Inference and Design in Kuba and Zillij Art with Shape Grammars

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Abstract

We present a simple method for structural inference in African Kuba cloth, and Moorish zillij mosaics. Our work is based on Stiny and Gips' formulation of "*Shape Grammars*". It provides art scholars and geometers with a simple yet powerful medium to perform analysis of existing art and draw inspiration for new designs. The analysis involves studying an artwork and capturing its structure as simple shape grammar rules. We then show how interesting families of artworks could be generated using simple variations in their corresponding grammars.

1. Introduction

Generative specification [1] of paintings has long been a topic of interest. Stiny and Gips' seminal works [2, 3] well demonstrates the potential and usefulness of this model of generating art forms. In this paper we illustrate the application of a generative model to enable us interpret the geometric structure inherent in Kuba[1] textile and Moroccan zillij type of art forms (see Figure 1).

1.1. Kuba and zillij Art. Kuba textile designs can best be described as compositions of "simultaneous diversity."[4]; women artisans incorporate spontaneity and improvisation in their designs to achieve uniqueness and individuality, part of their African aesthetics [5]. They are most often characterized by semi-symmetry. The semi-symmetry in Kuba weaving is achieved by the juxtaposition of distinct geometric motifs and by controlled variations in texture, scale, shape, orientation, and/or colour. (see Figure 1) [5]. On the other hand traditional zillij mosaics depict rigorous symmetry. They are characterized by their intricate interlace pattern and periodic tiling [6]. While their existence has been traced back to the twelfth century, they reached a zenith in Fez in the seventeenth century [7]. Some of the best examples can be found in the *madrasahs* (universities) in the old Medina.

These artworks are an aesthetic expression of culture, social context, and history of a region. Unfortunately not enough knowledge about this expression survives today. As noted by Kaplan et. al. [8], with their original design techniques for such art lost in the sands of time, analysis of these art forms generate considerable interests in the contemporary design and arts community.

1.2. Structural Inference. While Moroccan zillij art exhibits rigorous symmetry, Congolese Kuba cloth's complexity derives from semi-symmetric compositions of elementary geometric motifs. Although the richness of these two art forms is distinctly different, the challenge of inferring the structure of geometric patterns is present in both. In the context of this paper, by *structural inference* we mean classifying the structural composition of these art forms using computational tools. The desired classification should help

[1] African Kuba textiles are also referred to as BaKuba.

an artist, designer or historian understand the underlying generative process involved in creating the design. The emphasis is on finding simple geometric classification techniques so that the knowledge could be easily codified and conveyed to others. From our perspective, this objective presents many analytical challenges which are characteristic of classification and generation problems in pattern design. While continuing to experiment with various techniques [14, 15] like group theory, neural nets and pattern recognition, we have found that shape grammars provide a powerful paradigm both for representing the structural complexity in existing designs and for creating newer ones.

