Bridges London *Mathematics, Music, Art, Architecture, Culture*





BRIDGES Mathematical Connections in Art, Music, and Science





Conference Proceedings 2006

Reza Sarhangi and John Sharp, Editors

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Bridges London Mathematics, Music, Art, Architecture, Culture

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Contents

| Preface | |
|--|----|
| How Bridges Proceedings Come Together | |
| Collaboration on the Integration of Sculpture and Architecture in The Eden Project <i>Peter Randall-Page</i> | 1 |
| The Work of Foster and Partners Specialist Modelling Group Brady Peters and Xavier DeKestellier | 9 |
| The Borromean Rings — A Tripartite Topological Relationship Louis H. Kauffman | 13 |
| Cultural Insights from Symmetry Studies Dorothy K. Washburn and Donald W. Crowe | 19 |
| Non-Euclidean Symmetry and Indra's Pearls Caroline Series and David Wright | 25 |
| Love, Understanding, and Soap Bubbles Simon Thomas | 33 |
| Creating Penrose-type Islamic Interlacing Patterns John Rigby | 41 |
| Steve Reich'S Clapping Music and the Yoruba Bell Timeline Justin Colannino, Francisco Gómez and Godfried T. Toussaint | 49 |
| Illuminating Chaos — Art on Average Mike Field | 59 |
| Bridging the Gap — a Search for a Braid Language Jacqui Carey | 61 |
| Magic Stars and Their Components Sergei Zagny | 69 |
| Introducing the Precious Tangram Family Stanley Spencer | 73 |
| Sand Drawings and Gaussian Graphs Erik D. Demaine, Martin L. Demaine, Perouz Taslakian, and Godfried T. Toussaint | 79 |
| Symmetric Characteristics of Traditional Hawaiian Patterns: a Computer Model <i>Tony Cao and Jin-Ho Park</i> | 89 |

| Circle Folded helices Bradford Hansen-Smith | 97 |
|---|-----|
| The Taming of Roelofs Polyhedra Frits Göbel | 105 |
| A Program to Interpolate (and Extrapolate) between Turtle Programs Ken Kahn | 109 |
| The Programmer as Poet Russell Jay Hendel | 115 |
| Minkowski Sums and Spherical Duals John M.Sullivan | 117 |
| Polygon Foldups in 3D Kate Mackrell | 123 |
| Portraits of Groups Jay Zimmerman | 131 |
| A New Use of the Basic Mathematical Idea of Twelve-Tone Music Ward Douglas Maurer | 135 |
| A Braided Effort: A Mathematical Analysis of Compositional Options James Mai and Daylene Zielinski | 137 |
| On a Family of Symmetric, Connected and High Genus Sculptures Ergun Akleman, Ozan Ozener and Cem Yuksel | 145 |
| Affine Regular Pentagon Sculptures Douglas G. Burkholder | 151 |
| The Effect of Music-Enriched Instruction on the Mathematics Scores of Pre-School Children <i>Maureen Harris</i> | 153 |
| Teaching Arabesque Jean-Marc Castera | 157 |
| Slide-Together Structures Rinus Roelofs | 161 |
| Repeated Figures Susan McBurney | 171 |
| Seville's Real Alcázar: Are All 17 Planar Crystallographic Groups Represented Here? B. Lynn Bodner | 175 |

