

# Interactive Demonstrations with *Mathematica 6*

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## Abstract

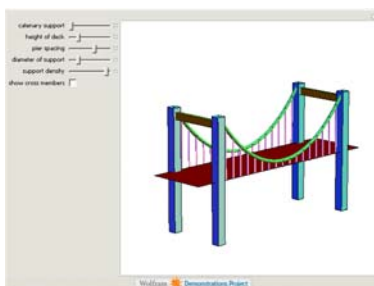
*Mathematica 6* was launched on 1 May 2007. The new features include a possibility of interactively manipulating 2D or 3D images. Wolfram Research Inc. maintains a site, where more than 2700 such demonstrations (April 2008) are available (<http://demonstrations.wolfram.com/>), among them are my own 230 demonstrations, from which I introduce several major categories (<http://demonstrations.wolfram.com/search.html?query=kabai&limit=20>). The Mathematica codes can be downloaded and used with the free MathPlayer also.

## The Catenary Curve

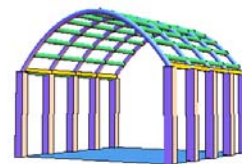
**Bridges and arches.** The curve a hanging flexible wire or chain assumes when supported at its ends and acted upon by a uniform gravitational force. The word catenary is derived from the Latin word for "chain." The equation was obtained by Leibniz, Huygens, and Johann Bernoulli in 1691 in response to a challenge by Jakob Bernoulli (Wolfram MathWorld). The application of the catenary to the construction of arches is ancient, the modern rediscovery and statement is due to Robert Hooke, who discovered it in the context of the rebuilding of St Paul's Cathedral, possibly having seen Huygen's work on the catenary (Wikipedia).

I prepared several demonstrations to show the application of catenary curves in bridges and structures. When the catenary is convex, you get an arch bridge configuration, with compression stress in the supports. When the catenary is concave, you can see a suspension bridge, with tensile stress in the supports, which would generally be made of stranded steel cables. Building frames can also be interpreted as special bridges. Two parallel rows of pillars are joined with beams shaped like catenaries to form a building framework.

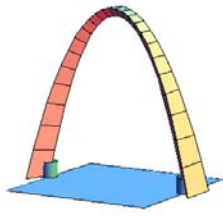
Three catenary curves are connected with polygons to form an arch with a triangular cross section. This is essentially the geometry of the 630-foot-tall (192 m) Jefferson National Expansion Memorial (sometimes called the Gateway Arch or The Arch) in St. Louis, Missouri. When two such arches are placed side-by-side, their appearance is very similar to McDonald's logo.



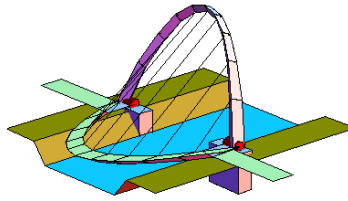
[Bridges with catenary support](#)



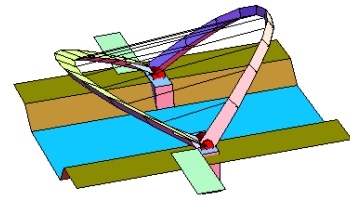
[Building frame](#)



[Jefferson National Expansion Memorial](#)



[Gateshead Millennium Bridge](#)