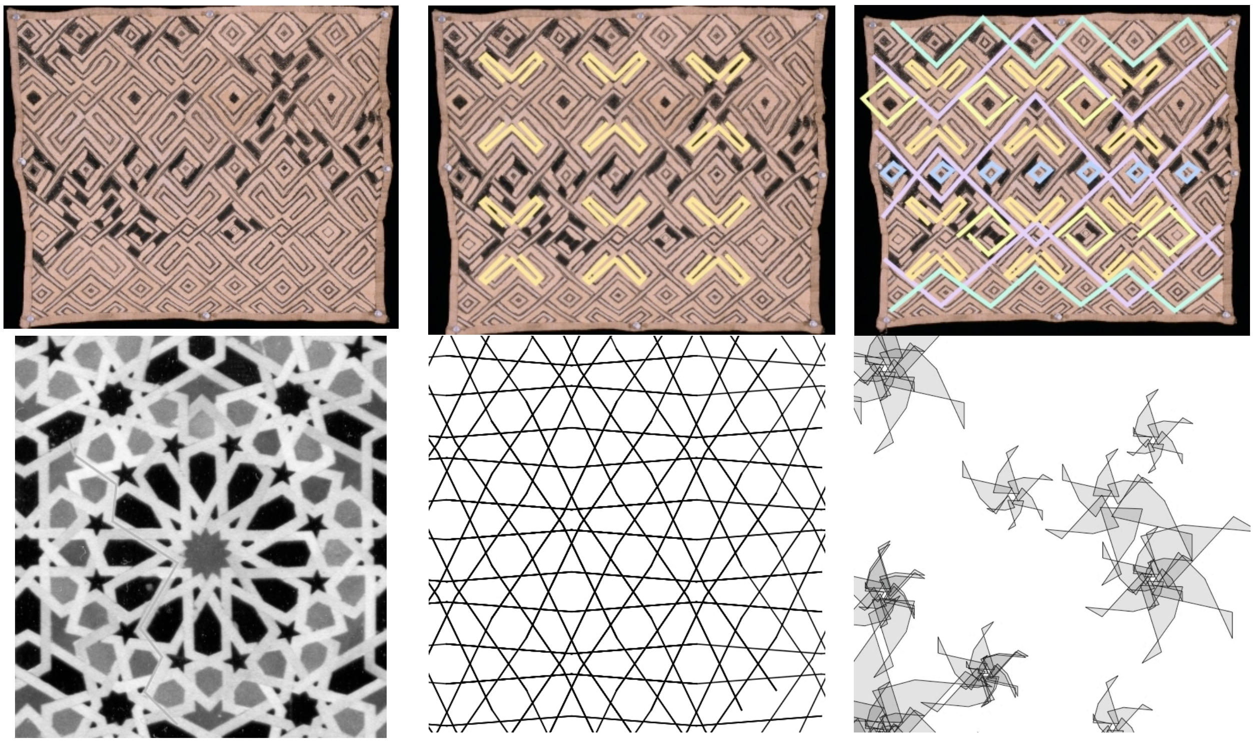

Figure 1: *Top row illustrates* ***inference (analysis)****; the image on the left shows an example of an African Kuba cloth design, the middle image has identical geometric motifs overlaid on the design and the right image demonstrates all the five different types of motifs inferred from the design overlaid on top. The overlaid geometric pattern of the motifs on the right image is generated using a simple shape grammar as discussed in section 3. Bottom Row illustrates* ***new designs (generation)****; the image on the left shows an example of a Moroccan zillij mosaic with an overlaid motif imitating the underlying strand shape. The middle image retains the underlying structure and markers while modifying terminal symbols. The rightmost design simplifies the rule application sequence while changing a terminal and adding scaling to create a radically different design. These images demonstrate new family of art inspired by simple variations in the grammar of the zillij mosaic (see section 5).*

1.3. Previous Work. Earlier research attempts addressing the structural inference problem in Kuba and zillij art are inspiring. Moser and Coxter [17] give a comprehensive treatment to the 17 wallpaper groups employing group theory. Grunbaum and Shephard [6] classified periodic Islamic patterns like zillij using tiling theory. They obtain a fundamental region in a given pattern and infer its symmetry group by observing the strand's movements in it. Abbas and Salman [9] confirmed the theoretic analysis on a large collection of historical designs. For Kuba textiles, Washburn and Crowe [10] confirmed that while 12 of the 17 possible ways a design can be symmetrically varied on a surface are found in these textiles, they are most often characterized by semi-symmetry. Most recently, Grünbaum [18] has developed symmetry schemes for pattern variations on a two dimensional plane.

In design creation, Ostromoukhov [11] proposed steps for Islamic art generation by extending Grünbaum's group theory analysis [6], Dewdney [12] outlined a method for constructing designs based on reflecting lines through a regular arrangement of circles. Kaplan and Salesin [8, 13] presented a software tool, Taprats, which carries out a tile-based construction. It generates the pattern over a fundamental region of the tiling and then subsequently replicates it over the entire plane using the symmetry of the tiling. Taprats allows users to specify the details of the different regular polygons of the tiling using stars and rosettes. For irregular polygons the software infers the pattern based on the existing specification of the regular ones. For Kuba art, Washburn and Crowe [10] present interesting analytical results, but there is little suggestion of how new Kuba like designs might be constructed.

1.4. Our Contribution. The focus of our research is to develop a single, powerful computational system for analyzing the symmetric zillij mosaics, as well as the semi-symmetric African Kuba cloths. This approach has the advantage over traditional methods of providing a uniform analytical approach to pattern analysis, independent of the presence of symmetry. In addition, the generative design mechanism helps in capturing the designs uniquely.

