Magritte: Analogies in Mathematical Reasoning

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Abstract

The work of Belgian artist Rene Magritte does not contain apparent mathematical quality, as, for example, the work of Maurits Cornellius Escher. Nevertheless, it is not uncommon to find his images among illustrations of mathematically related texts. We discuss the conceptual connections between mathematics and Magritte's work.

Modern art and mathematics as disciplines of human culture are braided not only by visual applications of symmetry, theory of perspective and geometry, but involve parallels of the conceptual level [4]. The work of Belgian artist Rene Magritte does not contain apparent mathematical quality, as, for example, the work of Maurits Cornellius Escher. Nevertheless, many mathematicians find Magritte's paintings especially appealing and provocative. The roots of this special attractiveness can be found in the very philosophy of Magritte's art. His work involves notions that are very close by nature to similar notions in mathematical science. In the following examples we record interpretations that can be evoked by these paintings in an exposed to mathematical culture viewer.

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To codify or to create

Almost every research project in pure mathematics starts with a question of the form "What if...?" What properties of an object would remain, if we change the definition of the object? What if we try to extend a theorem to a wider class of functions? What if we try to prove the statement in the reverse direction?

Let us consider mathematical foundations as an example. Historically, there are two different points of view on axiom systems of mathematics. The traditional activity, which consists of codifying real objects in axioms, was conducted by Euclid and Pasch for geometry and by Helmholtz for integers. Here the existence of objects in the real world dictates the set of axioms [5].

The other way to construct axiom systems has the only rule: consistency implies existence. Those systems of axioms do not describe the surrounding objects, but they create new ones. One of examples of a creative system of axioms is non-Euclidean geometry. In the classical treatise "Elements" Euclid postulated five axioms of geometry, which he called Postulates. The last Postulate was standing apart from the others, since its nature was different. One of equivalent versions of this axiom is called Parallel Postulate, and it is formulated as follows:

For any straight line and a point not on it, there exists exactly one straight line, which passes through that point and never intersects the first line, no matter how far they are extended.

There were many unsuccessful attempts to show that the fifth postulate is a consequence of the previous four. But in 1823 two mathematicians, J. Bolyai and N. Lobachevsky, independently posed the same question:

What if we try to construct geometry without fifth axiom? Would there be a contradiction?

And they both found a geometrical model that exists perfectly well without Parallel Postulate. Such spaces are called non-Euclidean spaces, and they are widely used in modern mathematics.

The idea that axioms are valid as long as they do not contradict one another was strongly advocated by David Hilbert. But even seventy years later after the discovery of non-Euclidean geometry, Hilbert's point of view was not widely accepted and created the polarity of opinions.

Nevertheless, during the same time as Hilbert and other mathematicians were revising the foundations of arithmetic, the group of surrealists was revising the foundations of realistic painting. A series of Magritte's paintings focuses on the processes and states like defiance of gravity, magnification, petrification, etc. The issues addressed by these paintings do not correspond directly to any mathematical problems. But the philosophy strongly reminds challenging the consistency of axiom systems. The artist moved the presentation from codifying the reality to creating a new one. And similarly to a mathematical research, Magritte poses the same question: "What if...?"

What if the things that are taken for granted are changed? How the world would look like?

"Not a question about circumstances, but a question which defined something through posting the thought of its being otherwise in a way that was impossible by definition... Given a door that exists so that people can walk through it, Magritte painted a door which looks as though someone had walked through it when it was shut. Given that clouds are vaporous masses floating in the sky, Magritte painted a rainy day with cumulus clouds spread over the ground. Given that a mermaid is a half woman and half fish, Magritte painted one who is half fish and half woman..." [1].

It is clear that Magritte is interested in paradoxes. His paintings juxtapose controversial properties in the same object: a heavy rock that defines the gravity and floats in the air, a peaceful scenery of day-night time, a horseman, passing behind and in front of the trees at the same time.

Figure 1: *Tracing of Carte Blanche, 1965. National Gallery of Art, Washington, Collection Mr.and Mrs.Paul Mellon*

Mathematicians are interested in paradoxes by the very nature of their science. Wherever is a contradiction, there is a mystery, a puzzle to resolve. Mathematics is the science of the law of the excluded middle. Any proposition in mathematics is either true or false. Paradoxes do not represent valid mathematical propositions, and contradiction is the tool to prove theorems: if a contradiction is obtained, the initial assumption was wrong, and the opposite of the assumption is true. On one side, Magritte's paintings have the power of intriguing mathematical questions, inviting to search for a resolution of a paradox. On the other side, the artist proved that those controversial things can be created at least on canvas, as Lobachevsky and Bolyai proved that there is geometry without fifth axiom.

Word pictures

Magritte's word pictures are well-known for their semiotic significance. Magritte was interested in the means of representation by words and images, and how the title of a painting interrogates of the painting itself.