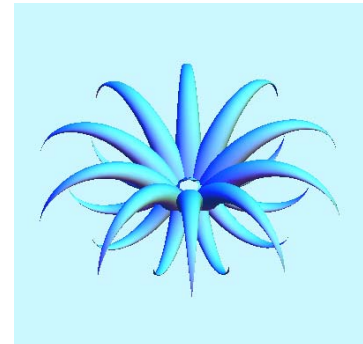
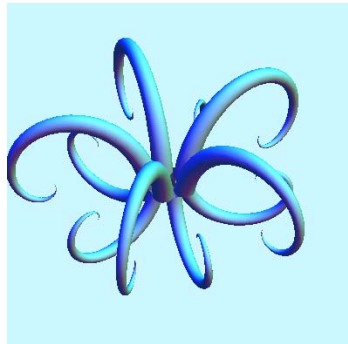
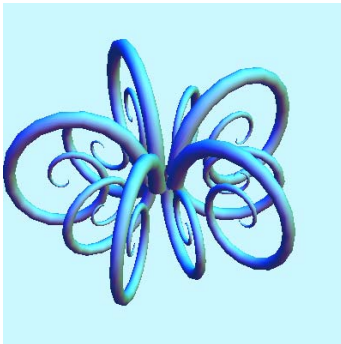


**Figure 1:** Bridges and arches

In case of the he [Gateshead Millennium Bridge](#), a pedestrian and cyclist tilt bridge, an inclined catenary arch supports a horizontal bridge deck with ropes. It spans the River Tyne in England between Gateshead on the south bank and Newcastle upon Tyne on the north bank. The arch and the bridge deck can be rotated together around large bearings, lifting the bridge deck to let small ships and boats pass underneath.

### Spirals

This is a reconstruction of the cover image of The Mathematica Journal, Volume 9, Issue 2. You can vary the length, diameter, number, and size of the spirals.



**Figure 2:** [Swirly](#)

**Snails** are produced by growing snails in pairs in 3D to produce ornamental structures. Use a central sphere to make the structure stable.

A **spiral** is revolved to form a surface **around a torus**.

A trapezoidal slab is selected between two adjacent polygonal spirals as a unit to construct **spirals of trapezoids**.

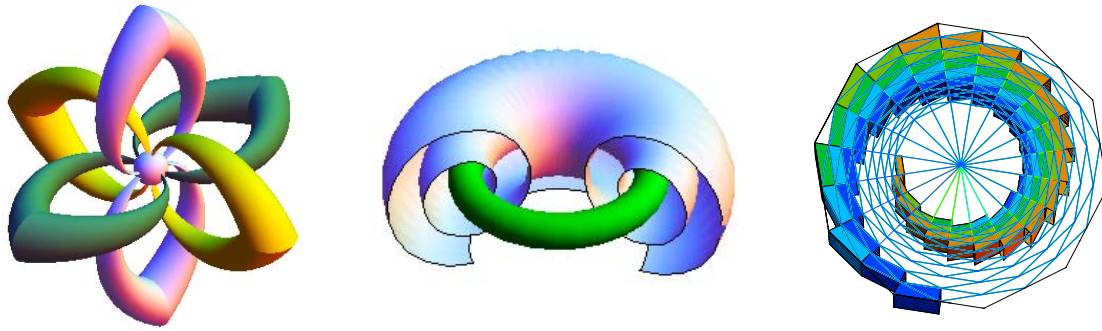


Figure 3: *Ornamental Snails, Wrapping a Torus and Spirals of Trapezoid*

**Pentagon spirals.** Draw a ring of ten pentagons. Place other pentagons inside or outside the ring, decreasing or increasing their size in proportion to the golden ratio.