The rest of the paper is organized as follows. Section 2 presents the needed background on shape grammars for specifying the structure. Section 3 shows how grammar rules help in structural inference and subsequently generating an entire family of new designs by simple variations in the parent design's grammar rules. Section 4 outlines the essential components of our Generative Design System. Techniques for creating new designs inheriting a parent's structural properties are given in Section 5. The paper concludes in Section 6 by exploring some opportunities for future work.

2. Shape Grammars

2.1. Pictorial Specification. If we consider the proposition that an artist executes a finite set of shapes and number of transformations in creating an artwork then the resulting painting, sketch or engineering plan could be defined using a *pictorial specification*. Stiny defines a pictorial specification [2] as a set of drawing instructions which can be implemented on a finite plane, in finite time. At the heart of a pictorial specification is an *arrangement of finite lines* (both curved and straight). Any *shape* is characterized by using a particular arrangement. Arrangements are formally captured as a finite sequence of *Euclidean transformations* (which are translation, rotation, scale and reflection). Next in turn we could use shapes as primitives and apply transformations to them to develop patterns based on geometry. This process could be carried out recursively till the desired picture is achieved. Colouring techniques in the artwork could associate as properties of basic shapes (closed ones). Texturing, blending, and fading are the primary coloring techniques employed. But these are not considered as part of a pictorial specification. Hence a pictorial specification should capture the employed shapes and the sequence of transformations on them.

2.2. Formal Grammar and Shapes. The concept of using a grammar to describe a family of artworks has its origins in linguistics. Noam Chomsky [19] defines a formal grammar to be an abstract structure that describes a formal language precisely: i.e., a set of rules that mathematically delineates a (usually infinite) set of finite-length strings over a (usually finite) alphabet. Formal grammars are so named by analogy to grammar in natural language. Formal grammars fall into two main categories: *generative* and *analytic*. A *generative grammar*, the most well-known kind, is a set of rules by which all possible string(s) in the language to be described can be generated by successively rewriting strings starting from a designated start symbol. A generative grammar in effect formalizes an algorithm that generates strings in the language. An *analytic grammar*, in contrast, is a set of rules that assume an arbitrary string to be given as input, and which successively reduce or analyze that input string and yields a final Boolean result indicating whether or not the input string is a member of the language described by the grammar. An analytic grammar in effect formally describes a parser for a language.

In short, an analytic grammar describes how to read a language, whereas a generative grammar describes how to write it. A *shape grammar* by definition is a generative grammar used to capture a family of pictorial specifications. It provides a structured mechanism to capture the recursion inherent in the artwork.

2.3. Definition of a Shape Grammar. Gips [2] and Stiny [3] define a shape grammar as a quadruple: $SG = \langle V_T, V_M, R, I \rangle$ where –

- V_T is a finite set of shapes. A finite arrangement of an element or elements of V_T in which any element of V_T can be used multiple times with the desired Euclidean transformations (translation, rotations, scale and reflection), gives the set V^*_T. Also we define $V^+_T = V^*_T - \phi$.
- V_M is a finite set of shapes such that shapes in the nonempty sets V^+_T and V^+_M are distinguishable.
- R is a finite set of shape rules of the form $u \rightarrow v$, where u and v are shapes formed by the shape union of shapes in V^*_T and V^*_M. The shape u must have at least one sub-shape that is a shape in V^+_M. The shape v may be the empty shape.
- I is the shape formed by the shape union of shapes in the power sets V^*_T and V^*_M. I must have at least one sub-shape that is a shape in V^+_M.

Shapes in the sets V^*_T or V^+_T are called terminal shapes (or *terminals*). Shapes in the sets V^+_M or V^*_M are called non-terminals (or *markers*). Terminals and markers are distinguishable, i.e., given a shape formed by the shape union of terminals and markers, the terminals occurring in the shape can be uniquely separated from the markers occurring in the shape. No shape in V^+_T is a sub-shape of any shape in V^+_M and vice versa.

