

Linkages to Op-Art

John Sharp
20 The Glebe
Watford, Herts England, WD25 0LR
E-mail: sliceforms@compuserve.com

Abstract

Many artists using mathematical curves to generate lines in their work use Lissajous figures or cycloids. There are many other curves which can be drawn “mechanically” and linkages do not appear to have been used in an obvious way. In my op-art period many years ago, I used a simple linkage and I have resurrected this to create some new ideas following a particular interest in the lemniscate.

Introduction

The mechanical generation of curves has been a feature of many forms of art. Harmonographs, based on swinging pendulums were invented in the nineteenth century and computer versions have been used by Bob Brill in his work on Lissajous figures [1]. There were many lathe or gearing mechanisms which also developed in the nineteenth century giving rise to Rose Turning machines which are essentially cycloidal mechanisms. More complex versions of these were used to engrave plates for banknotes, although this is now done using computer programs. My work, described here, is based on a simple three bar linkage. The mechanics of such systems were developed to control machinery in the Industrial Revolution. A. B. Kempe’s book of 1877 [2] on how to draw a straight line is the classic book, and mathematicians like Sylvester and Cayley were also bitten by the linkage bug. Sylvester’s notable contribution was the pantograph for enlarging drawings. Kempe is also famous for his “proof” of the four colour theorem. The straight line of the title is important since it enabled linear motion to be turned into rotary motion and vice versa. James Watt’s linkage of 1784 was fundamental to obtaining motion from beam engines.

The lemniscate from a linkage

The simplest linkage might be said to be a bar compass. One point of the bar rotates about a pivot and a pen is placed at another position on the bar. Kempe [2 p 4-5] describes how to build up complexity:

Turning to that apparatus, we notice that all that is requisite to draw with accuracy a circle of any given radius is to have the distance between the pivot and the tracer properly determined, and if I pivot a second “piece” to the fixed surface at a second point having a tracer as the first piece has, by properly determining the distance between the second tracer and pivot, I can describe a second circle whose radius bears any proportion I please to that of the first circle. Now, removing the tracers, let me pivot a third piece to these two radial pieces, as I may call them, at the points where the tracers were, and let me fix a tracer at any point on this third or *traversing* piece. You will see that if the radial pieces were big enough the tracer would describe circles or portions of circles on though they are in motion, with the same ease and accuracy as in the case of the simple circle-drawing apparatus; the tracer will not however describe a circle on the fixed surface, but a complicated curve.

His figure is shown in figure 1. However, it is simpler to show linkages diagrammatically in as figure 2. The two end pivots or pins are fixed and the others can rotate. The freedom of the motion is limited and

each point on the bars or links trace a curve. The linkage in figure 2 is sometimes called a three-bar and sometimes a four-bar linkage. The fourth bar is the fixed bar between the outer pins shown here. The central point of the middle bar traces a figure of eight curve known as a lemniscate when the distance between the end pins and the lengths are prescribed. If the short bars are of length $\sqrt{2}$, the long bar is 2, as is the distance between the foci, then the linkage generates a curve known as Bernoulli's lemniscate [3].

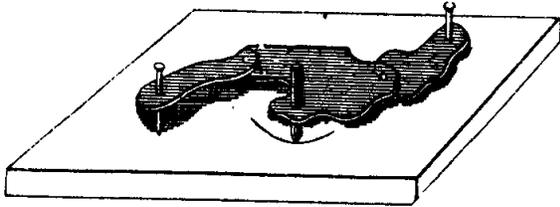


Figure 1, Kempe's figure

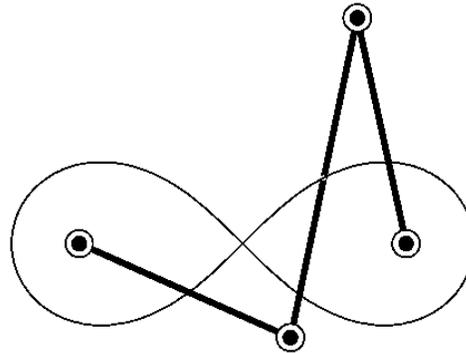


Figure 2, an "instrument" for drawing a lemniscate

The linkage is one of the simplest to make and a cardboard model is shown in figure 3. The bars of the linkage are three pieces of cardboard two of the same length and a longer central one with a hole in its centre for a pencil point. The pins or pivots are made using paper fasteners. The fixed, end pivots are drawing pins. Note how the lemniscate is incomplete. This is because the pencil hits the bars.

