

**Renaissance Banff II**

# **Bridges Banff**

**Mathematics, Music, Art, Architecture, Culture**



**2009**



The 12th Annual Bridges Conference

# Bridges Banff

Mathematics, Music, Art, Architecture, Culture



**Banff International Research Station for  
Mathematical Innovation and Discovery**

**The Banff Centre  
Banff, Alberta, Canada**



Proceedings 2009

Craig S. Kaplan and Reza Sarhangi, Chief Editors  
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# Bridges Banff

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**Cover:** *Limosa* is a work by Brian Evans, an Associate Professor in the Department of Art and Art History, at the University of Alabama. The full image is presented on the 2009 Bridges Art Exhibition website. The artist says: “Here we see a slice, through the time dimension, of an abstract animation (a mathematical visualization). We see how a single scanline changes as the animation unfolds. This 2D slice of a 4D object is then mapped into sound, so the image is actually a graphical music score. I sonify the score to create music that correlates sonically with what we see unfold visually in the animation. We hear the colors. We listen with our eyes.” Jeffrey Rutzky, the cover designer, selected a part of the artwork and harmonized the presentation of the words with the movement of the image.

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## Preface

2009 marks the 12th year of the annual Bridges Conference. This year we find ourselves returning to The Banff Centre, a wonderful facility nestled in the Canadian Rockies. It's hard to imagine a location better suited to the spirit of Bridges. The region inspires with its natural beauty. The campus itself has a longstanding reputation as a world leader in art, media and culture, and a tradition of interdisciplinary cooperation. Over the past decade, the Banff International Research Station (BIRS) has grown to become a focal point for mathematical interaction within North America and worldwide, and has made The Banff Centre an ideal place to explore ideas that lie in the intersection of art and mathematics. We are honoured and thrilled that BIRS has invited us back to Banff for Bridges 2009. and hope to make the trip again many times in the future.

As always, we make an effort to bring new and exciting experiences to Bridges. This year we will explore sights around the Banff region as well as some of the studios and facilities on campus at The Banff Centre. Three evenings will be dedicated variously to music, theatre, and film.

Like last year, we collected regular paper, short paper, and workshop submissions using EasyChair, a free conference management service hosted by the University of Manchester. Each submission was then assigned to two members of our international Program Committee for review. In some cases one or more additional reviews were provided by other Program Committee members or by external experts. The quality of the reviewing was quite high, with many reviewers providing fine-grained, detailed advice for improving a paper's content or presentation. In several cases authors wrote back to express their gratitude to the reviewers who helped improve their papers significantly.

Once the reviewing process was complete, Craig S. Kaplan, the chair of the Program Committee, worked with the reviewers and the Bridges organizers to arrive at a decision for each paper. Occasionally the decision process called for additional opinions or communication with the authors. Submissions of all kinds were either accepted as-is, accepted with revisions, or rejected. In addition, some regular paper and workshop submissions were recommended for inclusion in the short papers program. In the end, we arrived at a total of 38 regular papers (including four contributed papers from invited speakers), 24 short papers (including one contributed paper), and 11 workshops. These papers represent a wide range of multidisciplinary work in mathematics, art, culture and pedagogy, from an equally diverse group of authors from around the world. We are certain to have another stimulating week of inspiration and dialogue.

The Bridges Visual Art Exhibition is always a highlight of the conference. Robert Fathauer, the curator, collected submissions. He then assembled a Jury that decided on the works to include in the exhibition. This year we will feature the work of 40 artists working in a variety of visual media. One design by Brian Evans was chosen as the cover image for the Proceedings. The cover was expertly designed by Jeff Rutzky.

In organizing and producing a Bridges Conference, the Bridges Organization relies on the concerted volunteer efforts and general goodwill of a large number of people. The members of the board of the non-profit Bridges Organization work on aspects of conference planning and management throughout the year. Robert Fathauer has worked tirelessly to run the Art Exhibition, as well as hosting the registration website. Mara Alagic and Paul Gailunas oversee the workshops. This year we received support and participation from Mary Hofstetter, the president of The Banff Centre, Barry Schiffman, the Director of Music Programs, Kerry Stauffer, the Director of the Creative Electronic Environment, Leslie Vanderzwet, General Manager, and Kameko Higa, the manager of Conference Services. Also, Bridges would never have made it to Banff

without the support of Nassif Ghoussoub, the director of BIRS, along with BIRS Station Manager Brenda Williams, Programme Coordinator Wynne Fong, and Administrator Danny Fan. To these people we offer our sincere gratitude. They have helped make the conference an event that many of us look forward to all year.

Most of all, we would like to thank everyone who has made the trip to Banff to participate in this year's conference. We look forward to talking to you, seeing your work, exchanging ideas with you, and building one more bridge to another source of inspiration.

The Bridges Organization Board of Directors  
<http://www.BridgesMathArt.org>