For the *shape rule* $u \rightarrow v$, u is called the *left side* and v the *right side* of the shape rule. The shape consisting of all the terminals in the left side of the shape rule is called the *left terminal*. The shape consisting of all the markers in the left side of a shape rule is called the *left marker*. (The left marker has at least one sub-shape that is a marker.) *Right terminal* and *right marker* are defined similarly. The shapes u and v are represented in identical sized canvas to show the correspondence between them (in terms of their Euclidian transformations). I is called the initial shape. The shape consisting of all the markers in I has at least one sub-shape that is a marker. Euclidean transformations could be applied to any of the above shapes to facilitate creation and orientation. But only those that are applied to markers in a rule are applied to the subsequent generations employing that marker.

A shape is generated from a shape grammar by beginning with the initial shape and recursively applying the shape rules. The shape generation process is terminated when no shape rule in the shape grammar can be applied. The *sentential set* of a shape grammar, *SS(SG)*, is the set of shapes (sentential shapes) which contains the initial shape and all shapes which can be generated from the initial shape using the shape rules. The *language* of a shape grammar *L(SG)*, is the set of sentential shapes that contains only terminals. In other words, the language defined by a shape grammar L(SG) is the set of shapes generated by the grammar that do not have any sub-shapes which are markers. The language of a shape grammar may be finite or infinite set of shapes. The next section demonstrates examples.

3. Inference and new design

3.1. Developing Shape Grammars. We shall illustrate the application of shape grammars using the Kuba design from Figure 1. Kuba designs present a challenge to shape grammar based interpretation because the set of rules need specificity and detail to describe their complex compositions. Our aim is to enable development of simple shape grammars that do not have an overwhelming set of rules and yet have the full power to capture the structure of the design and its creational process. Figure 2 shows a simplified subset of V_T, V_M , and I of a shape grammar SG1 for the Kuba design in Figure 1. V_T contains a single element, a geometric motif traced out from the Kuba design presented in Figure 1. V_M contains an arrow shape as its only element.

The pattern generated using the four grammar rules of SG1 is shown in Figure 2. Application of rule 1 to initial shape results in the creation of a geometric motif (terminal) and an arrow (marker). Application of rule 2 and 3 causes a translation by x and $-x$ units respectively. Rule 3 causes a horizontal flip. The availability of a marker on the right side forces the continuation of the generation process till we apply a rule in which the marker is dropped from the right. With no marker in the canvas the generation process halts. The desired placement of our terminal symbols (as shown in Figure 1, middle) is achieved by applying rule 2 three times, followed by rule 3 twice, and finally rule 4 twice. Please note that the above generation is parallel in nature, i.e., whenever a shape rule is used it is applied simultaneously to every part of the shape to which it is applicable. Examples of new designs by modifying the parent's grammar are discussed in section 5.