| Math must be Beautiful Carla Farsi | 181 |
|--|-----|
| The Integrated Scale Desirability Function: A Musical Scale Consonance Measure Based on Perception | |
| Data Richard J. Krantz and Jack Douthett | 183 |
| Tiled Artworks Based on the Goldbach Conjecture Sharol Nau | 191 |
| Sculpture Puzzles George W. Hart | 195 |
| The Mechanical Drawing of Cycloids, The Geometric Chuck Robert Craig | 203 |
| Sashiko: the Stitched Geometry of Rural Japan Barbara Setsu Pickett | 211 |
| Literatronic: Use of Hamiltonian Cycles to Produce Adaptivity in Literary Hypertext <i>Juan B. Gutierrez</i> | 215 |
| Responsive Visualization for Musical Performance Robyn Taylor, Pierre Boulanger and Daniel Torres | 223 |
| The Necessity of Time in the Perception of Three Dimensions: A Preliminary Inquiry <i>Michael Mahan</i> | 231 |
| An Interactive/Collaborative Su Doku Quilt Eva Knoll and Mary Crowley | 237 |
| Patterns on the Genus-3 Klein Quartic Carlo H. Séquin | 245 |
| The Lorenz Manifold: Crochet and Curvature Hinke M Osinga and Bernd Krauskopf | 255 |
| Playing Musical Tiles Rachel W. Hall | 261 |
| Mathematics and the Architecture: The Problem and the Theory in Pre-Modern Cultures <i>Zafer Sagdic</i> | 269 |
| Towards Pedagogability of Mathematical Music Theory: Algebraic Models and Tiling Problems in Computer-aided Composition | 277 |
| Moreno Andreatta, Carlos Agon Amado, Thomas Noll and Emmanuel Amiot | 277 |
| Streptohedrons (Twisted polygons) David Springett | 285 |

| Fractal Tilings Based on Dissections of Polyominoes Robert W. Fathauer | 293 |
|--|-----|
| Vortex Maze Construction Jie Xu and Craig S. Kaplan | 301 |
| Models of Cubic Surfaces in Polyester Sergio Hernández, Carmen Perea, Irene Polo Blanco and Cayetano Ramírez "Tano" | 309 |
| "Geometry" in Early Geometrical Disciplines: Representations and Demonstrations <i>Elaheh Kheirandish</i> | 315 |
| Ant Paintings using a Multiple Pheromone Model Gary R. Greenfield | 319 |
| Verbogeometry: The Confluence Of Words And Analytic Geometry Kaz Maslanka | 327 |
| Zome-inspired Sculpture Paul Hildebrandt | 335 |
| Developable Sculptural Forms of Ilhan Koman Tevfik Akgün, Ahmet Koman and Ergun Akleman | 343 |
| Constellations of Form: New Compositional Elements Related to Polyominoes James Mai and Daylene Zielinski | 351 |
| Transformations of Vertices, Edges and Faces to Derive Polyhedra Robert McDermott | 359 |
| Chromatic Fantasy: Music-inspired Weavings Lead to a Multitude of Mathematical Possibilities Jennifer Moore | 365 |
| Asymmetry vs. Symmetry in a New Class of Space-Filling Curves Douglas M. McKenna | 371 |
| Modular Perspective and Vermeer's Room Tomás García-Salgado | 379 |
| On the Bridging Powers of Geometry in the Study of Ancient Theatre Architecture <i>Zeynep Aktüre</i> | 387 |
| The Gemini Family of Triangles Alvin Swimmer and Mary C. Williams | 395 |
| Taitographs: Drawings made by Machines Jack Tait | 403 |

| Photography and the Understanding of Mathematics Richard Phillips | 411 |
|--|-----|
| Inference and Design in Kuba and Zillij Art with Shape Grammars Ramgopal Rajagopalan, Eric Hortop, Dania El-Khechen, Cheryl Kolak Dudek, Lydia Sharman, Fred Szabo, Thomas Fevens and Sudhir Mudur | 419 |
| Green Quaternions, Tenacious Symmetry, and Octahedral Zome David A. Richter and Scott Vorthmann | 429 |
| Mathematics and Music — Models and Morals Meurig Beynon | 437 |
| Teaching Design Science: An Exploration of Geometric Structures Carl Fasano | 445 |
| More "Circle Limit III" Patterns Douglas Dunham | 451 |
| J-F. Niceron's <i>La Perspective Curieuse</i> Revisited <i>J. L. Hunt</i> | 459 |
| A Meditation on Kepler's Aa Craig S Kaplan | 465 |
| Approximating Mathematical Surfaces with Spline Modelers Stephen Luecking | 473 |
| The Lost Harmonic Law of the Bible Jay Kappraff | 481 |
| New Ways in Symmetry María Francisca Blanco Martín and Elena Elvira Nieto | 491 |
| Linkages to Op-Art John Sharp | 497 |
| D-Forms: 3D Forms from Two 2D Sheets Tony Wills | 503 |
| Visualizing Escape Paths in the Mandelbrot Set Anne M. Burns | 511 |
| The Math of Art: Exploring Connections between Math and Color Theory <i>Amina Buhler-Allen</i> | 517 |
| Celtic Knotwork and Knot Theory <i>Patricia Wackrill</i> | 521 |
| Islamic Art: An Exploration of Pattern Carol Bier | 525 |

