# Workshop on The Fusion Project: Bridging Art Museums and Middle School Math Teachers

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#### Abstract

The Fusion Project is a research program under development at the University of San Francisco, envisioned by Philip Wagner and directed by Benjamin Wells. With the collaboration of the Fine Arts Museums of San Francisco (and their de Young Museum), it seeks to bring art to the math classroom and math students to the art museum. In this Bridges Workshop, we shall explore the art of the de Young Museum and the mathematics it stimulates. Participants will also learn how to align test problems, outcomes, and works of art. Members of the workshop will have the opportunity to extend and improve Fusion Project instructional materials (and the program itself). The workshop is especially geared to middle school math teachers and to those who are interested in replicating or adapting this program.

## 1. The Fusion Project Workshop: Introduction and Purpose

Philip Wagner created The Fusion Project [1] (FP) as a collaboration between art museums and middle school mathematics classrooms. The main idea of FP is to engage middle school students in learning some critical math concepts by discovering them displayed in artwork and architectural components of a major art museum. In 2006 Phil approached the Fine Arts Museums of San Francisco (FAMSF) with the idea, and Sheila Pressley, FAMSF Director of Education, saw the value to students and to the museum. She immediately wrote a letter of support for FP. FAMSF defined the initial step of the project as an analysis of their permanent collections' potential for supporting mathematics instruction. They contracted with Benjamin (Pete) Wells to perform alignments of works of art with targeted California 7th grade math standards; these arose from aligning the standards with math test results. Thus began a unique program of using the art museum to enhance standard mathematics curricula in middle schools. The claim of uniqueness is based on surveying teachers, districts, and museums in many states, four countries, and numerous web pages. It is not an exceptionalist claim: we would love to find others doing this work. Indeed, spreading the ideas is the main purpose of this workshop. For more background on FP, see [2–5].

#### 2. The Fusion Project at FAMSF: The Rest of the Stories

Pete's FP report to FAMSF [4] comprises seven *Stories*, later expanded into *Encounters* [5]. These are described in [2], which includes material from three of the seven topic clusters. The remaining four Stories are given below, abbreviated and slightly reorganized from the original form. The reader should keep in mind that these are outlines, like screen treatments, not finished instructional materials. Two of the Stories/Encounters in [2] are much closer to that. All art is from the de Young Museum; works not pictured here may be seen in [2], and there are many additional works listed in [4].

**Story 2. Fractions, percent, and breakdown.** You enter the gallery that is the new home of the *Piazzoni Murals* (Fig. 1), set into identical alcoves. The *Land* side shows inland scenes, the *Sea* side gives coastal views. It is hard to grasp all of them, and small groups from your math class gather before different scenes. The teacher notices that 1/6 of the class is looking at 1/4 of the murals and these two numbers are the same. Luckily the number of mural wall alcoves is 6 + 6 = 12, and that makes for easy fractions. Harder fractions come from breaking down a mural into sky, water, and land portions.



**Figure 1.** *Piazzoni Murals Gallery* **Figure 2.** Dinner for Threshers<sup>1</sup> (Wood), Burning of LA (Petlin)

*Math questions related to artworks:* Problems, skills, and techniques exercised by answering the questions: arithmetic (and maybe some algebra) with fractions, percent, conversion of fractions to percent, easy addition of fractions with unlike denominators.

Piazzoni Murals Gallery by Gottardo Piazzoni (Fig. 1)

- A. If 1/6 of the class equals 1/4 of the mural niches, then how many students are in the class?
- B. One-sixth of your class would be how many niches? what fraction of niches? what percent?
- C. If we add on the niches next to the doors, what percent increase in the number of niches is this? Fractions with denominators 7 and 14 now arise too.
- D. Show different ways to pick 1/3 of the murals, 1/4 of them, 1/6 of them.

