

# Combinatoria Poetica

## Counting and Visualizing Rhyme Patterns in Sonnets

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### Abstract

In this paper visualizations of end rhyme patterns of poems, including sonnet sequences, are studied. First, a notation for the grouping structure and end rhyme pattern types of poems and sonnet sequences is developed. Then the rhyme patterns of poems with even rhymes are constructed and their counts computed. Cascading this construction over rhyme groupings leads to counts for a variety of sonnet forms. The structure and counts are visualized for two types of sonnets, as trees that are drawn in a circularly symmetric manner. Finally, systematically generated rhyme patterns for sections of sonnets as well as rhyme patterns of sequences of sonnets from the literature are visualized as color bands. These bands provide a holistic visual overview that can give insight into the structure of poem sequences that may span hundreds of lines. Different color assignments can also be used to exhibit and enhance the visual beauty inherent in rhyme patterns (see website [6]). Mathematica 6 is used to create counts, summary tables and images of end rhyme patterns.

### 1. Introduction

In this paper principles and algorithms are described that can generate, count and visualize the number of possible end rhyme patterns for poems with paired rhyme patterns in general and sonnets in particular. Imposing different restrictions on the sound patterns for end rhymes and on the total number of end rhyme sounds in a poem allows counting and tabulation of a wide range of possible constructions of poems and sonnets that fit the specified end rhyme patterns. Even though a number of poets across the centuries have lamented the constrictions that the sonnet form places on their creative powers, it turns out that types of end rhymes for sonnets, even those close to the traditional forms, permit large numbers of patterns. Indeed the numbers are so large that they appear to contradict this sentiment expressed by some poets. Thus two complementary goals guide this paper: first to understand why the number of possible end rhyme patterns is so large when the rules governing sonnets seem so strict, and secondly to provide visualizations for types of sonnets generally and for sequences of sonnets in the literature that permit one to see at a glance end rhyme patterns used by poets. This may provide a new tool for literary criticism.

In Section 2 the underlying concepts for the structure types of poems that are the topic of this paper are introduced. I cite a number of sonnets that demonstrate the flexibility of this poetic form and provide a hint of the surprisingly large number of end rhyme patterns that 14 lines permit. The sonnet form also sets the stage for developing a notation for the structure types of poems that takes into account the grouping into subsections as well as conditions for the end rhyme sound patterns. I will discuss interrelationships and dependencies between some of the types.

Section 3 first considers poems with an even number of lines and paired end rhyme sounds. Since a poem of  $2n$  lines can have between 1 and  $n$  end rhyme sounds, the counts of end rhyme patterns can be arranged to form a number triangle with rows denoting the length of a poem and columns denoting the

number of sounds occurring in the poem. I describe an algorithm to generate all end rhyme patterns for poems with even rhymes and show tables of end rhyme counts. Then I apply the algorithm to Shakespearean and Petrarchan sonnets and visualize their end rhyme structure and counts in trees.

Section 4 presents the visualization of end rhyme patterns as bands of gray scale strips. One end rhyme pattern is displayed on one line so that, for example, 14 horizontal squares visualize a sonnet, and the patterns of a sequence of poems are displayed as vertical columns of horizontal strips. I show bands for all possible patterns of three particular end rhyme pattern types as well as bands for the rhyme patterns of a sonnet sequence from the romantic period in English literature. These visualizations extend into greater depth the visualizations created in [3].

In Section 5 I list formulas related to the counts of end rhyme patterns for even rhymed poems and suggest directions for further investigations.

## 2. A Classification of End Rhyme Pattern Types in Poems and Sonnets

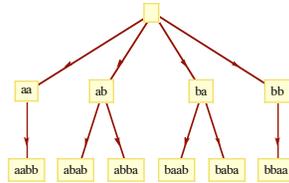
A sonnet is a poem of 14 lines. The analysis begins by assuming that an end rhyme sound is always paired with the end rhyme sound in another line. In some traditions end rhyme sounds may appear in triplets; we will consider those rhyme patterns also. I will follow the literary tradition that labels end rhyme sounds consecutively "a", "b", "c", etc., where "a" represents the first end rhyme sound in a sonnet, "b" the second, "c" the third, and so on.