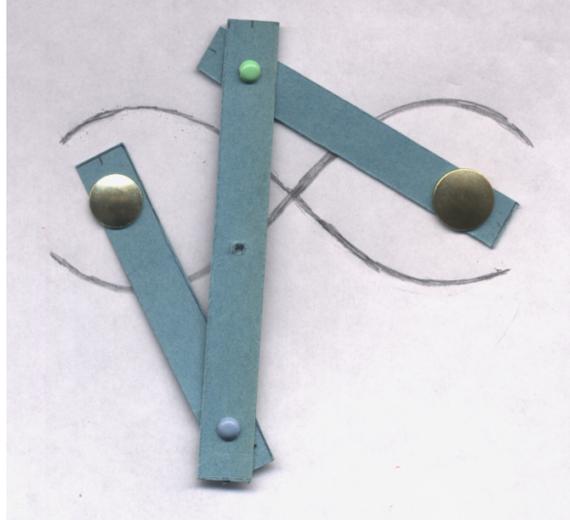


Figure 3, model of the lemniscate linkage

You can have a lot of fun by seeing what happens when you vary the distance between the positions of the fixed points. Wider distances than those for the standard lemniscate allow you to complete the curve. Figure 4 shows some examples of what happens. You could also change the relative lengths of the bars and obtain other curves or you could explore the family of curves that results from moving the pencil point along the central bar.

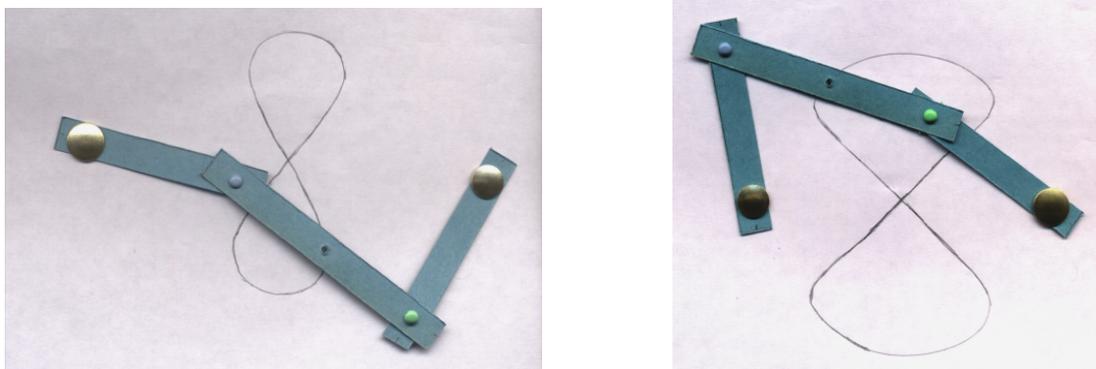


Figure 4, results of varying the fixed points

Some 25 years ago this is how I drew the curves for op-art pieces. It is now easier to work on the computer, plotting the equation for the lemniscate which in polar terms is:

$$r^2 = a^2 - [c \sin \theta \pm \sqrt{b^2 - c^2 \cos^2(\theta)}]^2$$

where the origin is in the centre, the length of the two short bars is a , the central bar $2b$ and the distance between the outer pins is $2c$. Figure 7 shows the curves for various values of a , b and c turned into op-art.

Dynamic Geometry output

With the appearance of dynamic geometry packages like Geometer's Sketchpad and Cinderella [4], it is possible to create the curves as loci by constructing the linkage in the computer. The latter is very good for producing output that can be imported into drawing packages. Using these packages has the added benefit that you can vary the positions of various parameters and construct the locus in real time. This gives much more of an artist's control compared with manipulating equations.

