An Interdisciplinary First Seminar on Symmetry

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Abstract

The concept of symmetry, including what it means in mathematics, how it occurs in other disciplines, and how the mathematical ideas are applied to symmetry in art and architecture, is investigated in an interdisciplinary first seminar at Allegheny College.

The Freshman-Sophomore Program at Allegheny College

Allegheny College requires each of its students to complete a sequence of three First-year/Sophomore seminars that emphasize writing and speaking. Each faculty member who teaches a first-year seminar may choose his or her own topic, and interdisciplinary topics are encouraged. My course, "Symmetry through the Eyes of Mathematics", investigates the interdisciplinary nature of symmetry.

What is Symmetry?

On the first day, I begin with an exercise asking students to define what it means for a figure to possess symmetry. While most students begin by focusing on bilateral symmetry, the discussion naturally leads to an introduction of the four rigid motions of the plane.

I also assign for reading and discussion the introduction to *Patterns of Symmetry* [3], which is the proceedings of the 1973 "Symmetry Festival" held at Smith College. The reading foretells many of the ideas we discuss in my course: the universal nature of symmetry, the rigid motions, groups, the idea of analyzing symmetries in a pattern, and the fact that there are only finitely many possible symmetry types.

The reading also primes students for the first major paper and speech assignment, which asks them to investigate how symmetry occurs in any discipline other than art and architecture (which is discussed later in the course). Examples of possible subject areas include biology, chemistry, physics, dance, music, neuroscience, psychology, theatre performance, and literature. Students address, for their chosen field, what the concept of symmetry means, where and how symmetry occurs, the significance of symmetry (its purpose or how it is interpreted), and how and why symmetry is broken.

The Mathematics: Rigid Motions and Symmetry Groups

The major mathematical goals of the course are to understand how rigid motions are combined (multiplied), the concept of a mathematical group, and how we can describe the symmetry type of a figure in the plane. Many of the activities I use in class are, or are inspired by, tasks in Chapters 2-5 in Farmer's book *Groups and Symmetry* [2].

I begin with worksheets that I developed which ask students first to determine where a particular combination of rigid motions moves a given triangle, and then to conjecture a general result. My primary goal is to investigate what happens when a reflection is followed by another reflection. We discuss why any translation and rotation can be described as a product of reflections.

The language of rigid motions provides us with the usual mathematical definition of what it means for a figure in the plane to possess symmetry. Beginning with regular polygons and other simple finite shapes, the students investigate the multiplication (or group) tables of the symmetries of the figures and classify figures as having either cyclic or dihedral symmetry type.

Moving on to strip (or frieze) patterns and wallpaper patterns, the students discover why there are at most $2^4 = 16$ possible symmetry types of strip patterns. After some experimentation, they conjecture that only 7 of the 16 apparent possibilities actually occur. The facts about rigid motions investigated earlier in the course enable us to explain why some of the other possibilities cannot occur. Students classify a variety of strip patterns, using IUC notation. Since working out why certain combinations of symmetries cannot be present in a wallpaper pattern is mathematically very difficult, we focus on using the standard flow chart to analyze the symmetry types of these patterns.

Connections

As mentioned earlier, the first major assignment asks students to investigate the meaning of symmetry in disciplines other than art and architecture. Patterns and symmetry in art or architecture are the focus of the second major assignment. Students choose an artist or architectural style to examine; some examples (many taken from Farmer's book [2]) include the art of M. C. Escher, William Morris, or Robert Adam; Islamic art (including the Alhambra); Turkish art or architecture; African art or textiles; American Indian pottery, rugs, or beadwork; Asian carpets or pottery; Amish quilts; and Mayan art and architecture. Students are asked to include a summary of the history of the culture, artist, architect, etc., in question. They must also include and analyze examples of at least six symmetrically different patterns that are appropriate to their topic; any combination of wallpaper patterns, strip patterns, or finite figures is permitted.

In an effort to reinforce the message that symmetry is ubiquitous and expose students to ideas from other cultures, I also require students to read a portion of the chapter "Symmetric Strip Decorations" from Ascher's book *Ethnomathematics* [1]. This chapter compares the strip patterns found on the rafters of marae, the community and spiritual meeting places of the Maori, to the strip patterns found on Inca pottery.

References

[1] Marcia Ascher. *Ethnomathematics: A Multicultural View of Mathematical Ideas*. Brooks/Cole Publishing Company, 1991.

[2] David W. Farmer. *Groups and Symmetry: A Guide to Discovering Mathematics*. The American Mathematical Society, Providence, 1991.

[3] Marjorie Senechal and George Fleck. Patterns of symmetry. In Senechal and Fleck, editors, *Patterns of Symmetry*, pages 3–19. University of Massachusetts Press, Amherst, 1977.