E. Fraction problems based on the ceiling slots, lights, and the window panels can also work.

*Dinner for Threshers* by Grant Wood (Fig. 2)

F. Fractions based on 12 again, or 6, 7, 14 by using one side of the table or the head and foot. Then 16 or 17 using the wait-staff, with more people in the wings.

*The Burning of Los Angeles, 1965-1967* by Irving Petlin (Fig. 2)

G. Fractions based on 13 strips in each of four panels. Provides parallel to weeks in a year. Also the figures are worth counting. There is only one white figure (or at most a few), so percent could be interesting. How about the fraction of strips with or without smoke?

**Story 5.** Lines and planes in space, ruled surfaces, and belts. There are only a few ways two lines can be placed in space: on top of each other, parallel, intersecting, or skew. Two or three planes also have limited ways to run along or into each other. With more lines, everything gets more complicated, but there are some neat curved surfaces (called *ruled surfaces*) that are built from lines. The skin of the Hamon Tower (see [2, Figs. 9, 10]) is such a surface. Another kind of ruled surface is a belt, where the lines run across the belt. These too turn up at the de Young Museum.

*Math questions related to artworks:* Problems, skills, and techniques exercised by answering the questions: how lines and planes interact in 3-dimensional space, recognition and qualitative analysis of ruled surfaces, different ways belts can be connected and run.

## *Raceme* by Daniel Clayman (Fig. 3)

H. A raceme is a flower shape, but this shape is clearly something more familiar. Cones are ruled surfaces, as are cylinders. Planes are ruled, so prisms and pyramids are too. Spheres?



Figure 3. Raceme (Clayman), UW84DC#2 (Deacon), Wuvulu ritual oil dish

*UW84DC* #2 by Richard Deacon (Fig. 3)

I. Multiple belts form this sculpture. If a belt is given a half-twist before it is joined in a loop, then it is a Moebius surface or band. This is used in industry for even belt wear, because a Moebius band has only one side. Does this sculpture have any one-sided belts in it?

Wuvulu ritual oil dish (Fig. 3)

- J. This shape is simple but it escapes simple description. The surfaces are not only ruled, they are also types of cylinders (the ruling lines are all parallel). Locate the three ruled surfaces. *Hovor II* by El Anatsui (Fig. 4)
  - K. This work is ruled in small areas, but is distorted overall by hanging and by variation in seal sizes. Find some ruled areas and some that are not.
- *Terrace overhangs/Mondrianic shadows* (Fig. 4)
  - L. Seen from the second floor, the huge Terrace overhangs cast shadows on the courtyards below. The shadows fall from bars arranged in space; how does that affect the lines in the shadows? How will the shadows change as the sun moves?



Figure 4. Hovor II<sup>2</sup> (Anatsui), Mondrianic shadows

Figure 5. 3 Gems (Turrell)—interior

**Story 6. Parallels, perspective, other projections, and the shining sun.** Parallel lines and planes abound in normal buildings. There are plenty in the de Young Museum too, but there are some surprising departures built right into the architecture, for example the *Hamon Tower* (see [2, Figs. 9, 10]). In making drawings look realistic, parallel lines (like horizontal ledges on buildings and road edges) are drawn so that, if extended, they meet at a point, called the vanishing point. This is the heart of creating *linear perspective*. Perspective drawing is a type of projection (think of the bulb in a film projector casting rays out in a cone through the film and onto the screen). There is plenty of perspective in paintings at the de Young Museum but there are several other interesting cases of projection too. One is the bright disk cast on the floor in *3 Gems* (Fig. 5); it is not an image of the sun, but an image of the oculus, the hole in the ceiling.

*Math questions related to artworks:* Problems, skills, and techniques exercised by answering the questions: Geometric and algebraic conditions for parallels; finding vanishing points in perspective drawings; visualizing objects in space; seeing projections and how projections can help us visualize the original objects.