| In Search of Demiregular Tilings Helmer Aslaksen | 533 |
|---|-----|
| Tribute to the Atomium Samuel Verbiese | 537 |
| RHYTHMOS: An Interactive System for Exploring Rhythm from the Mathematical and Musical Points of View Jakob Teitelbaum and Godfried Toussaint | 541 |
| Spidron Domain: The Expanding Spidron Universe Dániel Erdély and Marc Pelletier | 549 |
| An Introduction to Medieval Spherical Geometry for Artists and Artisans <i>Reza Sarhangi</i> | 551 |
| Fabric Sculpture — Jacob's Ladder Louise Mabbs | 561 |
| Eva Hild: Topological Sculpture from Life Experience Nat Friedman | 569 |
| Interdisciplinary Bridges: A Novel Approach for Teaching Mathematics <i>Gail Kaplan</i> | 573 |
| Concerning the Geometrical in Art <i>Clifford Singer</i> | 579 |
| Knot Designs from Snowflake Curves Paul Gailiunas | 581 |
| Asymmetry in Persian Symmetrical Art and Architecture Hourieh Mashayekh and Hayedeh Mashayekh | 585 |
| Cultural Statistics and Instructional Designs Darius Zahedi | 587 |
| Musical Scales, Integer Partitions, Necklaces, and Polygons David Rappaport | 595 |
| 1927: Two Processes of Creating Form in Music Veryan Weston | 599 |
| On Mathematics, Music and Autism Ioan James | 605 |

Bridges for Teachers, Teachers for Bridges

| Mathematics Investigations in Art-Based Environments Mara Alagic and Paul Gailiunas | 613 |
|---|-----|
| Moving Beyond Geometric Shapes: Other Connections between Mathematics and the Arts for Elementary-grade Teachers Virginia Usnick and Marilyn Sue Ford | 617 |
| A Geometric Inspection of Pennsylvanian Dutch Hex Signs Evan G. Evans and Reza Sarhangi | 625 |
| Creating Sliceforms with 3D Modelers Stephen Luecking | 631 |
| Paper Sculptures with Vertex Deflection Tevfik Akgün, Ahmet Koman and Ergun Akleman | 639 |
| Understanding the Mathematics Based Formulation on Dome Tessellation in Architect Sinan's Mosques Design Zafer Sagdic, Mujdem Vural and Gokce Tuna Taygun | 641 |
| Mandala and 5, 6 and 7-fold Division of the Circle Paul F. Stang | 645 |
| Mathematical Book Forms for Teachers Susan Happersett | 647 |
| The Aréte of Line Designs Michael Round | 649 |
| The Plato Bead: A Bead Dodecahedron Laura Shea | 651 |
| Building Simple and Not So Simple Stick Models Robert McDermott | 655 |
| Topological Mesh Modeling Ergun Akleman and Vinod Srinivasan | 661 |
| Vermeer's "The Music Lesson" in Modular Perspective Tomás García-Salgado | 663 |
| Zellij Multipuzzle Jean Marc Castera | 665 |

Preface

When a man is tired of London, he is tired of life; for there is in London all that life can afford. — Samuel Johnson 1777

In 1998 one man from the UK made a trip, all alone, half-way around the world to the small town of Winfield in the plains of Kansas, to the first Bridges Conference. On the first night there was a terrific thunderstorm with sheet lightening he had never seen before, which made him wonder what he had let himself in for. He does wonder, because now this man, John Sharp, is one of the organizers for what promises to be the largest Bridges so far. With many international faces, Bridges itself has travelled half-way around the world, visiting Spain and Canada, to be in London, a city centred on the river Thames which has many real bridges.