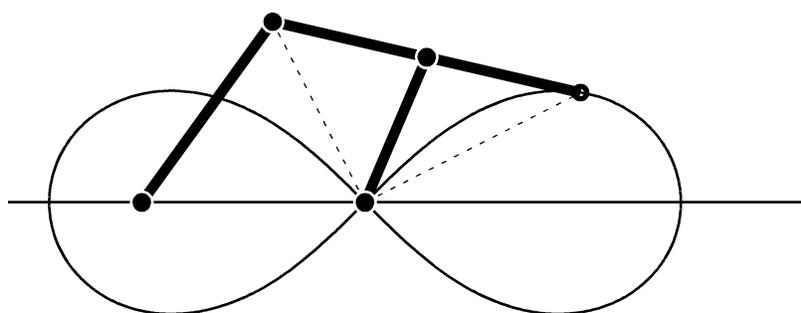


Figure 5, another lemniscate linkage

An alternative linkage for the lemniscate is shown in figure 5. A set of curves can be created by taking different positions on the top bar so that the lemniscate is transformed (figure 6). A set of further variations can be achieved by erecting a triangle on the top bar (figure 7). The results were used to create figure 9.

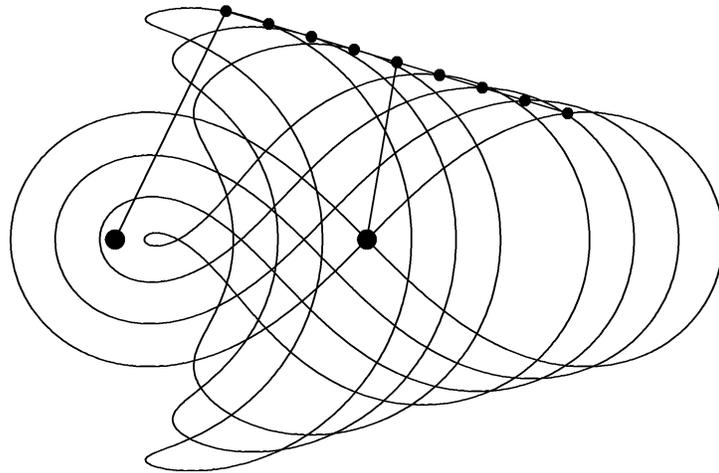


Figure 6, loci of different points on the top bar

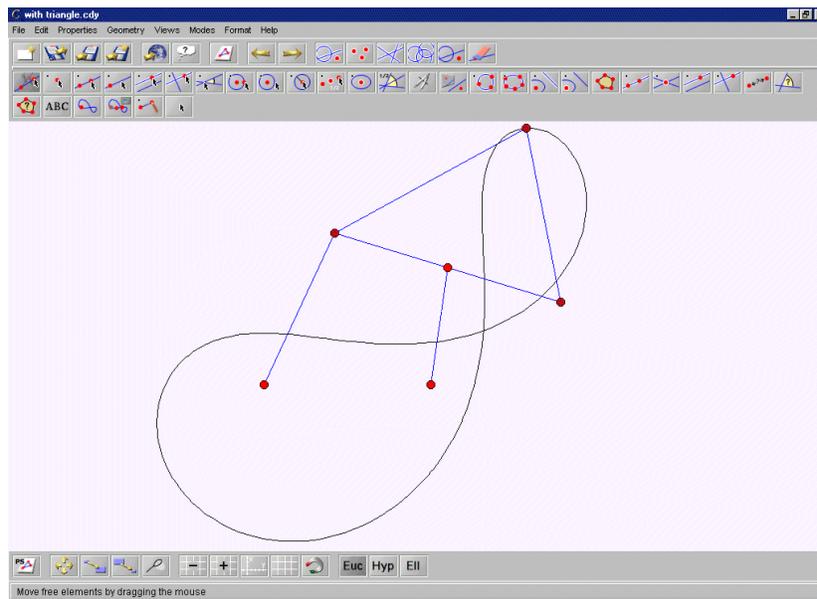


Figure 7, changing the locus (example showing Cinderella screen)