Figure 6. From the Garden of the Chateau (DeMuth), Rainy Season in the Tropics<sup>3</sup> (Church), Igbo door

3 Gems by James Turrell (Fig. 5)

- M. The rings around the central blue/green disk are parallel. Why?
- N. At certain times of day, the oculus (the hole in the roof) is projected by the sun on the floor. Because the sun is so far away, its rays are nearly parallel. So the image of the hole on the floor is really the same size as the hole seen along the sunbeams (why? because the oculus is parallel to the floor). What does this say about the size of the hole from below?

From the Garden of Chateau by Charles DeMuth (Fig. 6)

O. This painting purposely distorts perspective. It harks back to the days just before the Renaissance when artists knew something about parallels appearing to meet in the distance but made it more complicated than it needed to be. See if you can find three violations of the rules of perspective here.

Rainy Season in the Tropics by Frederic Edwin Church (Fig. 6)

P. Circles can be parallel too. When does that happen?

Q. The space between the two rainbows is called Alexander's dark band. Describe it.

- Igbo door (Fig. 6)
  - R. There are numerous lines and line segments in this carving. But there seem to be only three different slopes plus vertical segments. How can you figure out the slopes?

**Story 7. Uniformity, symmetry, chaos, confusion, and the feeling of being lost.** Sometimes art is all about uniformity, but more often it breaks up order and introduces some element of diversity, even chaos or confusion. The de Young Museum features art lying across this entire range. But now we focus on the extremes: great sameness and great differences. The mathematics of sameness is called symmetry, the geometry of chaos is called fractal geometry, and the logic of confusion is called paradox. Any of these extremes can give the sense of being lost. If everything is the same (say, you are in the middle of *Strontium*, [2, Fig. 3]), every place looks alike (but not every direction). If you are on the crack in *Drawn Stone* (Fig. 7), it looks rough, but pretty much like any other place on the crack. If you are looking at *Hovor II* (Fig. 4), you might be looking up, or down, or sideways. People talk about losing themselves in art; in these cases, the loss feels very real.

Math questions related to artworks: Problems, skills, and techniques exercised by answering the questions: finding symmetry and uniformity; finding variety and scaling roughness; finding impossibility and perplexity.

Strontium by Gerhard Richter (see [2, Fig. 3])

S. This work has symmetry: one place is much the same as many nearby places. In particular, there is no difference between one ball and its neighbors of the same size. The picture can shift left or right a column, or up or down a row, and still look the same, but it "resists" being turned sideways because the rows of large balls are different than the columns of large balls (same idea for the small balls, too). This type of symmetry is called *translation* symmetry. Can you see some other translation symmetries in this work of art?

*Eucalyptus benches* by architects Jacques Herzog and Pierre de Meuron (see [2, Fig. 7])

- T. The benches have rotation symmetry. How much do you have to turn a set of 6 benches until it looks like the original (ignore the woodgrain)? How about in the other direction?
- 3 Gems (Fig. 5)
  - U. There is rotation symmetry on the floor, and chaos in the mixture of colors in the central disk. If the benches continued all the way around, and the archways were closed, can you describe other rotation symmetries?



Figure 7. Drawn Stone (Goldsworthy), Shadow Frieze (Cook)

Drawn Stone by Andy Goldsworthy (Fig. 7)

- V. This crack is like a coastline, a scaling fractal, a kind of roughness that has the same wiggliness from the second floor window, standing above it, or getting your face close to it. But it is not completely random, for it runs fairly straight across the tiles then turns in order to pass through the benches. Is it a loop? What percent of the tiles does it break? What percent of the benches? How could the artist crack stone in such a controlled way? Did he?
- Shadow Frieze by Lia Cook (Fig. 7)
  - W. There are symmetries that slide the shapes (ignoring the colors) up to the right and up to the left, plus others we can explore. Up close, there is a lot of confused color. At a distance, both this symmetry and this chaos disappear into a coherent picture of draped cloth, which is paradoxical. Can you find a place to stand where all these extremes balance?

## 3. Outline of Workshop Activities

Several activities will be offered. It is likely that there will be some parallelism, but participants will determine their own preferences for progressing through the topics. We believe this introduction to FP can benefit participants' own classrooms as well as the Fusion Project.