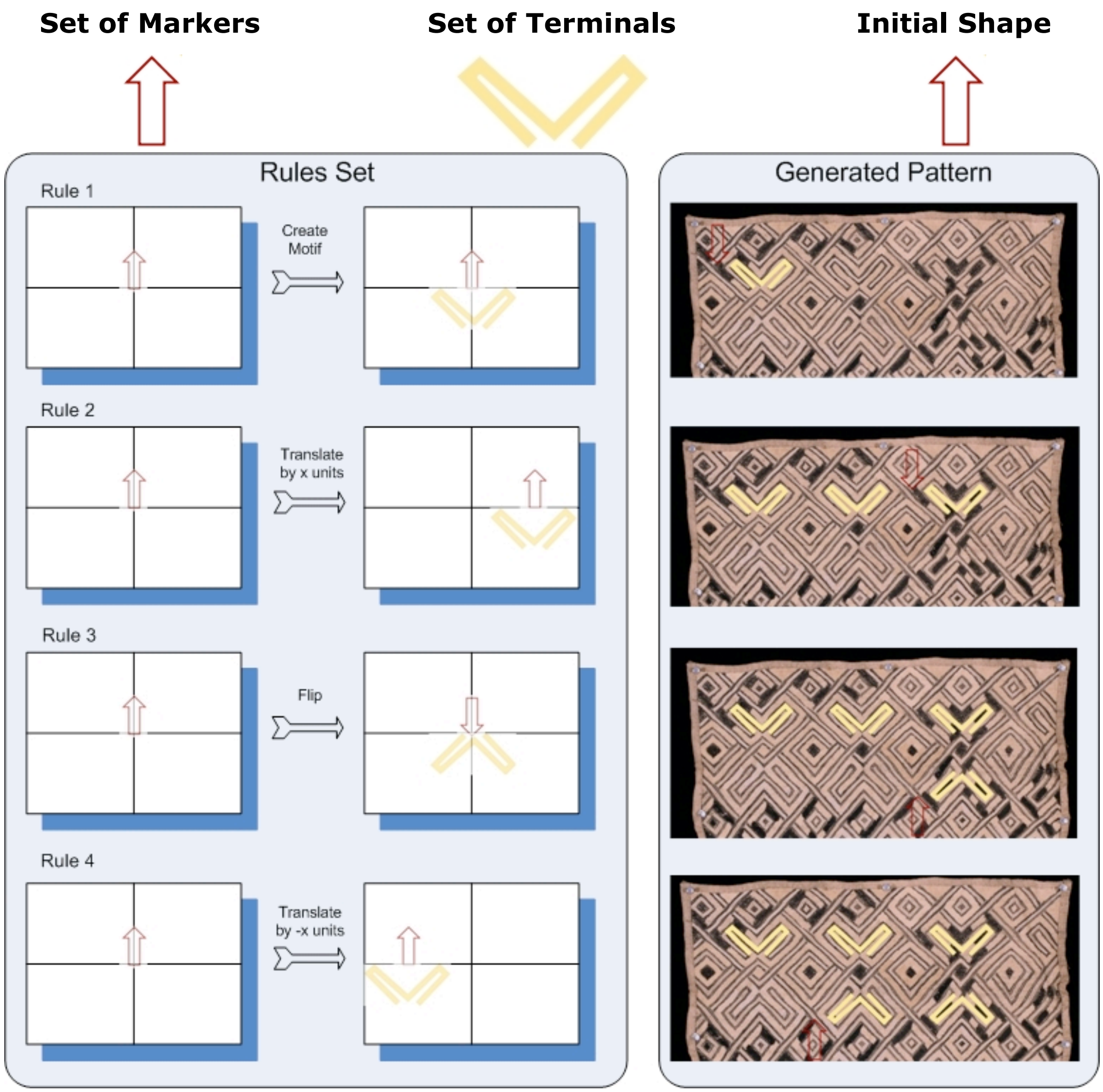

Figure 2: *SG1; Set of terminals, markers initial shape and rules for the Kuba Design in Figure 1.The overlaid pattern on the right is generated by the corresponding left hand side rule application on the design.*

4. Components of the Generative Design System

4.1. XML-based Specification. Computer implementation-wise, we capture the shape grammar for any design in XML with the future intent of being able to create further tools for structure analysis. Our XML schema for shape grammar specification is simple. It consists of three parts: *ShapeList*, *RuleList* and *SequenceList*. ShapeList captures the terminal and marker shapes in the design i.e. the sets V^*_T and V^*_M. RuleList captures the rule set R, of the grammar. For simplicity we use a unique marker as the start symbol. This facilitates modeling the set of initial shapes I, as additional rules. The SequenceList captures the sequence of rules applied to generate the design. Figure 3 lists the shape grammar SG1 (from Figure 2) in our XML. Please note that Euclidean transformations (translation, rotation, scale and reflection) are specified as an inherent property of the rule. This is in complete compliance to the shape grammar specification outlined in section 2. Colors could be associated with each shape. Transparency of the shapes can also be set. Transparency is a useful characteristic when it comes to blending shapes and creating new designs (refer to Figures 4 and 5).