Checker-boarding

Combining and overlaying a set of curves then forms an ideal starting point for checker-boarding in black and white to form op-art designs. This is much easier to do on a computer than with pen and ink or painting. Figure 8 shows the result of using the results from equation 1. Figures 9 and 10 show the result of using output from Cinderella. The curves are exported as a PostScript file which is then imported into a graphics program and shaded by filling. This is far easier than using paint or pen and ink and is a boon to the artist as painting fine detail becomes harder with age as one gets more long sighted.

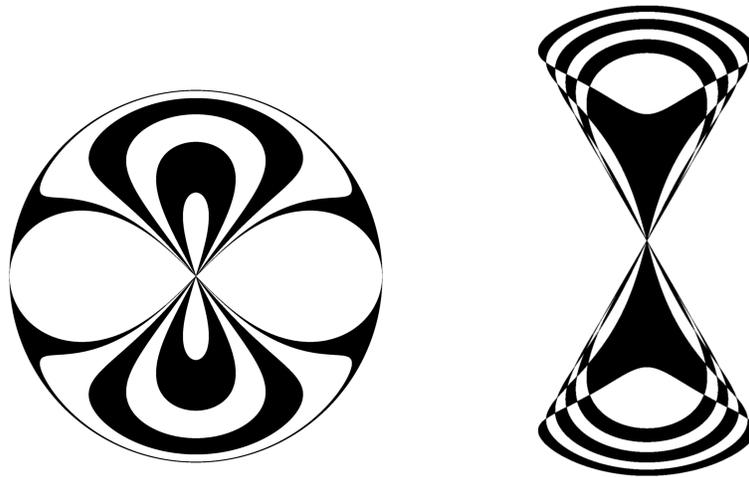


Figure 8, "Pinch" and "Hourglass"

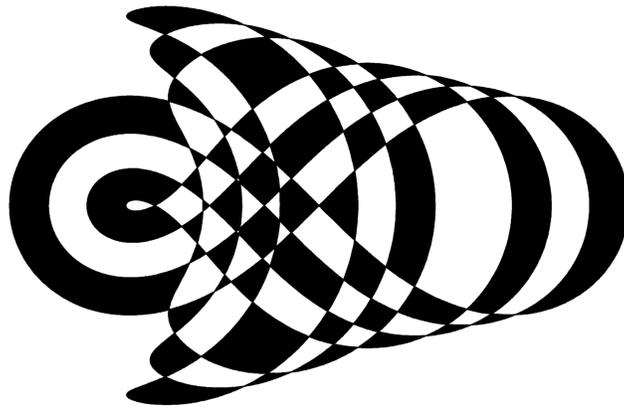


Figure 9, "Shockwave"

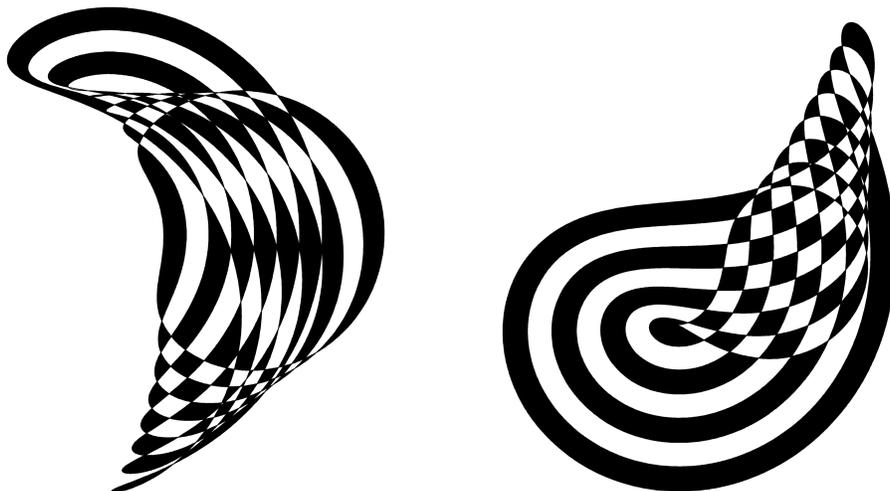


Figure 10, "Dancer" and "Falling Water"