**I. Math from art.** Examples of art from the FAMSF collection will be shown on slides and posters, and participants will be invited to identify interesting math principles, concepts, and topics that could be supported by or discovered in the artwork. The outcome may be something like the stories outlined above. Our personal experience from taking tours of the de Young Museum with middle school and college math students (more than 50), math teachers, college professors, and interested nonmathematical friends has shown us three things: (1) there is always more to find in the art, (2) students are remarkably insightful, with teachers next, and (3) FP benefits immensely from the visitors' observations.

**II.** Alignments. Workshop participants will be invited to make their own alignments between math test problems and typical standards, and then between the top standards and works of art. This will be good practice if members are interested in replicating FP in other localities. It can also give one confidence when colleagues make judgments that correlate highly with each other's (and with one's own).

Please see Figs. 8–10 for examples extracted from the FP-FAMSF alignments. These charts are fragments of the full worksheets. In Fig. 8, the problems from a standardized test are associated with the math standards [6] for which they have affinity. The affinity rating depends on the number of incorrect answers to a problem; the pink standards have emerged as targets. Fig. 9 shows an alignment between works of art at the de Young Museum and standards. Fig. 10 shows how the artwork and math topics cluster into stories. These figures are extractions from the full alignments, and the exercises in the workshop will be similarly limited.

\varTheta 🔿 🔿 📑 Br09 sample alignments.xls																								
$\diamond$	Α	В	C	D	Е	F	G	Н	Т	J	K	L	М	Ν	0	Ρ	Q	R	S	Т	U	V	W	Х
			ALIGNING TEST PROBLEMS AND STANDARDS	Problem description	jiven pt and 4 line eqns, on which does it lie?	jiven pix, change in x or run of line (pos slope)	r = 4x, qt vs. gal, related to slope and y-cept	lope of graphed line	tumber line solution set for $ x  > 2$	-cept from given graph	lope of line parallel to given eqn	dentify run-walk-rest scenario of graph of dist/time	jiven pix, what is line eqn?	t-cept of a line eqn?	iys linegn 2 var 2 egn y = in both	dentify graph of y = -x^2	olve  x - 3  = 4	:xpand (4x + 2)(x +3)	implify 4(2x - 3) + 6x / 2	implify (8x^2y^5z^4)/(4xy^4z^4)	implify ((3a)^2b^3)/(6ab)	$5(x^2 - 1) > 2x+2$ equiv to? (no parens only)	then x/2 + 1\geq 4, which is true (in words)?	dm blifv (3x ^2v^3)(4x ^-1v^4z^5)
2			Test item #:		43	27	29	28	26	40	45	22	44	42	48	26	35	49	20	24	25	38	32	21
3			affinity weighted by low scores: 3 = many misses	corr:	6	7	7	8	8	8	8	9	9	10	10	11	5	6	7	7	7	9	10	1:
4			Standards math t	ype:	AG	AG	AG	AG	AG	AG	AG	AG	AG	AG	AG	AG	AL	AL	AL	AL	AL	AL	AL	AL
5	2	NS1.2*	+,*,-,/ rational numbers (ints, fracs, & term dec; (a/b)^m	11														3	2	2	2	1	1	_
6	10	NS2.3*	Multiply, divide, and simplify rational numbers by using exp rules.	4																2	2			
7	13	AF1.1	Use var, ops to write expr, eqn, ineq, sys eqn/ineq to verbal desc	1									1											
8	14	AF1.2	Use order of ops to eval alg exprns such as $3(2x + 5)^2$ .	14	3													3	2	2	2	1	1	
9	15	AF1.3*	Simplify num exprs by applying laws of ratl numrs (assoc++)	11														3	2	2	2	1	1	
10	16	AF1.4	Use algebraic terminology (e.g., va, eqn, term) correctly.	0																			_	
11	17	AF1.5	Represent quant rels graphically; interpret part of graph	7					2	2	2	1											_	
12	18	AF2.1	Interpt + int powers as repeated *, -int as /; simp, eval exprs w/exps.	4																2	2			
13	19	AF2.2	*, / monomials; take powers and whole roots.	4														3				1	_	
14	21	AF3.2	Plot 3D vois as ich of edge length, blase edge length.	0	-		-	-	_		-											_	_	
15	22	AF3.3*	Chapminearions, note dy is same for given dx; rise/fun = slope.	16	3	3	3	2	_	2	2		1									_	_	
16	23	AF3.41	Plot quants whose ratios are constant (ft/in); fit line, interpret slope.	11		3	3	2	_	2	_		1									_	_	
1/			officity totals		6	6	6		2	6		-	3	0	0	0	0	12	6	10	10	4	2	,
10			annity totals:		0	0	0		4	0		1	3	0	U	0	U	12	0	10	10		3	