4.2. Grammar Interpreter. The grammar interpreter takes a shape grammar in XML as input and creates a tree based representation of the pictorial specification. Depending upon the sequence of shape rules provided it accordingly creates the design on the canvas.

```
<?xml version="1.0" encoding="UTF-8"?>
<!DOCTYPE ShapeGrammar SYSTEM "sg_ver_1.01.dtd">
<ShapeGrammar>
        <ShapeList>
                <Shape type="marker"  shapeID="ARROW">
                        <Polygon color="150, 0, 0" opacity="0.0" stroke="2.0">
                                <Point x="320.0" y = "300.0" />
                                <Point x="300.0" y = "320.0" />
                                <Point x="280.0" y = "300.0" />
                                <Point x="290.0" y = "300.0" />
                                <Point x="290.0" y = "250.0" />
                                <Point x="310.0" y = "250.0" />
                                <Point x="310.0" y = "300.0" />
                        </Polygon>
                </Shape>
                <Shape type="terminal"  shapeID="MOTIF">
                        <Polyline color="251,228,140" opacity="0.0" stroke="8.0">
                                <Point x="364.0" y = "361.0" />
                                <Point x="327.0" y = "324.0" />
                                <Point x="335.0" y = "317.0" />
                                <Point x="380.0" y = "357.0" />
                                <Point x="422.0" y = "316.0" />
                                <Point x="431.0" y = "325.0" />
                                <Point x="396.0" y = "362.0" />
                        </Polyline>
                </Shape>
        </ShapeList>
        <RuleList>
                <Rule ruleID="INITIAL">
                        <Left>
                                <CompositeShape><RuleShape shapeRef= "ARROW" /> </CompositeShape>
                        </Left>
                        <Right>
                                <CompositeShape>
                                        <RuleShape shapeRef= "ARROW" />
                                </CompositeShape>
                        </Right>
                </Rule>
                <Rule ruleID="TRANSLATE_ON_X">
                        <Left>
                                <CompositeShape><RuleShape shapeRef= "ARROW" /> </CompositeShape>
                        </Left>
                        <Right>
```

```
                              <CompositeShape transformation="translate(180, 0)">
                                     <RuleShape shapeRef= "ARROW" />
                                     <RuleShape shapeRef= "MOTIF" />
                              </CompositeShape>
                       </Right>
              </Rule>
              <Rule ruleID="TRANSLATE_ON_-X">
                       <Left>
                              <CompositeShape><RuleShape shapeRef= "ARROW" /> </CompositeShape>
                       </Left>
                       <Right>
                              <CompositeShape transformation="translate(-180, 0)">
                                     <RuleShape shapeRef= "ARROW" />
                                     <RuleShape shapeRef= "MOTIF" />
                              </CompositeShape>
                       </Right>
              </Rule>
              <Rule ruleID=" FLIP_ABOUT_X">
                       <Left>
                              <CompositeShape><RuleShape shapeRef= "ARROW" /> </CompositeShape>
                       </Left>
                       <Right>
                              <CompositeShape transformation="translate(0, 800); scale (1,-1)">
                                     <RuleShape shapeRef= "ARROW" />
                                     <RuleShape shapeRef= "MOTIF" />
                              </CompositeShape>
                       </Right>
              </Rule>
       </RuleList>
       <SequenceList>
                <Sequence ruleRef="INITIAL"/>
                <Sequence ruleRef="TRANSLATE_ON_X" count=”3”/>
                <Sequence ruleRef="FLIP_ABOUT_X"/>
                <Sequence ruleRef="TRANSLATE_ON_-X" count=”2”/>
       </SequenceList>
</ShapeGrammar>
```

Figure 3: *XML based specification of shape grammars. Please note the correspondence of the marker and terminal shape listed in the <Shape> tags with the sets V^*_T and V^*_M of SG1. Also note the four rules listed in the <Rule> tags and the sequences under <Sequence> tags correspond to the rule set and sequence of application respectively, in Figure 2.*

5. Inheritance and New Designs

5.1. Creating a Family of New Designs. As mentioned in section 1.4, once we capture the structure of a base design using shape grammars we can generate interesting families of artworks by simple variations in its grammar rules.

Figure 4 (first row) demonstrates the grammar rules for the zillij mosaic from Figure 1 (Being evident in the rules, the marker and terminal shapes are not shown separately). The rest of the designs in Figures 4 portray the results of simple variations in the shapes, rules and rule sequences of the grammar.

