Geometry and New Urban Order

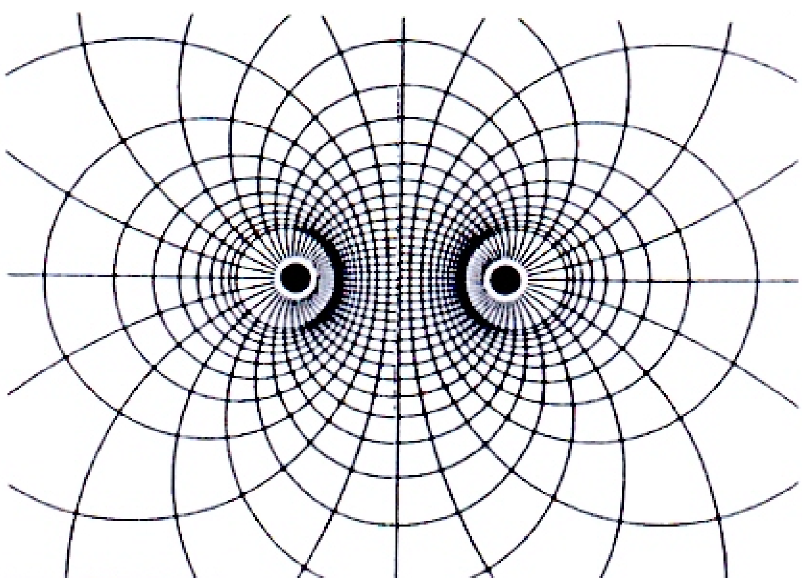
Cristina Argumedo[a], Mª Francisca Blanco[b], Dora Giordano[c] and Miriam Pisonero[b]
[a] Facultad de Arquitectura, Planeamiento y Diseño, Universidad de Rosario, Rosario, Argentina
[b] Departamento de Matemática Aplicada, E. T. S. Arquitectura, Universidad de Valladolid, Valladolid, España
[c] Facultad de Arquitectura, Diseño y Urbanismo, Universidad de Buenos Aires, Buenos Aires, Argentina
E-mail: crisar@fadu.uba.ar , fblanco@maf.uva.es , dora_giordano@hotmail.com and mpisoner@maf.uva.es

Abstract

In this paper, we propose the study of alternatives for resolving **complex situations** such as the expansion of the cities, areas which are exempt from the general order (hybrids) and connections between urban fragments. We start from the idea of presuming a reference to the existing design as a binding factor and, at the same time, we develop possibilities of transformation in the urban layout by means of geometrical criteria used in diverse situations resolution, seeking the potential of alternatives variability. The geometrical layouts proposed, seek to provide an answer to some of the objectives of contemporary town planning: to those which propose to generate **partial structures** for decentralising and identifying places in the continuity of a design.

Proposal of a master layout system, for the expansion or growth of the cities

The interdisciplinary mathematical-design approach is based, on recognising those complex problems, which require an overview of the specific incumbencies of those disciplines. We propose the study of alternatives to solve **complex situations** such as the perimetral expansion of the cities, interstitial areas which are exempt from the general order and connections between fragments. Mathematical models are a useful instrument for finding a **new systematic variability**, for transformations conceived as continuous process.