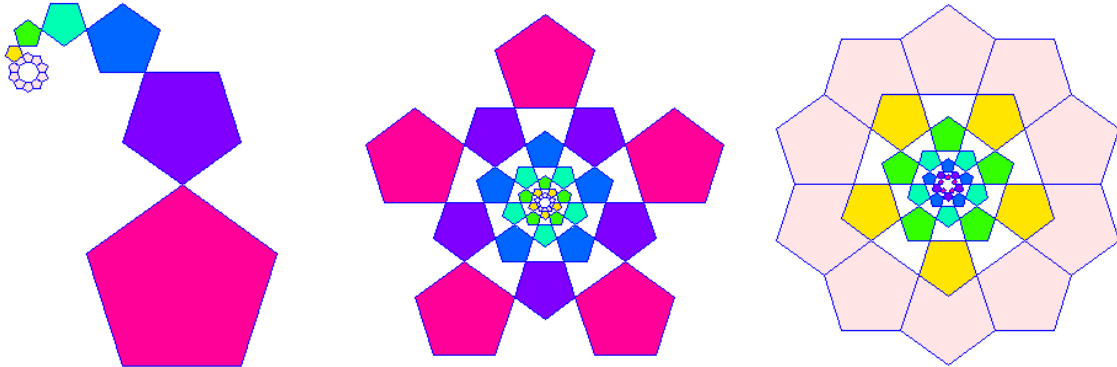


Figure 4: *Pentagon spirals*

Triangles are drawn so that their vertices are on the sides of the next larger triangle. This series of triangles is taken as a unit, which is distorted to fit  $k$  times in a  $k$ -sided regular polygon. The triangles can be replaced with another regular polygon, and the units can be moved in and out in a radial direction. Penrose kites are arranged in spirals within a pentagon; 12 such pentagons form the faces of a foldable dodecahedron

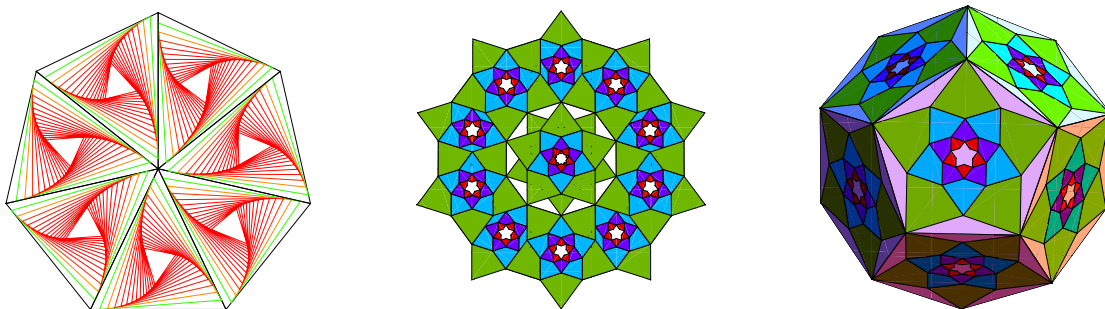
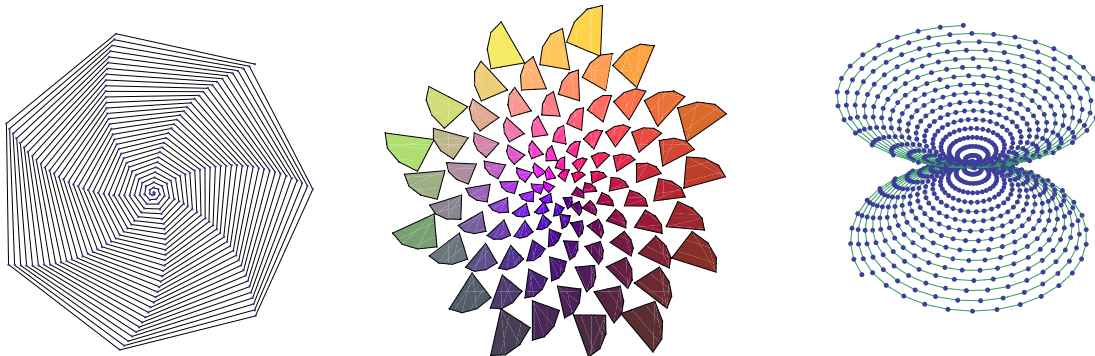


Figure 5: *Whirling Polygons and Kite Tiling and Dodecahedron*

**Points on a Spiral.** Points are placed on a spiral at regular intervals. You can vary the number of turns of the spiral (turn), the density of points ( $n$ ), and the advance angle of points (angle). Segments of a regular polygon are placed along a logarithmic spiral. The positions of the segments are determined by the

angular advance of the subsequent segments. The edges of the segments lie along various spirals. When the angular advance is proportional to the golden ratio, the number of spirals are Fibonacci numbers (8, 13, 21 in the default case). Many plants (including pine cones, leaves, and branches) exhibit this arrangement of segments



