Love, Understanding, and Soap Bubbles

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Abstract

As an artist my interest in mathematics has evolved through a love of nature and a desire to better understand the "nature of things". An evolving interest in natural efficiencies has recently led to a thorough investigation of soap bubble foam, where I have found the relationship between pressure differentials and geometric organisation of particular interest. Through this study I have developed a physical modelling system, which is the foundation of my latest Artwork(s).

Development

Fascination focused on an all embracing notion of nature remains the motivation behind my artistic activities. This fascination is nourished by that revealed from observations and investigations into the ways of nature, a process which more often than not will include a mathematical analysis of patterns, structures, and functions, found all around. Simultaneously love has developed a voice, expressed when successful as "poetry of space" within the marriage of spirit and matter. Over the years any distinction between these developments has become more and more blurred. I have come to consider mathematics not only as a language to describe the weave of life, but also as the thread of its fabric.

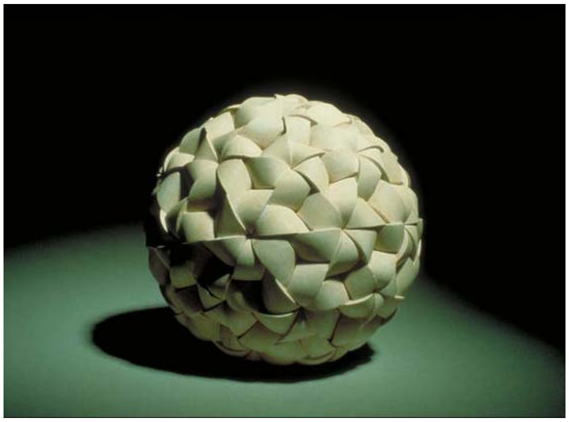

Figure 1 "Orb" by Simon Thomas.

For someone who learned more on the riverbank than in the classroom my connection with mathematics has evolved along less orthodox lines than some. My first brush with mathematical magic was at primary school, when with a compass I drew my first “flower”.

Many years later whilst a post-graduate student at the Royal College of Art I rediscovered geometry’s flowery gateway, and this time armed with compass, pencil, and ruler, embarked on a journey into the wonderland of divided space. In the early days I employed my new found knowledge and skills to create practical devices such as templates and the like, but soon enough I was seduced by the process itself, investigating the properties of various proportions, sequences, and symmetries.

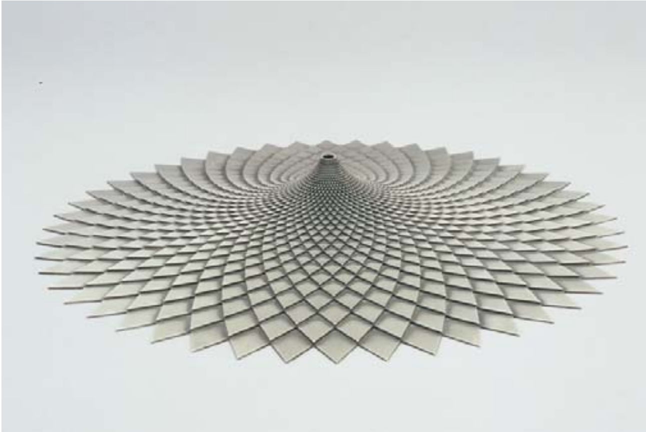

Figure 2 “Plane Liner” by Simon Thomas.

Humanity’s sense of beauty has long been a topic of debate, thought of by some as purely subjective, whilst others prefer the “reflection of God notion”. It is my belief that there is a strong relationship between natural efficiencies and our perception of what we call beautiful. Why are we predisposed to appreciate beauty in such things as the design and fabric of a dragonfly’s wing? Or, what about the dreadful and awesome sight of a sardine “bait-ball” under attack? Could it be that these things are fascinating because throughout human evolution we have learned to recognize “efficiencies” as a vital part of our survival mechanism; an appreciation of how to gain most through least possible effort?

Figure 3 “Eye” by Simon Thomas.