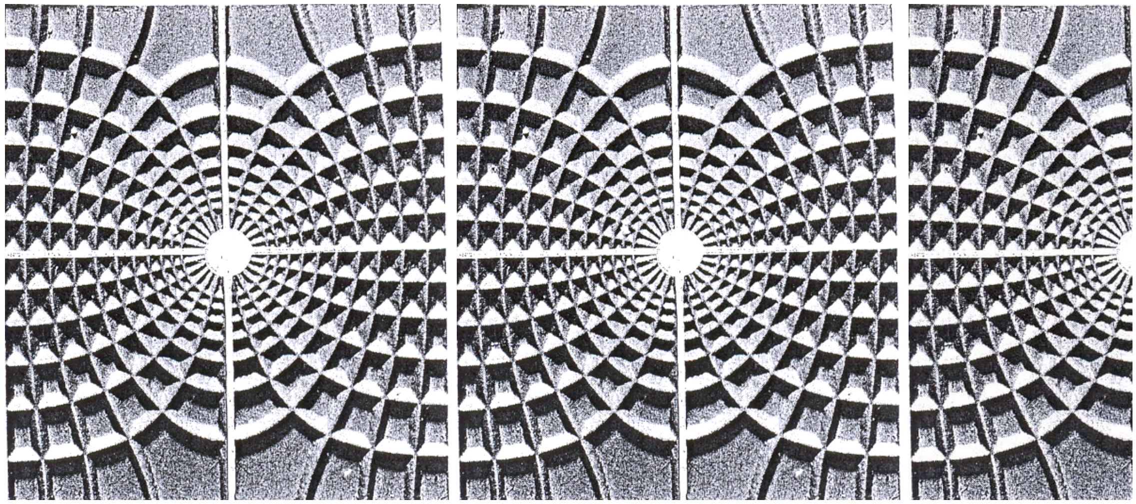


Figure 0

The geometrical layouts proposed, seek to provide an answer to some of the objectives of contemporary town planning: to those which propose to generate **partial structures** to identify new urban places.

The proposal consists of presenting geometrical criteria of variants in a **heterogeneous system**, related to the original design.

We are interested in assuming the complexity of the present in this field of study and in reformulating the urban approaches by means of other scientific methods. This implies having recourse, for example, to one of the methods mentioned by Abraham Moles in his book "The Scientific Creation". The method called by him "of emergence" consists of observing reflexively the phenomena of reality and defining exactly, from that observation, the "emerging concepts". It deals an intellectual attitude different from that of applying to reality objective concepts of existence "a priori". The trend is clear: those **emergent concepts** seek to withdraw from the dichotomy and to substitute it for a relationship generated from and towards the real and the existing, as well as stimulating, in the knowledge, the ability to understand the articulations in terms of a "**complex network"** of particular situations and circumstances in relation to a totality.

Until now, we have outlined a position with regards to the evaluative framework. To delimit the field of study in that framework and to confirm its sense, we start from the position of assuming the current situation of the cities in terms of: **abstract order and uncontrolled randomness**.

From the perspective which we put forward, the proposal would be inverse: *i.e.*, to consider the possibilities of generating those projects starting from layout criteria considered from the point of view of the city, so that the project is carried out in possible terms of difference, but also of integration.

The situation of the urban periphery is one of the most controversial questions regarding the problem of rapid growth of the cities. It is in this point where the different town-planning positions are most clearly evident, as regards the directions that this growth adopts. For us, that is to say, from our evaluative framework, the idea is that of generating new peripheral centres (as an emerging concept of the real circumstances in the periphery) starting from configurational layouts of **partial structures**, but at the same time **continuous** with the city (see figure 1). These structures would constitute **new urban centres** whose growth is foreseen in a similar way to their generation: *i.e.*, their limits once again become diffuse or permeable and give rise to the generation of centres on another scale.

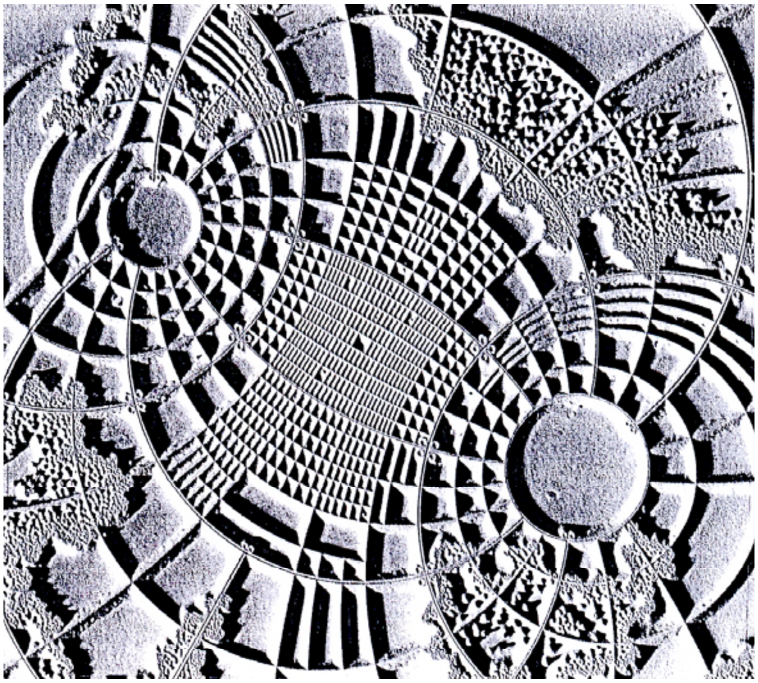


Figure 1: *Model of urban expansion with the option of symmetric and asymmetric growth.*