Figure 8. Selected data from FP-FAMSF alignment of standards with test problems (see [4])

**III. Sharing FP techniques.** Workshop facilitators will demonstrate FP enhancements (see [2, pp. 5–7] and invite participants to try them out in the workshop. Some of this is classroom-tested material used in

previous teachers' workshops, including the training of our FP Teachers Advisory Group (TAG). The TAG has been largely responsible for the detailed material we'll share. It is still under active development, so participants' reactions will be most valuable.

Based on FAMSF experience with their program *Get Smart with Art* (a) the de Young [7], it is anticipated that several iterations of refinement will be required for a working set of encounter and enhancement materials. Even before that, we are organizing summer workshops for teachers. Then we can evaluate the materials in classroom settings, obtain feedback from the users, and move to distributable printed products, with implementation already underway.

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$\diamond$	Α	В	С	D	E	F	G	Н	1	J	К	L	М	Ν	0	P	Q	R S	т
1						Artist	Goldsworthy	Richter	architecture	architecture	Deacon	El Anatsui	Piazzoni	Turrell	Turrell	DeMuth	Petlin	Thiebaud	
2			ALIGNING ARTWORK AND STANDARDS	Works of art at the	de Young Museum	Work	Drawn Stone	Strontium	Hamon Tower	Eucalyptus benches	UW84DC #2	Hovor II	Piazzoni murals	3 Gems benches	3 Gems pavers	Garden of Chateau	Burning of LA	Diagonal Freeway	High math content
4								6		9	2	~	00	4	~	m	m	4	
5			Standards																
6	2	NS1.2*	+,*,-,/ rational numbers (ints, fracs, & term dec; (a/b)^m	0	8	6		1				1	1	1	1		1	-	
7	3	NS1.3	Convert fracs to decs and %s; use in est, comp, applics	27	5	7		1		1		1	1	1	1		1	-	
8	4	NS1.4*	Differentiate between rational and irrational numbers.	5	0	0													
9	5	NS1.5*	Know every rati is either a terminating or repeating dec; term->rat	0	0	0													
10	6	NS1.6	Calculate the percentage of increases and decreases of a quantity.	0	3	4						1	1	1	1				
11	7	NS1.7*	Solve probs on discount, markup, commission, profit; s & c interest	3	3	2				1			1						
12	8	NS2.1	Understand negative whole-number exps. *,/ exp w/comm base	0	0	0													
13	9	NS2.2*	Add and subtract fractions by factoring to find common denoms.	0	8	4						1	1	1	1				
14	13	AF1.1	Use var, ops to write expr, eqn, ineq, sys eqn/ineq to verbal desc	5	0	0													
15	21	AF3.2	Plot 3D vols as fon of edge length, base edge length.	0	0	0													
16	22	AF3.3*	Graph linear fons; note $\Delta y$ is same for given $\Delta x$ ; rise/run = slope.	0	7	3		1								1		1	
17	23	AF3.4*	Plot quants whose ratios are constant (ft/in); fit line, interpret slope.	16	11	4		1				1				1		1	
18	27	MG1.2	Construct and read drawings and models made to scale.	3	0	0													
19	28	MG1.3*	Use measures expressed as rates & prods; use dim anal to check.	0	0	0													
20	29	MG2.1	Use fmls to find perim & area of 2-3D: tri, quadril, circ, prisms, cyls	0	3	5		1		1	1		1		1				
21	30	MG2.2	Estim & compute area of complex 2-3D figs by decomp.	10	4	7		1		1	1	1	1		1		1		
22	31	MG2.3	Comp perim, sa, vol of 3D objs from rect solids; scaling vol, sa.	1	5	4		1		1			1		1				
23	32	MG2.4	Relate the scale changes in measurement to units and conv.	3	0	0													
24	33	MG3.1	Identify, construct c+se geom figs (alts, mp, diag, bisects, circs)	0	4	3		1				1						1	
25	34	MG3.2	Coord graphs to plot simple figs; detmn ln, area; trans/refl image	10	6	2		1										1	
26	35	MG3.3*	Pyth thm and converse; appls; experverif by measuring.	3	0	0													
27	36	MG3.4*	Conditions for congruence (general); meaning for sides, angles	0	0	0													
28	37	MG3.6*	Elems of 3D objs; skew lines; 3-plane intersections	0	2	4	1		1	1						1			
29	38	PS1.1	Var disp of data sets; stem&leaf box&whisker cf. 2 datasets	3	0	0													
30	39	PS1.2	2 num var on a scatterplot; informal descrp of dist and reln.	4	0	0													
32			High consonance with targe	t stan	dards														