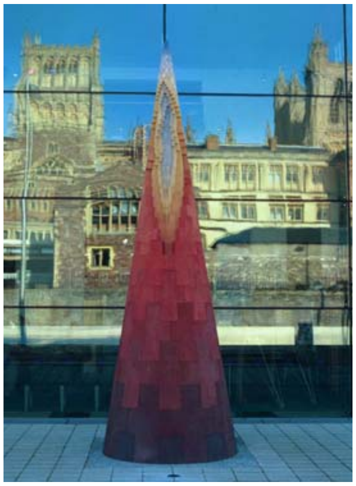

Figure 4 “Small Worlds” by Simon Thomas.

Studies in Soap Bubble Foam

Introduction

Throughout my study of soap bubble foam geometry I have been transported by a sense of wonder in the beauty of this optimized fluid structure. It is surely an unsurpassed expression of energy conservation, the qualities of efficiency and beauty inexorably interwoven. It is an example of a dynamic system where the parameters of geometric possibilities and the laws of physics co-exist in an ephemeral harmony.

Closed and open celled foams seem to be found across the scale range of existence, in organic and inorganic structures. From quantum foam at Planck length, right through to that state suggested in the research of cosmologist Margaret Geller, where the distribution of galaxies within the cosmos appear to be located within a foam structure.

Soap bubble foam is a randomly arranged congregation of air pockets varying in volume, encapsulated by films of detergent/water. Detergent lowers the amount of surface tension experienced by the water it is mixed with. If air is forced through the soapy water it will surface as a bubble. When this process is repeated many times, these bubbles find themselves positioned within a colony of neighboring bubbles, and through this coming together the original sphere bubbles transmute into polyhedral cells. The structure of this water matrix has long been a muse, and the numerous attempts at understanding its math and physics are testament to its charm.

Studying the history of this subject with its experiments and hypotheses has been totally captivating, inspiring me to join the chase in hunting down this particular grail. Rigorous observation in tandem with the relevant reading matter has been accompanied by various experiments to form my own research.

Within the apparent chaos of closed cell foam there are certain constants. Plateau's laws tell us about the shapes and connections of soap films:

(1) Films can only meet three at a time and they do so symmetrically, so that the angles between them are 120 degrees.
(2) The lines along which they meet are themselves joined in vertices at which only four lines (or six films) can meet. Again they are symmetric, so that the angle between the lines is 109 degrees (the tetrahedral, or Maraldi, angle).
(3) The films and the lines are curved in general: the average amount by which the films are bowed in or out is determined by the difference in pressure between the gas on either side (Laplace's law).
Note that the third law does not dictate that a zero pressure difference implies a flat film. Saddle-shaped surfaces can have zero mean curvature.

Now, you may ask, what is the role of an Artist in all of this? Well, I am in the process of creating an artwork which aspires to reflect the geometric conditions of this water matrix, and aims to celebrate the mesmeric relationship between beauty and natural efficiencies. In re-presenting experience the Artist, to a greater or lesser degree edits, orders, and interprets perceptions, entertaining certain compromises such as the use of a particular medium, or "freezing" a dynamic system. These compromises are an intrinsic part of the creative process, setting parameters, and where appropriate should be seen as opportunities. I also have to consider the various modelling techniques available to me, always with an eye on the practicality of any given process, its durability, and how they might impact on the overall "feel" I am aiming at.

Experimentation

To enable the creation of a credible Artwork concerning the structure of foam, the Artist should, at least, have a partial understanding of the subject. As mentioned earlier, I have sought to build my knowledge through observation, reading, and experimentation.

The most fruitful experiments I've conducted to date have been achieved with the help of a particular molecular modelling node. As I came to understand the structure of foam a little more, I realised that this node along with the use of flexible joining rods of differing lengths, could enable a very handy modelling technique, one I have subsequently found to behave in a very similar way to the edges of real foam.