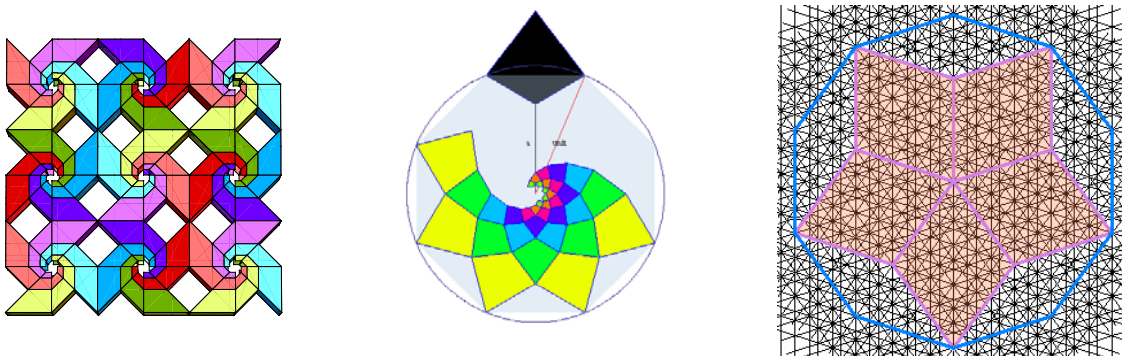
**Figure 6:** [Points on a Spiral](#), [Pine Cone](#) and [Point on 3D Spiral](#)

### Tiling

**Trapezoidal tiles** are arranged in a spiral, then four such spirals are used to form square tiles. Symmetry groups that can be observed in this tiling, includes  $p4m$ ,  $pmm$ , and  $p2$ . This tiling could also be used as a template to establish Escher style pictures with a head of horned animal as a unit.

**Kite tiles** are arranged so that two of its vertices coincide with two adjacent vertices of a regular polygon constructed on the unit circle. Let the other two vertices of the kite be on a radius bisecting that side of the polygon, one at an arbitrary distance "a" from the origin, and the other at a distance so that the different sides of the deltoid are in the ratio "1/a". Now place copies of this kite on each edge of the polygon, and tile the plane inside or outside the polygon with kites having sizes decreased or increased in proportion to "1/a".

**Pentagonal art template** is made from eight parallel lines are drawn with spacings  $\phi$ , 1,  $\phi$ , 1,  $\phi$ ,  $\phi$ , 1,  $\phi$ , where  $\phi$  is the golden ratio. Then more lines are added with the same spacings. These 16 lines plus a central line are repeated 10 times after rotating them by 36 degrees. The intersecting lines produce a network which can be used as a template for establishing various patterns having pentagonal symmetry (e.g., a Penrose tiling). The set of intersection points is similar to the X-ray diffraction pattern of certain quasicrystals.