Figure 9. Selected data from FP-FAMSF alignment of standards with artwork (see [4])

**IV. Building and truing FP.** Workshop members will be invited to contribute now and later to the growing body of FP materials. All those participating in the workshop will be urged to comment on FP and offer suggestions for improving it.

Our model proposed at the University of San Francisco is to pay teachers to take workshops at the de Young Museum in order to implement FP in their classrooms, with the assistance of FP-trained USF students and local peer teachers. We have a list of schools and teachers interested in such workshops, and by the time of Bridges 2009, we may already have held the first round of summer workshops. We hope

our Bridges workshop results in collaborations that can improve our USF program and lead to similar alliances with museums and educational institutions elsewhere.



Figure 10. Selected data from FP-FAMSF alignment of stories with artwork (see [4])

#### References

- [1] The Fusion Project, http://fusionprojectinfo.com/ —constructed by Cyndi Lott (Align PR Agency)
- [2] B. Wells and P. Wagner, *Bridging Art Museums and Middle School Math Classrooms*, submitted to Bridges 2009.
- [3] B. Wells and P. Wagner, *The Fusion Project: Bridging the Art Museum and the Middle School Math Classroom*, submitted to the Journal of Mathematics and the Arts.
- [4] B. Wells, *Math Stories at the de Young Museum*, private report to FAMSF, 2007, rev. 2009. Contact author for a copy. Some materials used are not yet circulating.
- [5] B. Wells, C. Potter, and D. Schattschneider, *Math Encounters at the de Young Museum*, private report to USF College of Arts and Sciences, 2008. Contact author for a copy.
- [6] California Department of Education, *Grade Seven Mathematics Content Standards*, in Mathematics Content Standards for CA Public Schools, Sacramento CA, 1997, pp. 31–35.
- [7] Get Smart with Art @ the de Young: http://www.famsf.org/fam/education/subpage.asp?subpagekey=19

<sup>&</sup>lt;sup>1</sup> Photo credit: Janet Haven, http://xroads.virginia.edu/~ma98/haven/wood/threshers.html

<sup>&</sup>lt;sup>2</sup> Inset photo credit: CRASH-candy, http://flickr.com/photos/crash-candy/2363892635/

<sup>&</sup>lt;sup>3</sup> Photo credit: Sillieee Bunnieee, flickr.com/photos/sillieeebunnieee/2758468099/