A grammar can generate related designs within its language, or the grammar can be modified. A modified grammar can inherit terminal shapes from a parent source and rearrange them (Figure 4, row 2; column 2), perhaps changing its symmetry group (Figure 4, row 4; columns 2 & 3), adding details, transforming terminals or modifying their arrangement (Figure 4, row 4; column 1), or retain some or all of its arrangement of markers but change terminals to build a new design with the same fundamental structure (Figure 4, row 3). A child grammar may even retain the exact same pictorial specification but modify colouring rules, yielding a new design through different blending, line traits or colour assignments (Figure 4, row 2; columns 2 & 3).

By traversing grammars' languages or reworking grammars, artists, designers and historians can wander freely in the "neighbourhood" of an existing design, exploring variations and discovering what elements, for them, define the character of an existing pattern. By modifying a pattern to replicate another existing design, the artist or researcher can get a hands-on feel for the similarities and differences between them. The end product of such work may be new patterns, or simply a better understanding of the ones under examination.

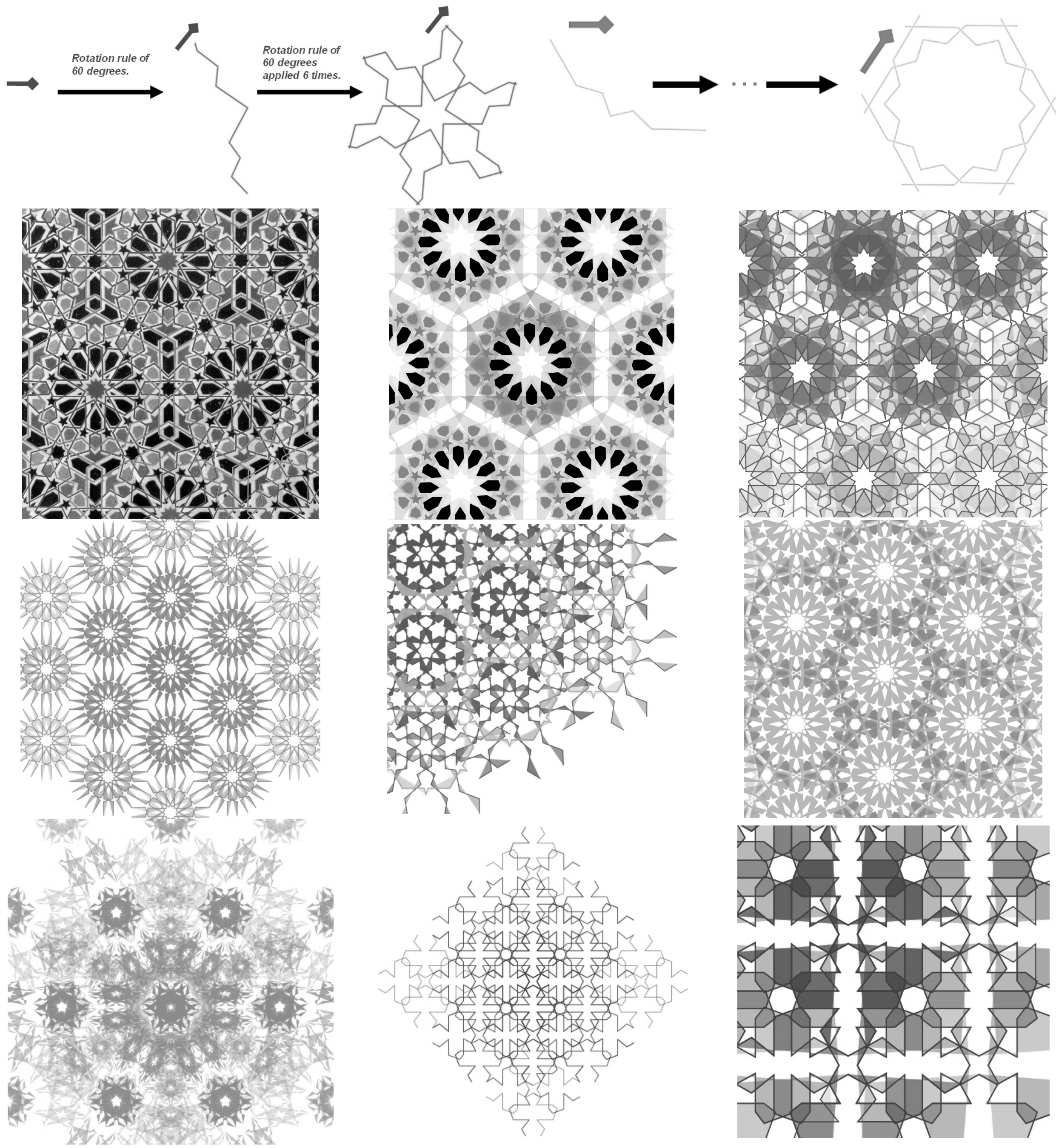

Figure 4: *Row 1: It illustrates the two different strands (terminal shapes) of the zillij mosaic from Figure 1 and their respective shape grammar rules. Row 2: the first image illustrates the recursive application of the rules to achieve the structural inference; the next two images show simple changes in colouring schemes and use of additional motifs adhering to the same grammar rules. Row 3: these designs retain the rules and sequences described in the row 1 but use different terminal shapes (strands). Row 4: the first image has one of the strands of the original mosaic with a non-uniform scale in the rule, generating a non-periodic design; the next two designs are produced by replacing the 6-fold dihedral symmetry in one rule with a 4-fold one, and modifying a terminal symbol.*

6. Conclusion and Future work

Kuba cloth and zillij mosaics seem to be on opposite ends of a design spectrum from the apparently arbitrary deviations within Kuba motifs to the characteristic repetitions of a zillij pattern. These designs present a challenge to shape grammar based interpretation because the set of rules need specificity and detail to describe their complex compositions. Our aim is to enable development of simple shape grammars that do not have an overwhelming set of rules and yet have the full power to capture the structure of the design and its creational process.