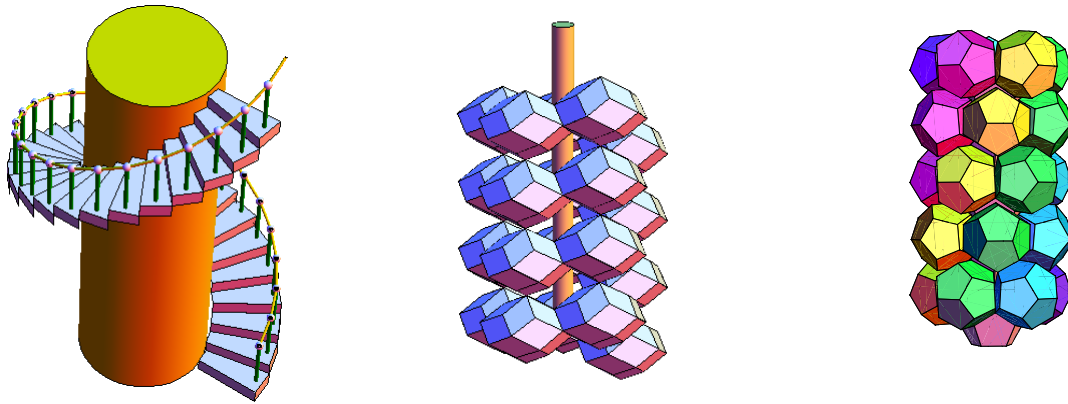
**Figure 7:** [Horn tiling](#), [Kite Tiling](#) and [Pentagonal art Template](#)

## Helices

A **helical staircase** is constructed around a central cylinder. You can control the size of the central cylinder, the height and number of steps, as well as the height of the railing.

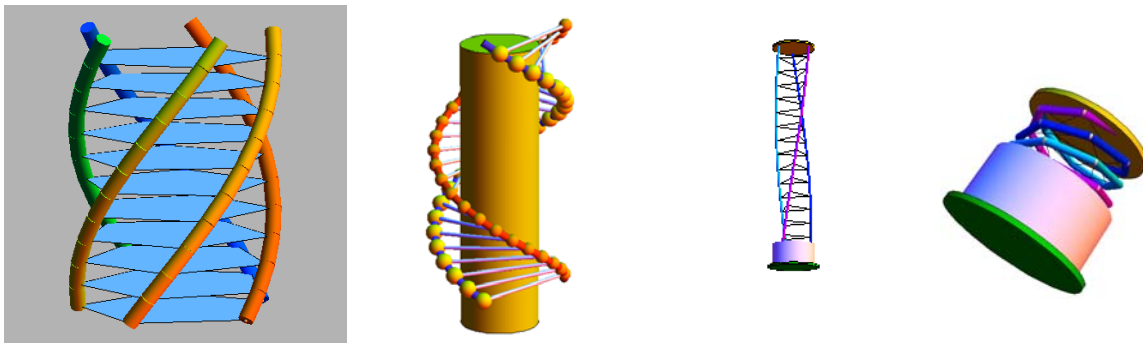
Two different **helices can be assembled from rhombic icosahedra** attached face to face. The default values of the controls produce a helix with small pitch. In this case the rhombic icosahedra are also aligned in five columns with connections at their tips. With a helix of rhombic rhombohedra the assembly becomes tubular. To produce the helix with large pitch, set the three controls (align, diameter, expand) all the way to the left.

Starting with a central **tower of dodecahedra** placed on top of each other, attach further dodecahedra around the tower. The outside dodecahedra seem to be aligned along helices.



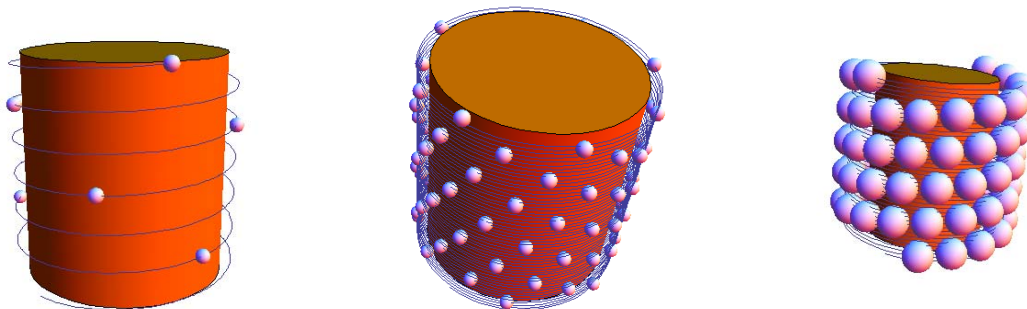
**Figure 8:** [Helical Staircase](#), [Rhombic icosahedra Helices](#) and [Tower of Dodecahedra](#)