Further possibilities

Computer output has the further advantage that it can be manipulated, for example with various filters. Figure 10 of using a bas relief filter and an impressionist filter with “Falling Water”. More could be done with colour and varying the line thickness of the output. Because I am colour blind, I tend to be happier with black and white.

Another option I mean to explore is that each of these curves is a slice of a surface. With many more linkages than I have described here and the variations I have mentioned, there is a lot of potential for many forms of art.

The linkages I have described here are very basic. More complex ones use sliding bars and many more linkages and pins. With increasing complexity, the variables increase, so the scope is enormous. Many books on linkages and mechanisms do not concern themselves with the curves. References 4-6, however are particularly good for the mathematics.

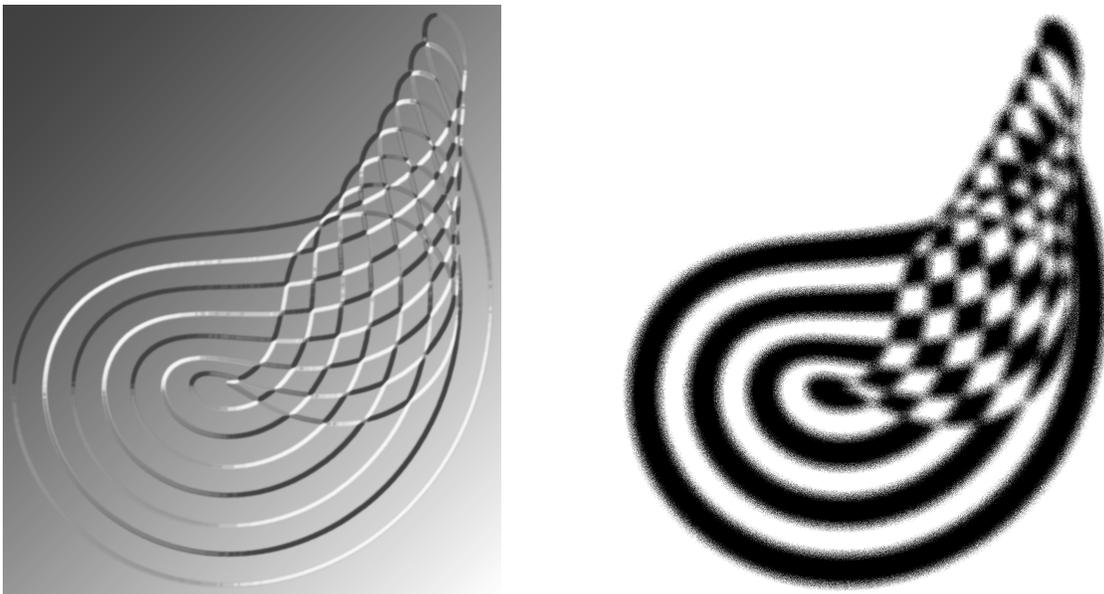


Figure 11, “Falling Water” with bas-relief and impressionist manipulation

References

- [1] Bob Brill, “The Endless Wave”, Bridges Proceedings 2002, p 56.
- [2] A B Kempe, “How to draw a straight line: A lecture on Linkages”, London 1877
- [3] John Sharp, “The infinite Lemniscate”, *Infinity* 2006/4.
- [4] Details of the Cinderella package can be found at www.cinderella.de
- [5] R C Yates, “A Handbook of curves and their properties”;
- [6] R C Yates, “Tools, A Mathematical Sketch and Model Book”, LSU Press 1941
- [7] Eugene V Shikin, “Handbook and atlas of Curves”; CRC Press 1995