In my past experience of working alongside Mathematicians, one trick learned in understanding difficult spatial problems is to whenever possible reduce the issues involved by spatial dimensions. So, for instance, when investigating the properties of a sphere, where possible simplify things by applying the question to a circle. When appropriate this approach can enable an understanding where previous attempts proved overly complex.

The benefits of dimensional "downsizing" (by which I reduce the foam to only edges and corners), allied with the development of a simple modelling technique where I have utilised a particular molecular node, has been invaluable. The node I am referring to is that associated with the fourfold carbon bonds found in diamond, the one where the four equally spaced valency pegs would fit into the four corners of a regular tetrahedron. The angle between pegs is approximately 109 degrees and is known as the "Maraldi". This is the same symmetry we find at the very heart of the four edged corners of foam.

Figure 5. Carbon atom nodes with "Maraldi" angles.

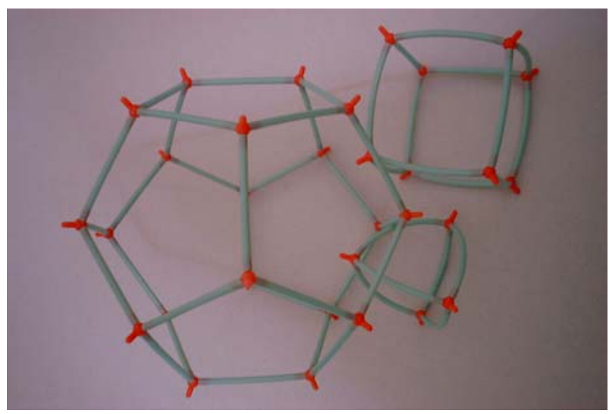

Figure 6. Polyhedra indicating relative edge curvatures.

While working with this modelling method I have developed an interest concerning the role of geometric parameters in fashioning the structure. The nodes, in combination with flexible bond rods of various lengths, create a network where the rods are obliged to deform through curvature, seemingly just as the edges of foam cells do. The positions and alignments of the nodes, and the way rods set in certain arrangements correspond to certain curvatures, is striking in its resemblance of the cell edges and corners of real foam.

There is a general scaling associated with the characteristic curvature and geometric arrays in relation to cell sizes, and this is tied up with the effects of surface tension. The larger cells have proportionally less pressure than smaller ones, and therefore are spatially invaded by the latter, causing a concaved face on the larger cell. This state of affairs is due to the surface tension being weaker over a larger surface area. The surface tension therefore exerts proportionally less constrictive force on the larger encapsulated air pockets or cells.

Larger individual cells can have convexed, concaved, flat and saddle shaped faces, all part of the same cell. This indicates a rule that wherever a cell is larger in volume than a neighbour, the face it shares will always be concaved into the larger one.

The relationship between cell size/pressure, and its geometric expression, is very much influenced by the Maraldi angle sat in the cell corners. The smaller cells of higher pressure create the most convexed curvature. Development of this modelling system with it's 109 degree angle, and through observations of real foam, have informed me that the maximum convexed curvature in rods (or edges) is witnessed when they form part of triangular faces as found on tetrahedra. A little less convexed edge curvature is found as part of a square face, approximately no edge curvature with pentagonal faces, and edges gradually increase in negative curvature (concaved) for faces with more than five edges. Of note here is that an edge is not exclusive to one face, or "ring" of edges. With each edge sharing three faces, and the faces having an angle of 120 degrees set between them, it is likely that the curvature of one edge is the mean of these three sets of force.