Another growth alternative could be that of the union between the city and the nearby conglomerates, by **absorption** or **permeability** of the limits. This implies, at the same time, various alternatives with respect to the layout:

- the criteria of peripheral layout foresee surrounding them or integrating them;
- the criteria of peripheral layout generate a situation of **nexus**, whether it be by means of new structures or by a connecting "bridge" with other places, in terms of continuity.

When criteria are defined, a variable system is proposed according to each circumstance.

The anthropological place is, at the same time, the beginning of sense for those who inhabit it and the beginning of intelligibility for the person who observes it (Marc Augé). Every urban place is constituted by means of the "relational" features which identify it, perhaps more than the static visual images. The geometry, adds Augé, provides us with familiar features, such as itineraries, axes, crossroads and paths which lead from one place to another.

Arch. Viviana Colautti has a different point of view: *The new geometrical proposals (of the layout) lead to the incorporation of the diverse networks which interact in a pull of systemic forces, translated into strata where the geometry pertaining to the environmental pre-existences, the fundamental lines derived from the strong urban or natural marks, the limits co-exist simultaneously which emerge from the topography of the ground where the proposal is carried out and the geometry of the original chess board which contemplates the layout*.

The master layout is indicative of the strategic criteria appropriate for urban expansion or growth. The system is proposed in terms of a "conceptual scheme" with possible developments with respect to real alternatives. The circumstantial strategies will define, in each case, the traffic circulation, the green belts, squares, parks, crossroads, the density of the urban fabric, the relationship with the topography, etc

The system contemplates the complexity in terms of variable sub-systems with reference to a constant general conception.

Figure 4 represents the possible expansion of existing cities, whether it deals with those whose original structure is radial (Valladolid) or reticular (Rosario). In either case, there is a reference to the centre but, the real starting point is the grid.

We develop a proposal that overlaps several geometric systems:

1. A **rectangular system**, which corresponds with the old historic city centre.
2. **Two** orthogonal and concentric **hyperbolic systems**, as a first extension of the rectangular system.
3. A concentric **super elliptic system** , which surrounds the rectangular system and meets the hyperbolic systems creating new possibilities of urban growth. An elliptic system is a particular case of this type of systems, or equivalently, a super elliptic system is a generalization of an elliptic system.

The intersections of these systems allow, on one hand joining to the new system different urban agglomerations, that before were disconnected from the centre, and on the other hand the creation of new urban areas connected between them. Let us note that this proposal permits the incorporation of close urban agglomerations to the system preserving their particularities and the creation of new poles of growth and extension of the centre.

Rectangular system: Let assume the old historic city centre follows a rectangular system, otherwise we can consider a rectangle that includes this centre. In what follows, we will consider the reference system formed by the centre of the rectangle that includes the city centre as origin, denoted by O, and the symmetry axis of the rectangle as coordinate axes, being the x-axis the parallel to the largest side of the rectangle. Let also assume that the rectangle is $2a \times 2b$ with $a \geq b > 0$. Note then that a is half of the length of the largest side of the rectangle, parameter that will be used on the definition of the other systems.

Hyperbolic systems: Let H_a be the equilateral hyperbola given by the reduced equation

$$H_a \equiv \frac{x^2}{a^2} - \frac{y^2}{a^2} = 1.$$

The elements of H_a are: center O, vertices $(\mp a, 0)$, foci $(\mp \sqrt{2}\, a, 0)$ and the eccentricity is $\frac{\sqrt{2}\, a}{a} = \sqrt{2}$. Observe that the vertices are the intersections of the x-axis with the initial rectangle; the eccentricity does not depend on a and the asymptotes are the diagonals of the square of side $2a$, centered at O and sides parallel to the coordinate axis, which contains the initial rectangle. Let H'_a be the conjugate hyperbola of H_a . This hyperbola can be obtained rotating 90º about O the curve H_a , its reduced equation is