While I show in this paper that the possible variations in the 14-line rhymed poem are immense it is useful to mention the two major types of sonnets described in standard accounts of this poetic form. In the **Petrarchan** sonnet pattern the 14 lines of the sonnet are commonly organized into a leading 8-line **octave** usually end-rhymed "abbaabba" followed by a final 6-line **sestet** that permits more variety but often is rhymed "cde cde" with three sounds or "ccd cdd" with two sounds. In the **Shakespearean** sonnet pattern the 14 lines of the sonnets are commonly organized into three 4-line **quatrains**, usually end-rhymed "abab cdcd efef", followed by a **couplet**, typically rhymed "gg". I will refer to the end rhyme patterns stated above, "abba abba cde cde" and "abab cdcd efef gg", as the standard patterns. If all poets adhered to these standards there would be no need for or interest in counting or studying all possible variations or actual poetic forms. However, a variety of rhyming patterns have been used in both traditions as well as patterns that depart sharply from these forms, demonstrating that there are venues of expression within the sonnet form that are not apparent at first sight. Indeed the number of possible rhyme patterns in a 14-line poem that a reader will recognize as a sonnet is surprisingly large as will become clear.

Despite conventional accounts of the two standard sonnet forms, in practice the rhyme patterns in sonnets have varied widely. A sonnet may not rhyme at all or use up to seven sounds. Each of these cases actually have been used by poets. Here are examples of each of the eight possible counts of end rhyme sounds with brief references:

No rhyme sounds:	abcdefghijklmn	Christian Gryphius	(see [1] p. 106)
One rhyme sound:	aaaa aaaa aaaa aa	Sir Philip Sidney	(see [2] p. 171)
Two rhyme sounds:	abba abba bba abb	Philander von der Linde	(see [1] p. 104)
Three rhyme sounds:	abab baba acca cc	Sir Philip Sidney	(see [2] p. 187)
Four rhyme sounds:	abba abba cdd ccd	William Wordsworth	(see [7] p. 187)
Five rhyme sounds:	abba abba cde cde	Wolfgang von Goethe	(see [1] p. 137)
Six rhyme sounds:	abba cdcd ebf ebf	Rainer Maria Rilke	(see [8] p. 30)
Seven rhyme sounds:	abab cdcd efef gg	William Shakespeare	(see [11] p. 387)

Some terminology is required in order to be able to classify different types within the sonnet form. The first quatrain in a sonnet will be called the *leading quatrain* and any subsequent quatrain will be called an *interior quatrain*, even if it closes a sonnet. When a sonnet is structured into an octave and a sestet then the octave tends to be the leading section of the sonnet. Therefore, I will follow this literary preference and consider octaves as leading octaves. In order to understand the complexity of counting end rhyme patterns I start with a visual arrangement of rhyme patterns for leading and interior quatrains with two end rhyme sounds “a” and “b”. The left half of the symmetric tree represents the three possibilities for a leading quatrain. Since an interior quatrain may start with sound designator “b” rather than “a” it is represented by six possibilities shown in the entire tree.



**Figure 1:** *Tree for Quatrains with Two End Rhyme Sounds.*

Counting possible end rhyme patterns for sonnets becomes much more intricate than the tree in Figure 1 may suggest since sonnets are divided into a number of sections of various lengths that determine the constraints on the end rhyme patterns. The following five examples of the leading octave sectioned into two quatrains or taken as a whole with two (Examples 1, 2 & 3) or with four (Examples 4 & 5) end rhyme sounds, respectively, demonstrate the structural possibilities and large variance in counts. The counts were computed using the algorithm described before Table 2 in the next section.

- 1: two paired sounds in each quatrain: 18 possibilities from "aabb aabb" to "abba bbaa"
- 2: paired sounds in each quatrain: 31 possibilities from "aaaa aabb" to "abba bbbb"
- 3: two paired sounds across the octave: 63 possibilities from "aaaaaabb" to "abbbbbba"
- 4: two paired sounds in each quatrain: 9 possibilities from "aabb ccdd" to "abba cddc"
- 5: four paired sounds across the octave: 105 possibilities from "aabbccdd" to "abccddcb".