Vertical cylinders are mounted on the vertices of a horizontal polygon to form a unit (floor), and such units are stacked. Then the cylinders are tilted so that their tops follow the circumscribing circle of the polygon. This is a geometrical model of extendable structures in industry, engineering, and biology. The structure resembles the geometry of DNA's double helix when there are 10 cylinders in a turn, the relative angular position of one helix to the other is 144 degrees, and the pitch is about 0.5. This is the basic geometry of the AstroMast® device, produced by Astro Aerospace, a business unit of Northrop Grumman Space Technology. It is used to deploy antennas in space.



**Figure 9:** [Helices](#), [Double Helix](#) and [Astromast](#)

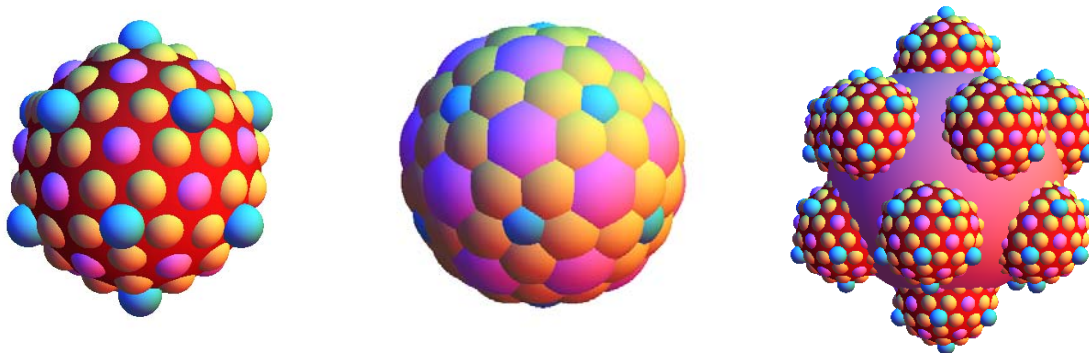
**Spheres** are placed with equal angular spacing **on a helix**. At various angular spacings the spheres seem to be arranged along opposite helices independent of the generating helix, showing how the parts of certain biological structures are arranged on a cylinder.



**Figure 10:** *Spheres on a Cylinder*

### Clusters of spheres

92 equal spheres are arranged on an icosahedron: 12 blue at vertices, 20 red at the face centres, and 60 yellow at the edges. Number and distances of spheres can be varied. Copies of this assembly are placed on a larger sphere again, forming the beginning of a fractal structure. This is similar to the morphology of lotus leaves that are remarkably self-cleaning due to the surface roughness produced by bumps on bumps.



**Figure 11:** *Cluster of 92 Spheres and : Lotus Effect*

### Clusters of polyhedra

**60 rhombic triacontahedra (RT)** are connected face-to-face in two different clusters. In one of them the RTs are at the vertices of a truncated icosahedron (buckyball), like in case of C60 molecule, and in case of the other the RTs are at the vertices a small rhombicosidodecahedron, showing the possibility of another arrangement of 60 atoms.

**Rhombic hexecontahedra (RH)** are placed at the vertices of a dodecahedron so that each RH meets two others at vertices. The author managed to produce polyhedral **clusters made of foam plastic**. The picture shows an example which is a physical realisation of the Mathematica graphics.

