaching and learning. Therefore we organized a lot of workshops for teachers. In numerous talks and workshops (figure 4) we demonstrated how to teach with the help of Eugen Jost's paintings.

Together with the Museum of Technology in Berlin we organized a special exhibition *"Everything is Number"* (figure 9.1) which will be shown until December 2010. Among some of the original paintings from Eugen Jost (figure 9.2) we also present several roll-ups, a video show and hands-on experiments for visitors particularly students. The Museum of Technology organizes workshops for teachers to spread the idea of combining mathematics and arts. Many teachers revisit the exhibition with their classes.

Conclusion

The interplay of mathematics and arts in the project *"Everything is Number"* is fascination and inspiring. The teamwork of artists, researchers, sponsors, teachers and students opens new perspectives in mathematics education. The project will be continued!

Figure 9: *Museum of Technology – Special Exhibition, Berlin, 2009 (1), Eugen Jost (2)*

References

[1] P. Baptist, *Alles ist Zahl,* Kölner Universitätsverlag, Vol. 2. 2008.
[2] P. Baptist, B. Bocka, C. Miller, *Alles ist Zahl – Dokumentation der Wanderausstellung,* Universität Bayreuth. 2009.
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[4] German project website, *www.mathematik-und-kunst.de*
[5] International project website, *www.everything-is-number.net*
[6] Museum of Technology Berlin, *www.sdtb.de*