As noted above, the 14 lines of a sonnet usually are divided into several sections, for example, into the 4-4-6 or the 4-4-4-2 groupings of the Shakespearean sonnet, or into the 8-6 or 4-4-6 groupings of the Petrarchan sonnet, and these divisions determine different kinds of rhyme patterns. In the remainder of this section I introduce a formal description for these (structured) divisions, count them and describe in more detail dependencies for those closely associated with sonnets.

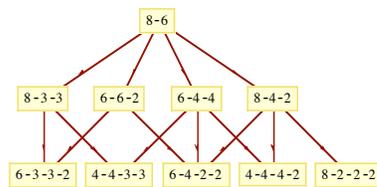
A poem may be thought of as a sequence of groups where a *group* is a sequence of contiguous lines. For a sonnet specifically, each group may be of length 2, 3, 4, 6, 8, 10, 12, and 14 with the constraint that the sum of the group lengths adds up to 14, the length of a sonnet. I will refer to any such sequence of groups as a *grouping type* or, for short, a *type* for a sonnet. In mathematical terms, a type is a partition of the number 14 into a set of  $k$  positive numbers  $\{g_1, \dots, g_k\}$  such that their sum equals 14. For example, 4-4-4-2 and 8-3-3 as well as 7-7 or 9-5 are partitions of the number 14 that have been used by poets as groupings in sonnets although the last two appear only rarely, see ([5], pp.210-211 and 283-284).

Of specific interest are grouping types that contain group lengths from the set  $\{2, 3, 4, 6, 8\}$ . These five numbers represent group lengths that are commonly used in sonnets. The computational task is to produce all possible ways, subject to the constraints imposed by the set of group lengths to be used for a type, to partition the number 14 into groups of smaller numbers that add up to 14. In Table 1 the rows represent six sets of group lengths that may occur in a grouping type for a sonnet. The five columns represent the maximum number of groups that may occur in a type and the last column indicates that any number of groups is permitted. Each table entry is the number of different types for a given set of group

lengths and a maximum number of groups in a type. For example, there are 7 different types using lengths from the set  $\{2, 4, 6, 8\}$  and consisting of at most 4 groups such as 8-6, 6-4-4 and 4-4-4-2. Note that the counts of types in successive columns across a row from left to right subsume the counts from the previous column.

	1	$\leq 2$	$\leq 3$	$\leq 4$	Any
4, 6, 8	0	1	2	2	2
3, 4, 6, 8	0	1	3	4	4
2, 4, 6, 8	0	1	4	7	11
2, 3, 4, 6, 8	0	1	5	10	17
2, 4, ..., 12, 14	1	4	8	11	15
1, 2, ..., 13, 14	1	8	24	47	135

**Table 1:** Sonnet Types Classified by Groups.



**Figure 2:** Dependency Structure of Some Sonnet Types.

In order to see the different structures in the sonnet form I visualize the 10 common grouping types for sonnets represented in row 4, column 4 of Table 1 through their dependencies. The fact that in this case there are 10 types of sonnets demonstrates the wide variety of structural constraints possible in this short poetic form. I will use the concept of partial ordering together with its standard pictorial representation in Hasse diagrams to render the 10 types as a directed graph in Figure 2. Grouping  $g_2$  is said to be *directly below* grouping  $g_1$  if  $g_2$  can be obtained by partitioning one number of  $g_1$  into two numbers. For example, grouping 4-4-4-2 is directly below 8-4-2, but it is not directly below 8-6, though it is below 8-6 in a two-step refinement. I normalize the notation for groupings so that the group lengths decrease or stay constant as one proceeds from left to right through the grouping type. Therefore, I write 6-4-4, the type of the Petrarchan sonnet, that is written more commonly as 4-4-6 indicating the actual quatrain-quatrain-sestet sequence of the poem. In the partial order of Figure 2 each vertex is labeled with its grouping pattern and the directed edges represent each direct refinement step.