For the conference logo, Phillip Kent, the young and enthusiastic co-organizer, whose involvement has enriched the conference greatly, cleverly drew some lines on a photograph of the bridge most commonly seen in tourist photographs: Tower Bridge. The logo is a set of five lines, which is a play on the diagram of Euclid's Book I, Proposition 5, commonly known as the *pons asinorum* or "Bridge of Asses". The logo can also be suggestive of construction lines for drawing in linear perspective. Apart from its obvious symbolism as a bridge, the story goes that only if you could pass over this bridge (understand the proof) was there hope for you as a mathematician. Perhaps because it is a humorous allusion only geometers will understand, no one seems to have noticed it! Maybe Bridges participants do not study Euclid any more. They obviously do study modern geometries as is evident by many of the papers in this book.

The range of topics in connections among mathematics and the arts in this book brings hope for a rich exchange of ideas during the conference and aftermath. The Bridges logo subtitle is "Mathematical Connections in Art, Music and Science". Over the years there has not been much science. Some papers in previous years have been very much at the pure mathematics end of the spectrum. This year we have made a conscious effort to pull them back from the edge if they did not have an artistic element. This is not to say that there are papers at the extreme both ways. However, this is not where the cross fertilization occurs best. Artists need to be made aware of mathematical possibilities; artists whose work has a mathematical element which is "intuitive" often are not able to include detailed mathematics. There are many papers this year that are collaborative efforts in the true spirit of Bridges. There are many mathematicians who are expressing their work in an artistic way. There are historical papers, and there are cutting edge descriptions in architecture. Analyzing the papers yields a tangled web of interactions that would take many pages to describe. The *Bridges Proceedings* are one of those books which one can come back to again and again, dipping in and finding something new every time.

Since the first Bridges, the size of the Proceedings has more than doubled with a wider variety of topics than ever before. The increase in submissions, of course, caused more work for the referees and editors. No words can express our appreciation for the work the referees have done to enable publication and also to those who helped some authors prepare and format their papers. In order to keep the book a reasonably manageable size, some authors were requested to shorten their papers. The complexities of bringing together so many ideas and the amount of work that goes on behind the scenes is described by John Sharp in the following pages. Because of the varieties of subject and author experience, Bridges is unique in the way it accepts such a large proportion of papers. In effect this changes the attitudes of the people submitting the papers even before they are published and the conference then strengthens the bridges as well as building them.

The front cover is by Carlo Séquin who has been a valued advisor to Bridges since the beginning. He is very much an "ideal" member of the Bridges community in the way he worked with artists like Brent Collins both to learn from them and to collaborate. His efforts symbolize the way art and mathematics is interconnected.

The *Bridges Visual Art Exhibit* is the result of Robert Fathauer's hard work in communicating with a large number of artists in order to carefully select the artwork and properly set them up for the exhibit. His job (together with fellow jurors Nat Friedman and Anne Burns) has been harder this year because there have been a record number of submissions. The website pages assembled by Anne Burns, which are also on the CD that accompanies this book, record the images presented at the *Bridges Visual Art Exhibit*.

Although we do not include the details of the conference in the Proceedings, it is worth mentioning that Bridges London has taken some new directions this year. We have more excursions since we are in one of the world's major cultural centers. We are also aiming this year to reach out to the general public with two free and open events. We will have a concert, or Musical Event, mixing music and mathematics by Bridges participants and well-known UK mathematicians. The workshop papers at the end of the Proceedings show how *Bridges for Teachers - Teachers for Bridges* helps the educational specialists. This has been organized by Mara Alagic and Paul Gailiunas.