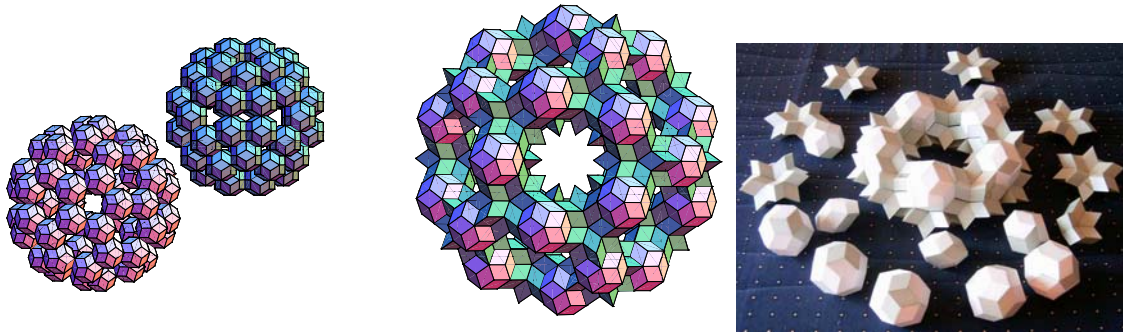


Figure 12: [60 Rhombic Triacontahedra in Two Different Clusters and Dodecahedral Cluster](#)

Placing a polyhedron at the vertices of the same kind of polyhedron is a way to produce polyhedral sculpture.

Cluster of icosahedra are placed on the verticals of a larger icosahedron in a fractal structure.

Place a cube on each face of a rhombic triacontahedron to produce a cluster of 30 cubes, in which each cube touches four others at its vertices.

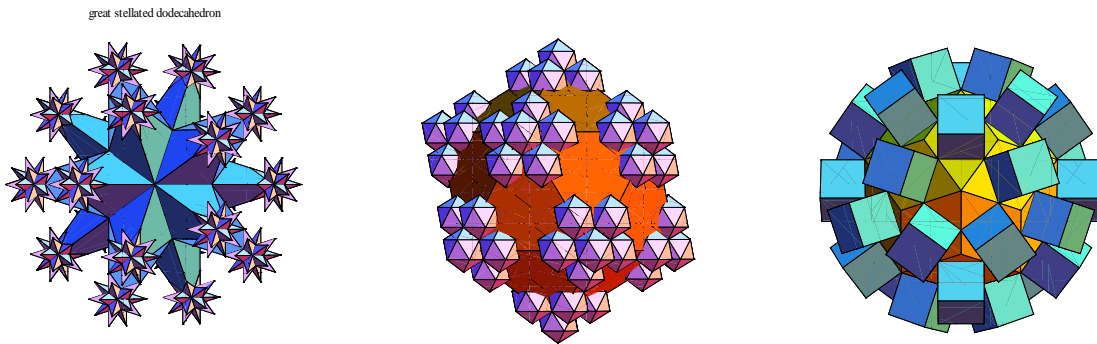


Figure 13: [Polyhedra Copied to the Vertices of the Same Kind of Polyhedron, 12x12 Icosahedra and Cluster of 30 Cubes](#)

### Concepts for geometric sculptures

Cut out all but an S-shaped piece from each of the 30 faces of a great rhombic triacontahedron (GRT) to produce this geometrical sculpture. Using this arrangement balls and sticks are aligned along the faces of a great rhombic triacontahedron (GRT) to show possible designs for geometric sculptures These demonstrations was inspired by the works of George W. Hart.

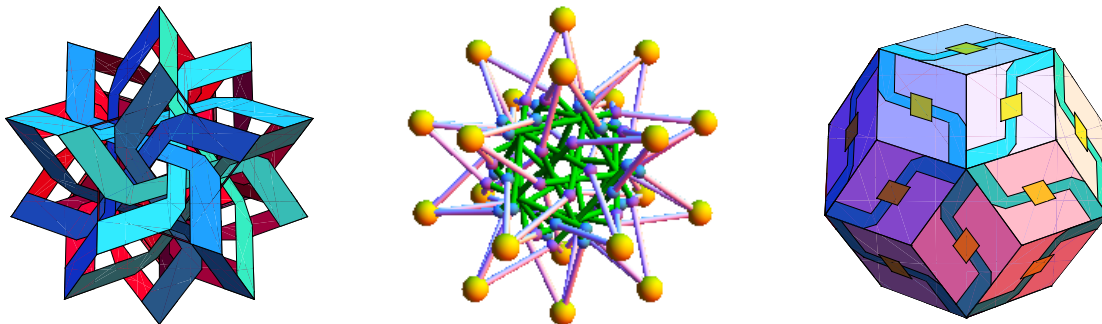


Figure 14: [Great rhombic triacontahedron sculpture](#) and [Balls and Sticks Geometrical Sculpture](#)