$$H'_a \equiv \frac{y^2}{a^2} - \frac{x^2}{a^2} = 1$$

and its elements are: center O, vertices $(0, \mp a)$, foci $(0, \mp \sqrt{2}\, a)$ and eccentricity $\sqrt{2}$. The four branches of hyperbola, that we say "orthogonal", on one hand represent the border of the old historic city center. On the other hand, they determine a new way, different from the traditional rectangular grid, of possible growth for the city: from the hyperbolas H_a and H'_a we generate two concentric equilateral hyperbolic systems by increasing the length of the semi-mayor axis a certain positive step p_i

$$H_{a+p_i} \equiv \frac{x^2}{(a+p_i)^2} - \frac{y^2}{(a+p_i)^2} = 1, \qquad H'_{a+p_i} \equiv \frac{y^2}{(a+p_i)^2} - \frac{x^2}{(a+p_i)^2} = 1.$$

Figure 4 shows a system with five hyperbolas for $a=2$ and steps $p_1=2$, $p_2=4$, $p_3=6$, $p_4=8$ and $p_5=10$.

Elliptic system: Let E_a be the ellipse whose vertices are the foci of the hyperbola H_a and the vertices of the hyperbola H'_a ; *i.e.* the ellipse of reduced equation

$$E_a \equiv \frac{x^2}{(\sqrt{2}\,a)^2}+\frac{y^2}{a^2}=1.$$

The elements of E_a are: center O, vertices $(\mp\sqrt{2}\,a,0)$ and $(0,\mp a)$, foci $(\mp a,0)$ and the eccentricity is $\frac{a}{\sqrt{2}\,a}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$. Note that the focus of E_a are the vertices of H_a and that the eccentricity of E_a does not depend on a and is half (equivalently the inverse) of the eccentricity of the hyperbolas H_a and H'_a , for this reason this ellipse is said to be reciprocal of the hyperbola H_a , and vice versa. The intersections of the tangent lines to E_a at its vertices form a rectangle of sides of length $2\sqrt{2}\,a$ and $2a$. The proportion of this rectangle is $\sqrt{2}$, value that is equal to the eccentricity of all the hyperbolas from the hyperbolic systems. The ellipse E_a contains the old historic city centre and we use it to generate a concentric family of ellipses reciprocal of the hyperbolic family H_{a+p_i} , *i.e.*

$$E_{a+p_i} \equiv \frac{x^2}{(\sqrt{2}(a+p_i))^2}+\frac{y^2}{(a+p_i)^2}=1.$$

Piet Hein called **super ellipses** in 1959 to all the curves of the family

$$\frac{|x|^n}{c^n}+\frac{|y|^n}{d^n}=1,$$

where c and d are positive numbers, $n > 2$ and $|x|$ means absolute value of x. As this Danish poet and scientist said *The super ellipse solved the problem. It is neither round nor rectangular, but in between. Yes it is fixed, it is definite, it has a unity*. He proposed this curve with $n=2.5$ and $c/d=6/5$ as a solution for Sergel Square in Stockholm, see figure 3. Hein did not discovered these curves, the mathematician Gabriel Lamé studied them for n > 0, but he is the responsible of its use and popularity in Architecture and Design. Note that for $n = 2$ the above equation is an ellipse (the dotted line in figure 2) and as n increases the curve tends to the rectangle of vertices $(c,\pm d)$ and $(-c,\pm d)$. Observe also that these curves always contains the points $(\mp c,0)$ and $(0,\mp d)$.

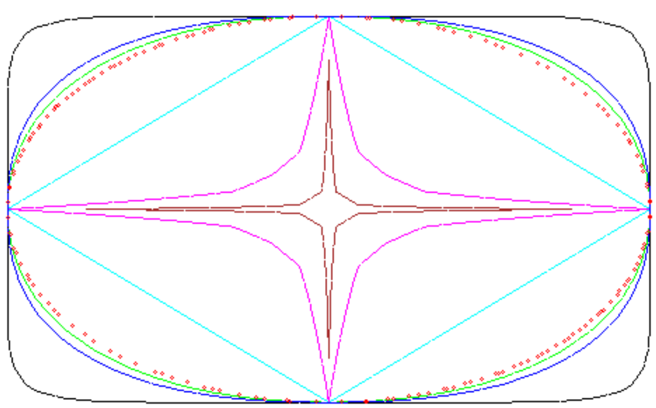


Figure 2: *Super ellipses and Lamè curves with c = 5, d = 3 and n =* ***0.1***, ***0.4***, ***1***, ***2***, ***2.2***, ***2.5***, ***10***.