The comparison between the periodic, elegant designs of the zillij mosaics and the seemingly arbitrary, improvisational Kuba cloth suggests opportunities for generating an entire class of artworks by simple variations in the grammar rules. As illustrated in Figures 4 and 5 the concept of inheritance from parent design is an interesting contribution of our work. It gives rise to many questions yet to be answered such as what extent of change to the grammar maintains a clear inheritance relationship to the parent design? Radical changes in symmetry group and design elements are supported in our system: how should those changes be considered to support contemporary design practice? How close is the mapping between our grammar process and the development of designs by traditional artisans?

Currently we are exploring the following areas:

- Explorations are being undertaken to create variations, possibly by using heuristic based approaches in the zillij rule set and create an analogue for understanding the geometry of the Kuba cloth patterns.
- Our current efforts are also geared towards building an "artist-friendly" system [16] that facilitates the structural analysis of these artworks through the interactive processes of creating, modifying, comparing and searching for descriptive shape grammars. While the back end of such a system is a databank of artworks in different states of analysis, the front end is a simple to use graphical user interface that provides intuitive interaction techniques for typical shape grammar tasks such as grammar based retrieval, production rule specification or modification, visualizing the evolution of the design as the production rules are applied or modified and structural similarity checking through shape grammar comparisons.
- As our generative system does not assume a regular underlying tiling, interlacing of strands cannot be performed on fundamental regions as proposed in [6]. We are still trying to arrive at intelligent ways to perform interlacing based on the information provided in the grammars.
- We are also looking at the possibility of deriving the tiling of a periodic generative specification. We believe that the shape grammar, if well-defined, has the flexibility of predicting the symmetry group of the generated pattern and hence it inherently contains the classification of the tiling in one of the 17 wallpaper patterns [17]. This classification is useful since it allows one kind of comparison between existing and newly generated patterns. This is currently one of our main research foci.
- We are exploring the range of variation in grammar rules and its effect on the inheritance relationship with the parent design. It is not certain where culturally or structurally clear inheritance relationships end and novel designs borrowing from a parent artefact begin. Nevertheless, the inheritance of shape grammar elements can be applied in elegant and novel ways to build designs which draw on an existing, rich heritage.

7. Acknowledgements

This work is supported by grants from NSERC (Science and Engineering Research Council, Canada) Discovery grant programme, FQRSC (Fonds Quebecois de la recherché sur la société et la culture) Appui à la recherche-création programme, SSHRC (Social Sciences and Health Research Council, Canada) Research/Creation in Fine Arts programme, Hexagram and Concordia University Faculty Research grants.

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