### 3. Counting Rhymes in Poems and Visualizing Rhyme Count Structures through Trees

The central topic of this section is the generation and enumeration of end rhyme patterns of poems that have an even number of lines in which end rhyme sounds have *even multiplicity*, that is, each sound occurs on some even number of lines in the poem. Such rhymes are called *even rhymes*. When an end rhyme sound occurs exactly twice that rhyme sound is called a *paired rhyme*. Using the grouping notation introduced in the previous section, the sets of poems of  $2n$  lines, for some integer  $n$ , that have exactly  $k$  or at most  $k$  end rhyme sounds are denoted by  $(2n)_k^e$  and  $(2n)_{\leq k}^e$ , respectively, where  $1 \leq k \leq n$  and the upper index “e” denotes “even”; the absolute value expressions  $|(2n)_k^e|$  and  $|(2n)_{\leq k}^e|$  denote their counts. When  $k=n$  all rhymes must be paired and a numeric upper index is used such as  $(4)_2^2$ .

The grouping types in the center of the partial order in Figure 2, particularly the two dependency chains 8-6 to 6-4-4 to 4-4-4-2 and 8-6 to 6-4-4 to 4-4-3-3 have special significance since these end rhyme patterns for sonnet types include and generalize the Shakespearean and Petrarchan sonnet forms. Furthermore, many poets who write rhymed verse use even multiplicities of the rhymed sounds with pairs dominating. This suggests the following computing and visualizing tasks of possible end rhyme patterns in the sonnet form and their literary use:

- (1) Generate all possible end rhyme sound patterns for all poems of  $2n$  lines that contain between 1 and  $n$  end rhyme sounds of even multiplicity, that is, compute the set  $(2n)_{\leq k}^e$  for  $1 \leq k \leq n$ , specifically for  $n \leq 14$ , and provide the associated counts for the end rhyme sound patterns.
- (2) Generate all end rhyme patterns for the Shakespearean and Petrarchan sonnet forms. Visualize counts of end rhyme patterns for those sonnet forms in a structural manner by trees.
- (3) Visualize all possible end rhyme patterns as colored bands for sections of poems such as sestets and

octaves as well as for end rhyme patterns of sequences of sonnets from the literature.

For the first task I begin with a description of the generating process of end rhyme patterns. The process starts with the set consisting of the single one-line pattern “a” and successively generates longer “partial” poems until the desired number of lines,  $2n$ , for some chosen value of  $n$  is reached. The essential test for extendibility of a partial poem  $pp$  with sound  $s$  consists of checking that (i) sound  $s$  either already occurs in  $pp$  or that it is the (next alphabetic) sound after the maximum sound occurring in  $pp$ , and (ii) that adding sound  $s$  still permits matching evenly all sounds in  $pp$  and sound  $s$  when the designated maximum length  $2n$  for the poem has been reached. For example partial poem “abba” may be extended with either “a”, “b”, or “c”, but not with “d”, while partial poem “abcade” may be extended only with “b” or “d” if maximum length of the poem is 8 and end rhymes must be paired.

Implementation of this algorithm produces the list of all end rhyme patterns. Grouping the list by number of end rhyme sounds and counting the sizes of those lists creates the triangle of numbers displayed in Table 2. Since the numbers in the table represent poems of even length up to 14 lines the last row in the table includes most types of evenly rhymed sonnets. The entries in the last column of the table are the sums of all entries in the respective rows.