Figure 3: *Sergel square in Stockholm.*

The elliptic system can be substituted by a **super elliptic system**, with similar properties than the elliptic one, whose family of curves, for $n > 2$, are described by the equations

$$nSE_{a+p_i} \equiv \frac{|x|^n}{(\sqrt{2}(a+p_i))^n} + \frac{|y|^n}{(a+p_i)^n} = 1\,.$$

The combination of these systems divides the city in four quadrants. In each of the quadrants, generated by the system of hyperbolas, different situations are proposed (see figure 4):

Quadrant 1: generative expansion and connection with a new urban pole, at the same time in "concentric" (in a certain sense) growth. Intersection with the concentric system of the ellipses with reference to original centre. Connection with the system of hyperbolas, which orientate the aperture. This sector contains a circular subsystem formed by non concentric circumferences, centred on the x-axis and with radius $r = 2$, $2r$, $3r$ and $4r$ respectively.

Quadrant 2: the existence of a nearby urban conglomeration which is incorporated into the system. The generation of another pole in the possibilities of growth. This area includes a subsystem formed by the intersection of the subsystems obtained as follows:

- reflecting, respect to a tangent line of a curve from the elliptic system, the branches of the hyperbolic system contained in this quadrant;
- reflecting, respect to a line parallel to the previous reflection axis, elliptic arcs from the elliptic system;
- a rectangular grid bordered on the tangent line and the reflected hyperbolic branch closest to the centre.

Quadrant 3: the existence of various conglomerates with their own centre (shared). These preserve their particularity but become integrated into the system. This area includes an orthogonal grid limited by two ellipses. One of these ellipses is simultaneously tangent to an ellipse and to an hyperbola from systems considered. The other ellipse is simultaneously tangent to two of the ellipses of the elliptic system.

Quadrant 4: the generation of new developments which grow progressively and the prolongation or displacement from the original administrative centre. This sector contains three urban centres, two of them follow a rectangular grid and are bordered on two ellipses of the elliptic system. The third urban centre is limited by two hyperbolic branches obtained rotating 120º and 240º about its vertex the outer branch of the hyperbolic system in this quadrant.