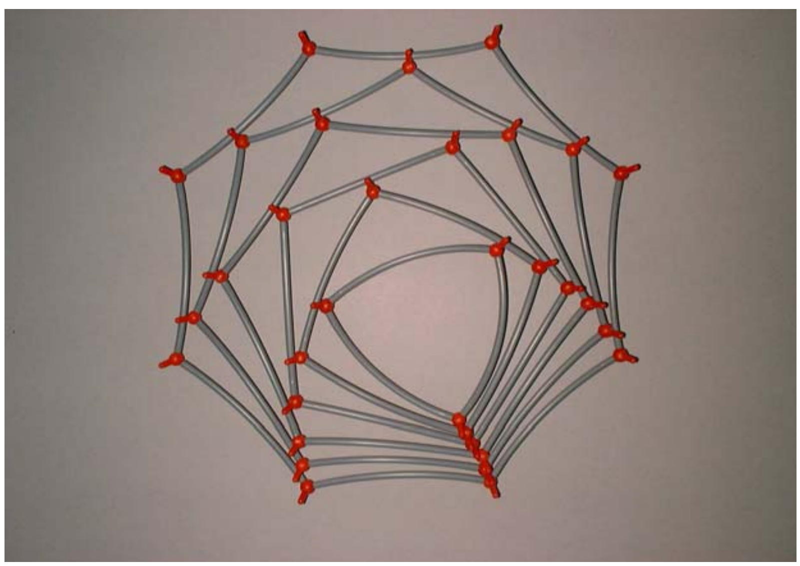

Figure 7. Clustered "Rings of edges" showing positive, neutral, and negative curvature.

When modelling in this way and as a foam bank is evolving, decisions on comparative rod lengths become more critical and choices are taken to eradicate any compound curvature in the rods, which seem to seldom occur in real foam. It is an intuitive process where the "trial and error" approach eventually pays off. The high degree of freedom in this modelling technique apparently corresponds to the character of cell edges and corners in real foam, where countless combinations of cell shapes and cell sizes will meet.

Worth bearing in mind here is that as with real foam, the “cells” on the periphery of the colony are different from those within the bulk of the complex. In the modelling system, cell edges surrounded by neighbours have full influence from the three “rings of edges” which they are part of (rings of face edges in real foam) whereas those cell edges not totally enclosed have rods of a more “relaxed” nature. Also of note is that where soap cells meet the internal surface of a containing vessel, cell edges linking that 2-D “vessel surface design” to the foam within, inwardly project at tangents of approximately ninety degrees. This is where the transition of spatial rules between three and two dimensions is taking place.

My modelling process is always accompanied by a large glass vessel filled with soap foam, so that I may compare as I progress.

Summary of findings

Foam expresses the dynamic equilibrium formed when the forces of pressurised air pockets co-exist with the surface tension of water, forcing an accommodation of each other’s energy. So, in simple terms we see a dual system of air pressure versus water tension, although in reality they are inextricably involved in each other’s status; and other lesser forces are involved.

Understanding the transition from a wet foam to a dry foam, is essential for a clear view of the geometric characteristics of a closed cell dry foam.

Imagine the moment when a wet foam is being formed, many spheres of air are surrounded by adequate water so that no contact is made with neighbouring air bubbles. As gravity drains the excess water from around the bubbles they will soon come to make contact. The first thing to happen is that they will kiss, that is to say that they touch at a point. These points rapidly open out laterally on the surfaces of neighbouring bubbles, forming a disk of contact. The disks enlarge until other disks similarly generated around the bubbles meet. This is the moment that edges and ultimately corners are created, changing the bubbles into polyhedral cells. Surface tension is the cause of this constriction, and the process ceases when the pressure within the air pockets is strong enough to resist further progress.

What were once independent bubbles are now a colony of interdependent cells. The characteristics of corners, edges, and films, are arranged by default in accordance with underlying geometrical efficiencies necessary to sustain a structural integrity. The most interesting aspects of this geometric default is the consistency of Maraldi angles (approximately 109 degrees) at the corners of cells, and the 120 degree angles emanating between film faces “spline-like” along the curved edges.

These particular angles are present owing to their fundamental nature. The Maraldi, or tetrahedral angle, is the first symmetrical division of 3-space when emanating from a point. It has four directions equally spaced, the least possible. The three directional symmetry associated with film co-angles, is a consequence of this corner angle, the cell walls could not be arranged in any other way.

These constant corner angles, where four edges and six films meet within a random colonisation of various sized air pockets, cannot join up in straight lines (edges). Curvature along the edges in between these corners is how this problem is overcome, and similar forces replicate these curves in the modelling system. By subtracting the cell faces from the equation, leaving only corners and edges, the difficulty of linking these angles is taken up by flexibility in the rods.