	1	2	3	4	5	6	7	sum
2	1	0	0	0	0	0	0	1
4	1	3	0	0	0	0	0	4
6	1	15	15	0	0	0	0	31
8	1	63	210	105	0	0	0	379
10	1	255	2205	3150	945	0	0	6556
12	1	1023	21120	65835	51975	10395	0	150349
14	1	4095	195195	1201200	1891890	945945	135135	4373461

**Table 2:** The Triangle of Even Rhyme Patterns by Number of Distinct Sounds.

A number of columns and diagonals in this Triangle of Even Rhyme Patterns are initial segments of sequences listed in The On-Line Encyclopedia of Integer Sequences (OEIS) [10]. If the entries in Table 2 are labeled  $w(n, k)$ , where  $n$  represents poems of length  $2n$  and  $k$  the number of sounds in the poems, correspondences between some number sequences in the triangle and initial segments of sequences listed in OEIS exist as follows:

column 2	$w(n, 2) / 3$	$4^{n-1} - 1$	sequence A002450,
column 3	$w(n, 3) / 15$		sequence A002451,
sum column	$\sum_{i=1}^n w(n, i)$		sequence A005046,
diagonal	$w(n, n)$	$(2n-1)*(2n-3)*\dots*3*1$	sequence A001147,
sub-diagonal	$w(n+1, n)$		sequence A001880.

For the remainder of this section I want to turn my attention to the rhyme pattern counts in the Shakespearean and Petrarchan sonnet forms. The next two tables show counts of rhyme patterns for poems such as sonnets that are constructed as sequences of blocks. The numbers were calculated by cascading the function that generates end rhymes patterns for a single group over the sequence of groups that form the indicated type of poem. The two tables demonstrate the huge variety of end rhyme patterns that are possible even under the constraints of quatrain and sestet sequences.

Table 3 shows the rhyme counts for poems that consist of sequences of quatrains  $(4)_2^2$  that have two sounds in each quatrain with repetition of sounds across quatrains permitted, that is, the poems satisfy the grouping type 4-4-...-4. The rows in the table indicate the number of quatrains and the columns the number of end rhyme sounds in each grouping. The counts in the table are listed through sequences of four quatrains, that is, 16 lines at maximum, since the counts for rhyme patterns of Shakespearean and

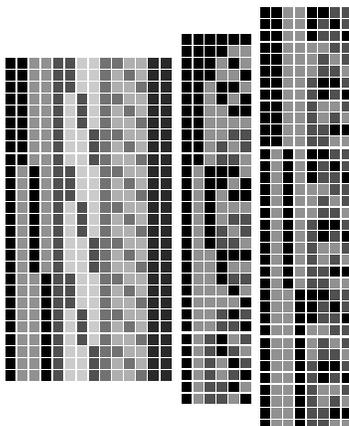


They visualize the internal multiplicative and additive structure of the counts of the end rhyme patterns for the Petrarchan (4-4-6) and Shakespearean (4-4-4-2) sonnet forms. The images are drawn as concentric trees with the root in the center representing the leading quatrain and each ring representing the next group in the grouping type. Each node in the trees contains two indicators. The upper set denotes the number of end rhyme sounds occurring in the sequence of groups that the node represents and the lower number gives the respective total count of patterns for that sequence of groups. For example, the node  $\begin{bmatrix} \{2,2,3\} \\ 720 \end{bmatrix}$  in Figure 3 denotes a count of 720 for sonnets with two sounds in the leading quatrain, two sounds across the first two quatrains and three sounds total for the sonnet. Adding counts in appropriate nodes in the outermost rings of the two trees produces the counts in the first two rows of Table 4.

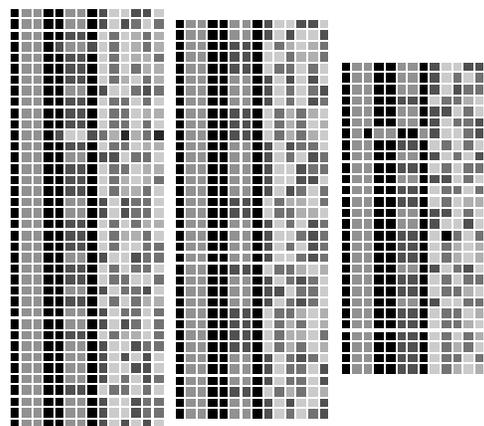
#### 4. Visualizations of End Rhyme Pattern Types and Sonnet Sequences as Bands