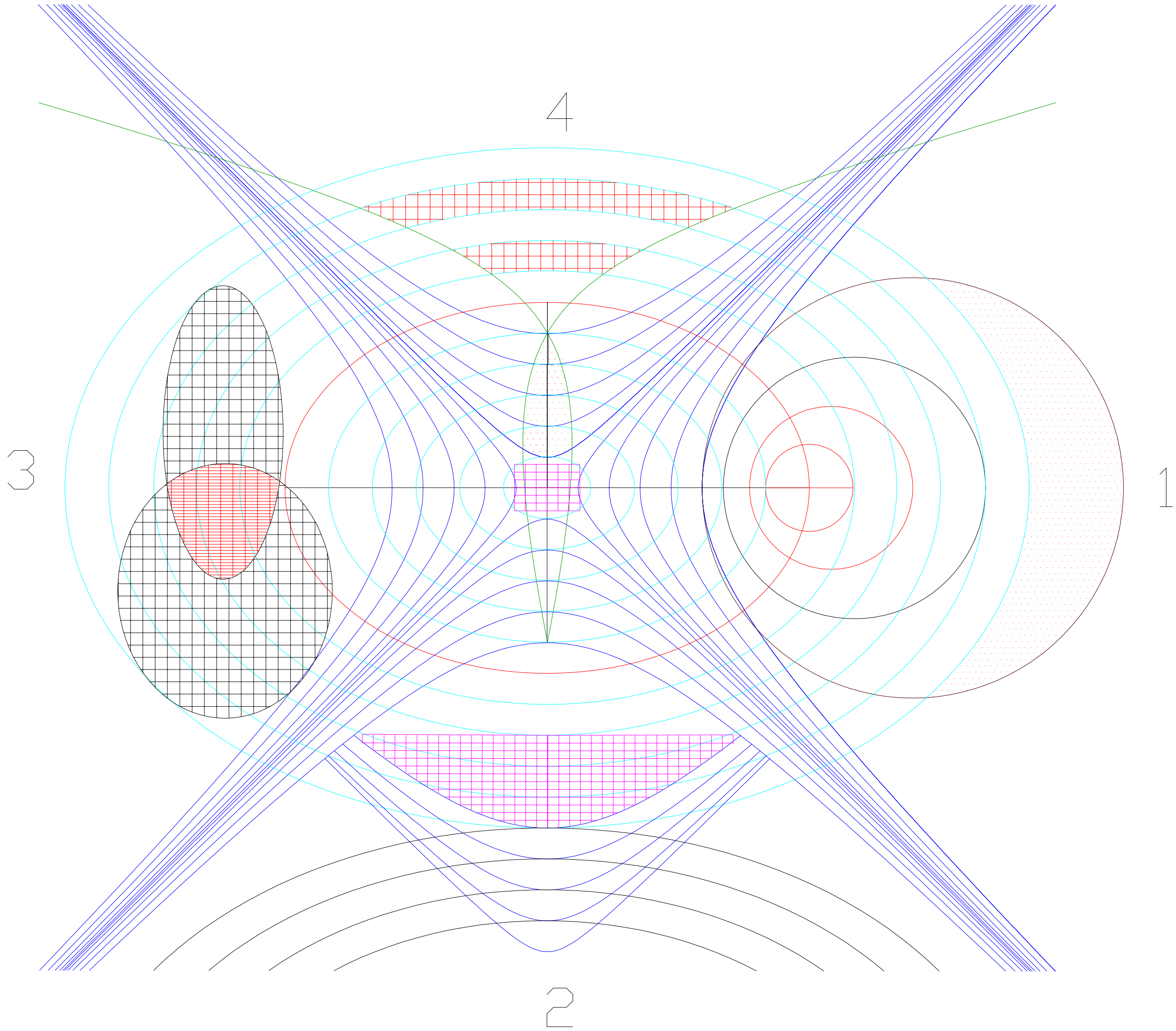


Figure 4: *Combination of the rectangular, hyperbolic and elliptic systems.*

Conclusions

- Our study is based, precisely, on the possibility of discovering operational ways starting from a system which lead to transformational alternatives and variable growth.
- The identity of the "whole" will depend on the significant constants, not on the repetition. This implies making the unpredictability of the variables compatible with the systematic constancy.
- Contemporary town-planning assumes the periphery, not as a limit, but rather as a potential value and sense of growth.
- The urban identity always refers to an "essence" that could be geometrical: for example, the centrality of an original structure which expands until it finally dilutes. This happens when the distances make us lose the evocation of that centre and it is now in the periphery.
- The structure which expands is not an unlimited succession similar to the origin. It is an "organic systematicity", changing according to the development strategies.

- The growth and transformation in this type of systematicity are produced as coherent and continuous processes.
- The city as a totality does not refer to its limits but to the systematic constant as a permanency of relationships.
- The inter-linking of different structures is developed within the system, in the form of strategic connections in a continuous process.
- The origin of the system could be a central structure, but the possibility also exists of various central starting points which, on expanding, become mutually related. This implies thinking of intersections in order to constitute a connected totality and, at the same time, of a process of growth.

References

[1] J. M. Alvarez Pérez, *Curvas en la historia*, Vol. 1 & Vol. 2. Nivola ediciones, España 2006.

[2] C. Argumedo, M. F. Blanco and D. Giordano, *Combination of Geometric Systems,* J. of Mathematics & Design, Vol. 1, Num. 2, pp. 3-12. 2001.

[3] M. Augé, *Los "no lugares". Espacios del anonimato,* Editions du Seuil, Colection La Librairie du XX siècle. Editorial Gedisa SA, España 1993.

[4] E. Catalano, *La Constante. Diálogos sobre estructura y espacio en arquitectura.* Cambridge Architectural Press/ Editorial Universitaria de Buenos Aires, Argentina.

Partially supported by A/5594/06.