We are privileged to have been able to collaborate with the Royal Institution of Great Britain. This organization has been a major player in scientific progress in the last 200 years, with Sir Humphrey Davy, Michael Faraday and James Dewar amongst its scientific members and it is famous for its annual Christmas Lectures on scientific topics delivered to school students. It is noteworthy the 1978 Lectures were titled 'Mathematics into pictures' and were given by Professor Sir Christopher Zeeman, FRS, famous for his work on catastrophe theory. Building on these lectures, in 1981 he launched the Royal Institution Mathematics Masterclass program, which aims to mathematically engage and inspire 12 to 14 year olds. We are delighted to celebrate the 25th anniversary of these classes at Bridges by having him and three other expert masterclass presenters as part of a Bridges Family Day to be held on the last day of the conference. This again is open to the public and includes a Maths Fair which allows Bridges participants to get involved in small workshops, either as contributors or participants.

Special thanks must also be given to the staff at the Institute of Education Conference Office and the School of Mathematics, Science and Technology and the London Knowledge Lab. We also gratefully acknowledge financial support of the conference by the London Mathematical Society, Sibelius Software, the Institute of Education and the London Knowledge Lab. We also thank Aida Jones for her time and effort to assist in the reviewing and registration process, and Barbara Kaiser at Southwestern College in Winfield, Kansas who generously spent days to deal with the registration process again this year and everyone else who has helped in their own way, not least the participants.

How Bridges Proceedings come together

This book is product of many people's time and creativity. The results are obvious once it is completed. The path to those results is long and tortuous in many ways, not all of them obvious. The following is an attempt to describe how this takes place in a way which shows why Bridges is different from most other conferences. Although I have reviewed and edited papers in previous years, I have never been involved to the level I have this year and it has been illuminating to see how much work is necessary.

Looking through the papers, it is not easy to define where art ends and mathematics begins, and vice versa. There is also the question of "what is art" which is why Bridges is about the connections between mathematics and the arts in all its forms, from the visual and the decorative to music, architecture and many more. Pablo Picasso said that "great artists don't borrow, they steal". Artists certainly copy and some look through mathematics books to gain ideas. This book has a wealth of material to inspire, copy and steal for both the artist and mathematician. Why do I say all of this? Because it shows the problems presented in bringing Bridges together. Because defining Bridges other than through the people who make it up is very difficult.

A Bridges conference starts in earnest as soon as the previous one finishes. There is publicity and announcements about when and where and what to submit. When the papers start rolling in the next job is to review them for suitability. These reviews are carried out by a group of anonymous (to those submitting the papers and all but the editors) reviewers. The choice of reviewers for a paper (usually three, but often two this year with so many papers) is based on the experience of the editors and is not easy. Sometimes it is not easy for the reviewer. Because a reviewer might not be as expert in the field as the writer, not all errors may be picked up. Some reviewers are not necessarily aware of how they should approach the job. They may give opinion rather than a check for accuracy. Some write a few lines and some another paper as their review with many more references, but on the whole they write a set of points that need to be fixed, which the editors can then pass back to the author before the paper is published. In some cases, though not many, the reviewers conflict with one saying accept and one reject. In some cases both suggest rejection and rejections can be quite strongly worded. Then editorial diplomacy comes into play.

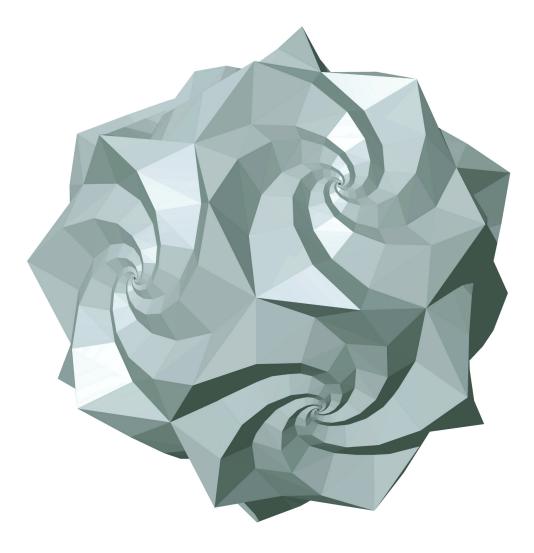
Regular conference participants will recognise this as fairly typical, but there are other aspects which make Bridges different. Many participants (usually at the mathematical end of the spectrum) are academics used to conferences and often relying on getting papers published to further their careers and thus are used to preparing and formatting papers. There is a strong contrast with artists and others who have never been to a conference before or written a paper. The latter are equally likely to be unfamiliar with the finer points of formatting in a Word Processor or presenting their ideas in a structured way, good though their ideas may be. This is very evident with artists who speak visually, and are not used to working with words and they might not know the correct technical words, or how to use them in the right way. Mostly this is picked up in the review process, but even so the editors have had to explain and edit some texts before they were suitable for publication. There is also the issue of someone who might be good at creating a picture but not technically aware of how to prepare it for publication. This means we have had a few pictures which do not have enough resolution and so appear blurred because we have not been able to get them improved, usually because of time constraints.