"All commentators agree that Magritte's drawings and paintings incorporating words are in some sense 'about' the relationship between verbal and pictorial signs. Where commentators disagree is over what is being said, or shown, or suggested or implied about that relationship" [2].

The artist stated some concepts of relations between words and images in the illustrated article "Les Mots et Les Images", which appeared in "La Révolution surréaliste", no 12, 1929. Probably, the most discussed statement of those is that

"The image and the object represented by the image are not the same thing."

As R.Harris [2] points out, "Magritte's famous picture of the pipe with words 'Ceci n'est pas une pipe' is often regarded as a classical example of deliberate verbal and visual ambiguity, where the artist has set out to puzzle us as to which of various possible interpretations we should opt for." He also adds: "Some commentators seem hard put to it to find any ambiguity at all. According to Hughes [3], Magritte's verbo-visual message is simple: 'a painting is not what it represents'. That would be a message of which simplicity is matched only by its stunning banality."

Despite this discouraging comment, we would like to refer to cited above statement from "Les Mots et Les Images" in the literal sense, since this way it evokes an analogy of very familiar relation between an object and its image in pure mathematics.

The opponents of Hilbert argued that there is only one world and so only one geometry, and any system of axioms that does not reproduce that reality is meaningless. But it is already questionable for any axiom system how closely it represents the world around us. For example, let us look at the notion of a number. Surprisingly, this handy and seemingly simple thing, mathematical number, does not exist in the real world. The length of a stick is not a number. Rather, the length can be described by a number relative to some specified unit of length. The same is true for mass, time, volume, anything that can be measured. Hence, mathematical number is an ideal image of real notion. And number is different from the things that it represents.

Another example is a mathematical point. A point in geometry is an ideal object: it has no dimensions, it has no weight, it is not made of any material. This is an idealized image of what we can observe in real life. We can mark a tiny point on a piece of paper, but still it would have some measurements, it would be made of tiny pieces of graphite. Ideal point would share some properties with the point-like objects that it represents, but it is very far from being the same thing. Mathematics is full of other examples of notions that perform similar relations with the "real objects" that they represent. Their properties are purified and idealized. A ball would not fall to the ground by a perfect parabola, smooth functions do not exist, and nobody saw absolutely flat infinite surface of a plane.

Going back to visual arts, Magritte was not the only person who realistically depicted objects that do not exist. But he was one of the first artists to question relation of image to the object. Even if an object is represented in the most careful way, it is not the object itself. And it makes as much sense to argue whether a painting of floating rock is less realistic than a painting of a rock on the ground, as to try to choose the most “realistic” system of axioms.

But this is not the only bridge between Magritte’s word paintings and mathematics. We would like to move in different direction and compare word paintings with mathematical formulas.

One can say that images usually represent the objects, working by resemblance, and words by arbitrary association. Generalization of notions in mathematics follows similar scheme. Particular cases, the mathematical objects that are similar in nature, are generalized under new mathematical notions. To illustrate this connection, let us recall development of algebra from arithmetic. Dealing with similar number problems, we would like to do all “hard” part only once. So more general object - variables, letters, are introduced to substitute particular numbers. By introduction of variables we use "arbitrary associations" instead of particular numbers.

Magritte's word paintings, such as L’Espoir Rapid (1928), represent verbo-visual formulas. In L’Espoir Rapid, instead of dealing with concrete images of a horse, a cloud, a tree, a village, Magritte introduces undefined forms with words - variables. As a result, the viewer sees the painting, where he has the freedom to substitute his own values-images of the variables-words.

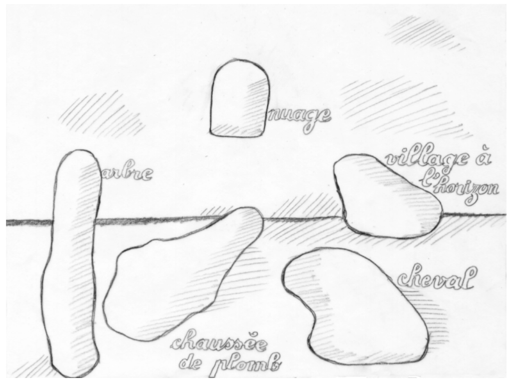

Figure 2: *Tracing of L’Espoir Rapid, 1928. Hamburger Kunsthalle.*

It is a curious exercise to replace "word" with "variable" and "image" with "number" in each of the statements of "Les Mots et Les Images" to see, which of the resulting sentences still would make any sense. Of course, not all of them would be literally translated in the world of algebra, but some parallels are remarkable. Here are the most successful examples:

"In a picture words are of the same substance as images."

"(In an equation) variables are of the same substance as numbers."

"Sometimes the name of an object takes place of an image."

"Sometimes a variable takes place of a number."

"An image can take a place of a word in a sentence."

"A number can take a place of a variable (in an equation)."

"Sometimes words denote themselves"

"Sometimes variables denote themselves."

References

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