Certain parts of a rhombohedron are used to form a ring; various shapes are then attached to it to imitate the appearance of an imaginary flying saucer.

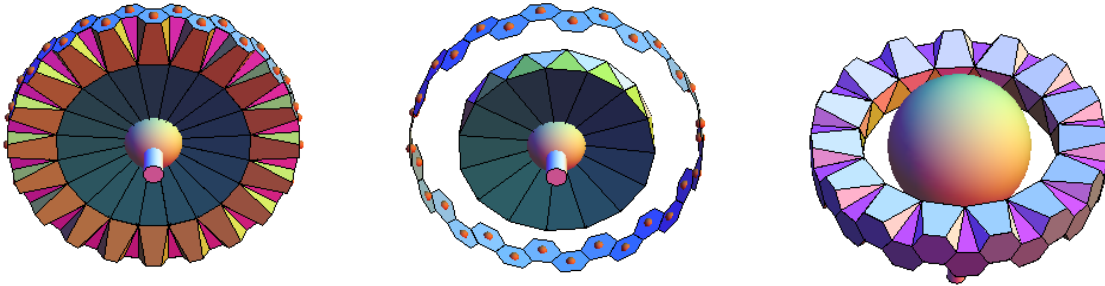


Figure 15: *[Flying saucer builder](#)*

Expandable rings are placed parallel to the faces of a cube and connected near the cube's edges. (This can be done with polyhedra other than the cube, too.).

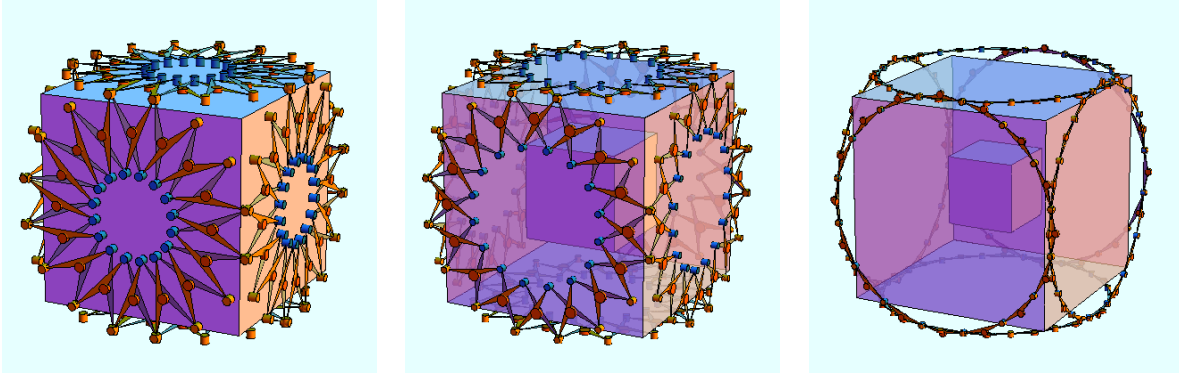


Figure 16: *[Hoberman Cube](#)*

I use Mathematica calculations to develop geometrical sculpture models cut out from plastic, wood, or with patterns printed on the surface of polyhedra.



Figure 17: *Geometrical sculpture models*

## References

- [1] <http://demonstrations.wolfram.com/>
- [2] [www.kabai.hu](http://www.kabai.hu)