Whilst understanding that the films (faces) are remnants of original independent bubble surfaces, in the context of a cell complex I prefer to see films as simply spanning “rings” of edges. These films are minimal surfaces, indicators of pressure differences yes, but also subject to the powers of geometric default in the corners and edges of that network.

The Artwork

Foam structure is an eminent expression of energy conservation, and in my opinion beautiful because of this. This mysterious structure where these qualities of efficiency and beauty are interwoven is a real wonder, and fruitful meditation on this subject will draw the viewer in contact with elemental geometric rules, and their physical consequences.

Now, for me the science of the subject is only half of the story, I also have to evolve an artwork, which above all has presence and meaning. The reader may appreciate that building a durable structure which is closely related to soap foam is not going to be an easy ride. It seems likely to be fraught with endless technical problems, inhibiting geometric parameters, and visual flaws; and so it is! I have worked through quite a few ideas concerning the look and build-technique of such a sculpture, and within the particular circumstances of my current commission I have decided to more or less stick with uncomplicated look of the modelling system, a design I feel is both technically plausible, and visually arresting. One thing I have done in order to fortify the visual impact of the piece is to exponentially increase the cell sizes in the vertical direction, adding a sense of the foam "billowing forth".

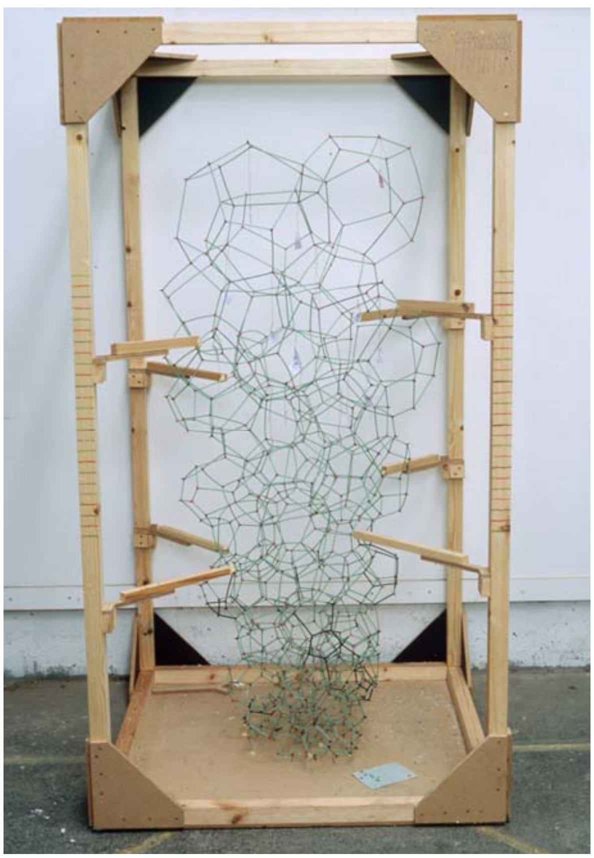

Figure 8. Modelled foam complex in frame.

Figure 9. "Cell" being cast.

In this way the Artwork will describe the edges and corners only, of a foam complex. Fabricated in 6mm stainless steel rod, every weld will be carved /ground back to form an "oily" minimal surface look. The feel will be one of a continuous homogeneous surface over the whole network, not lots of spars "spot welded" at their ends.

In order to transfer accurate co-ordinate information from the plastic model to the sculpture itself I have taken a cast from every one of the 71 "cells". These are composed of a heat resistant plaster based material (developed on the job) and will be used to weld upon. The rods will be rolled to the correct curvatures before being incorporated into the final design. Finally, the frame within which the plastic model has been located will be utilised to offer datum points, ensuring a faithful representation of the forces freely expressed by the tensile plastic model.

Figure 10. Seventy one "cells/ formers", the edges of which are to be used for welding upon.