So while the editorial job has often been one of reformatting or re-writing a paper there is a more important situation where changes have been required which makes Bridges different from other conferences. Many people submitted papers which were generally aimed at the idea of making connections between art and mathematics but did not make the link as strongly as they could have done. Some did not even do so at all, even though the author was aware of the link, otherwise they would not have submitted the paper. This is further complicated by the need for mathematicians to say "here are some ideas which I think artists can use, but I am not an artist so I can't write about that side". At the other end of the scale artists can be mathematically intuitive and see things before the mathematicians have found them. They can use mathematics in a way which is too "free" for some mathematicians. Then there are authors who come from what one might call "the fringes" and their original submissions were not of the required standard, especially when compared with the more academic papers. So the editors have had to perform the role that a conductor of an orchestra would and mould many papers to fit the conference. Rather than reject a paper, there has been much discussion with some authors who were not in tune with Bridges, most (but not all) of whom have not been to a previous conference. Moreover, since Bridges is unique in bringing so many disciplines together, the authors may not have been exposed to an audience that ranges so widely as Bridges. So while they might be distinguished mathematicians or artists, and used to communicating in their own peer group, their work has to be focussed in a different way. With so many papers and so many new faces, the orchestration has been a greater task than in previous years and a full-time job in parts of the process.

Many conferences publish rejections rates. It is not Bridges policy to do this. The main reason is because this would invite comparison of situations which are not equivalent. Bridges, by its very nature needs to spend time with the author, to coax them if necessary. So Bridges is unusual in not rejecting outright in the first instance. Whereas a reviewer might suggest rejection many papers that might have been rejected have been "brought into line" to a certain extent. Authors have been very accommodating in this respect and in some cases produced papers which are excellent examples of building the kind of bridges that the conference is aiming at. There are papers here which might have been more suitable for a pure mathematics conference. There are papers which mathematical readers will find incorrect or even confused and we do not have the luxury of many iterations to get a paper correct; with a journal this can take years and we only have a couple of months. Mathematicians can be very polarized (binary) in their outlook and often intolerant of what does not conform to the rigour of their subject. So if you are reading with a supercritical eye remember that an artist is not necessarily a mathematician and vice versa. Before being too critical ask yourself if the author has seen something that you haven't. After all that is what we have had to do when we have brought these papers together for Bridges and with such a short time to do so, how Bridges comes together sometimes feels like a miracle.

Bridges always produces the Proceedings in time for the conference. This benefits both the person giving the paper and the participants listening to the talk. Many people read the papers they are especially interested in before going to the talk. Participants can really get involved and not meet the ideas for the first time out of the blue and the paper comes more alive at the conference. Since there are only a few months to accomplish this, the method has its pros and cons. The ideas might not be polished, but then the author gains a chance to iron out any points that are not clear before, possibly, taking it further for journal publication. Bridges has many gems; often they are uncut and unpolished; many will get cut into diamonds which later appear in journals. There may be some ideas akin to common stones as well as gems, but even those have their uses and they might not see the light of day with another approach. The success and reputation of Bridges needs variety above all.

Bridges London The Papers



Dodecaspidroball by Dániel Erdély, Marc Pelletier and Walt van Ballegooijen,