In this section I show visualizations of two types of end rhyme patterns. In one visualization all possible patterns subject to stated constraints are shown side by side. The other visualization presents an actual sequence of sonnets from the literature to show how one poet used some of the rhyme patterns. Each end rhyme sound in a poem is associated with one color and the same color assignments for the sequence “a”-“g” of end rhyme sounds is used across sequences of poems. Each poem is shown as a horizontal strip of colored squares with the color for the end rhyme sound of the first line on the left and for the last line on the right. A sonnet will be represented as a strip of fourteen squares in at most seven colors. The visualization of a sequence of poems is a vertical band of these strips. Since the first end rhyme sound in a poem is always represented by sound symbol "a", a visualization of all poems in a sequence starts with the same color or gray tone when using a fixed color or gray scale assignment for each sonnet in a band.

Figure 5 shows the gray scale bands for three different end rhyme pattern types, arranged from smallest to largest number of possibilities. The left-most band represents the 27 possible rhyme patterns for sonnets of type  $(4)_2^2 - (4)_2^2 - (4)_2^2 - (2)_1^2$  that have seven end rhyme sounds (see Table 4, row 2). The band shows the Shakespearean rhyme pattern “abab cdcd efef gg” right in its center as the 14-th strip. It is interesting to speculate why Shakespeare chose the middle pattern #14 in the lexicographic ordering of the band. One reason may be that it is the one pattern among the 27 rhyme patterns with the maximum number of 12 sound changes given the structural constraint of a final couplet in the sonnet. The middle band shows the 31 possible rhyme patterns for sestets of type  $6_{\leq 3}^e$  with 1, 2 or 3 paired end rhyme sounds (see Table 2, row 3). The right-most band shows the 36 possible rhyme patterns for a pair of quatrains  $(4)_2^2 - (4)_2^2$  with three rhyme sounds (see Table 3, row 2).



**Figure 5:** 7-Sound 4-4-4-2 Sonnets,  $\leq 3$ -Sound Sestets & 3-Sound Quatrain Pairs



**Figure 6:** Wordsworth’s “Ecclesiastical Sketches” (as published in 1822)

Figure 6 shows the gray scale bands for the three parts of William Wordsworth's "Ecclesiastical Sketches", published in 1822 (see [7], pp. 137-204). The parts consist of 38, 36 and 28 sonnets, respectively, and the parts are shown as separate bands side by side. This image represents 102 x 14 = 1,428 lines of verse. It is immediately apparent that the poet used the same rhyme pattern, with a single exception (III.7), for the leading quatrain and two patterns, with three exceptions (I.4, I.12, III.7), for the second quatrain. The expressive rhyme variations occur in the concluding sestets of the sonnet sequence.

## 5. Further Work and Acknowledgements

In conclusion I would like to suggest two directions for further mathematical and literary inquiry. The first is motivated by the numbers  $w(i+k, k)$ ,  $0 \leq i$  and  $1 \leq k$ , in Table 2. The non-zero entries in the  $k$ -th column of that table appear as the initial values of the coefficients in the following power series:

$$G_k(x) = \prod_{j=1}^k \frac{2j-1}{1-j^2x} = \frac{1}{2^{k-1}k!} \times \sum_{i=0}^{\infty} \left( \sum_{j=1}^k (-1)^{k+j} \binom{2k}{k+j} \cdot (j^2)^{k+i} \right) x^i = \sum_{i=0}^{\infty} w(i+k, k) x^i \text{ for all } 1 \leq k.$$

For example, coefficient  $w(2+5, 5)$ , that is, the value in row 7 and column 5 of Table 2 evaluates in closed form summation as:  $w(2+5, 5) = \frac{1}{1920} (210 * 1^7 - 120 * 4^7 + 45 * 9^7 - 10 * 16^7 + 25^7) = 1,891,890$ . The

numbers of even rhymed patterns need to be related to the closed-form expressions for the coefficients in this power series by a combinatorial argument. The second direction would be to extract end rhyme words from poems and sequences of sonnets from a broad cross-section of the literature, to construct their associated colored bands and use the patterns evident in the bands as basis for comparative and interpretive studies.

Finally, I would like to thank the reviewers for their comments and suggestions that helped me improve this paper significantly.

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