Dynamic Geometry/Art in Mathematics Classroom

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Abstract

This paper is our follow-up on the article Fostering Understanding of Mathematical Visualization [1]. It is focusing on mathematical visualization of two-dimensional geometric objects within the dynamic geometry environment. Furthermore, it attempts to bridge the gap (if one exists) between the geometry and art, suggesting dynamic geometry/art activities as central for students' understanding of geometric transformations.

1. Introduction

Arcavi suggests that "*visualization* is the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understanding" [3, p. 56]. How is Arcavi's definition of visualization related to artistic expression? *Visualizing* encompasses both the process and the product of creating, producing and constructing pictures, applets, images, icons, even symbols, in order to represent what we perceive to be relevant for an understanding and representing that understanding. Both mathematics and art have their own language, form, structure and mode of expression; mathematics, as well as art, requires creative problem-solving skills, facilitation of both informal and formal skills in order to develop and support creativity and intuition. Developing expertise in selecting appropriate tools, media and approaches are common, in their own contexts, to both mathematics and art apprenticeship [16].

Cox and Brna [5] have shown that when people are learning complex new ideas it helps to interrelate/manipulate various visual representations like diagrams, graphs and animations. If the learner can integrate information from representations with different formats then they often acquire a deeper understanding of the concept. On the other hand, if the learner fails to make the connection between the different kinds of information, then many of the benefits that multiple representations provide may not occur (e.g. [18]). Furthermore, *multiple representations* for certain concepts have been linked with greater flexibility in student thinking (Ohlsson [11] as cited in [9]) and visualizations have a particular place in acquiring adequate representations.

This paper provides a couple of randomly selected practical illustrations for fostering understanding of geometric visualization, as a means for both understanding certain mathematical processes and developing one's artistic expression. A full range of activities and illustrations will be prepared for the conference and available on a digital medium.

2. The van Hiele Levels of Geometric Thought

While Piaget and Inhelder [12] suggest that the development of perception as described by the types of geometry are sequential (i.e. Topological, Projective, Euclidean), other researchers believe that all types of geometric thinking continue to develop over time and become increasingly integrated. The stages of development suggested by Piaget & Inhelder are similar in that they demonstrate the child's naturally increasing ability to perceive and represent the geometric complexity of our three-dimensional world. Both sets of stages emphasize the importance of comprehending spatial relationships between objects and finding ways to show these relationships through 'perspective' drawing techniques.

Originally there were five van Hiele levels, which have been adapted and renamed by various researchers, but now van Hiele concentrates on the three levels that cover the regular schooling time. The *visualization* begins with 'nonverbal thinking'. Shapes are judged by their appearance and generally viewed as 'a whole', rather than by distinguishing parts. At the *analysis* level, students can identify and describe the component parts and properties of shapes. For example, an equilateral triangle can be distinguished from other triangles because of its three equal sides, equal angles and symmetries. Students need to develop appropriate language to go with the new specific concepts. However, at this stage the properties are not 'logically ordered', which means that the students do not perceive the essential relationships between the properties. At the i*nformal deduction level* properties of shapes are logically ordered. Students are able to see that one property precedes or follows from another, and can therefore deduce one property from another. They are able to apply what they already know to explain the relationships between shapes, and to formulate definitions. For example, they could explain why all squares are rectangles. Although informal deduction such as this forms the basis of formal deduction, the role of axioms, definitions, theorems and their converses, is not understood. Students can see (establish) interrelationships between figures and derive relationships among figures. Simple proofs can be followed but not understood completely. Students at the *deduction* level understand the significance of deduction and the role of postulates, theorems, and proofs. They can write proofs with understanding. Students understand how to work in an axiomatic system. They are able to make abstract deductions. Non-Euclidean geometry can be understood at this highest, *rigor* level ([17], [18]).

These stages of learning are significant in providing a framework for instruction aimed to develop understanding of the material or skills to be learned [4]. The main idea is that a learner cannot achieve one level of reasoning without *having passed* through the previous levels. What "having passed through" means in this case is achieving deeper understanding of concepts and relationships attached to that level of reasoning.

> You can say somebody has attained higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these operations on new objects. The attainment of the new level cannot be effected by teaching, but still, by a suitable choice of exercises the teacher can create a situation for the pupil favorable to the attainment of the higher level of thinking [37, p. 39].

3. Visualizing with Dynamic Geometry.

Students' exploration with a variety of representations when building their conceptual understanding of mathematical ideas is very important and has been studied by many authors ([8], [6]). Demana and Waits emphasize "the ability of students to operate within and between different representations of the same concept or problem setting is fundamental in effectively applying technology to enhance mathematics learning" [6, p. 218]. Schultz & Waters [14] suggest careful consideration for selecting representations that will facilitate students learning. Visual representations in technology-augmented activities support mathematical connections in at least three ways: (a) linking multiple representations of the same mathematical idea, enhancing the context for reflective abstraction, (b) interconnecting mathematical topics and (c) connecting mathematics to real-world phenomena. Appropriate use of technology supports learners, both teachers and students, in bringing together multiple representations via intermediate representations and explicating connections among different representations of mathematical phenomena.

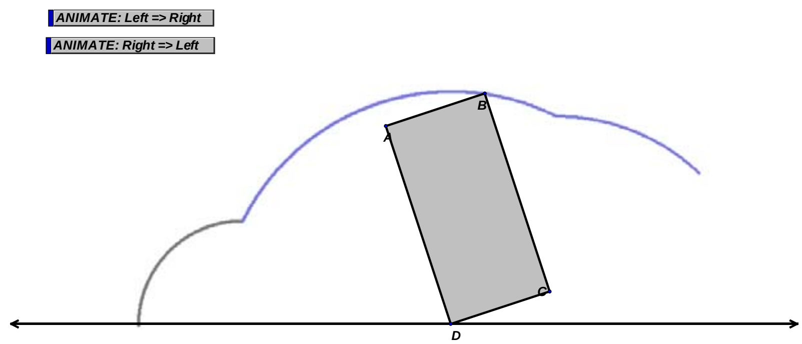

Figure 1: *Rolling rectangle tracer*

A visual solution to a problem can engage students with "meanings which can be easily bypassed by the symbolic solution of the problem" and can bring geometry-based representations to the aid of what seem to be purely symbolic processes [3, p. 62]. For example, one can symbolically study systems of equations with two variables without making a connection with geometry-based representation of the situation; relationships of two lines in a plane ([1]).

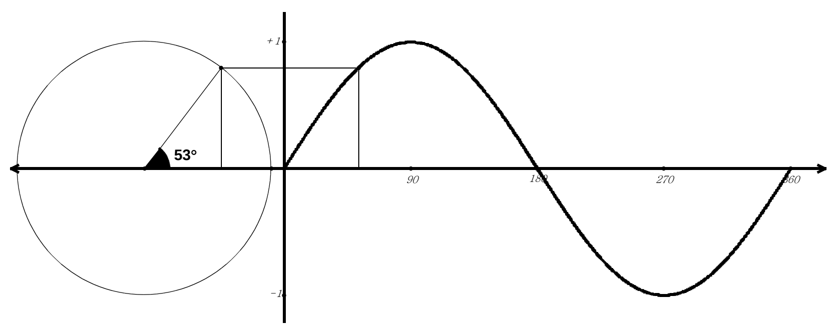

Figure 2: *A caption of an animated visualization of the sine function*

Dragging/animating a geometric figure across a computer screen illustrates dynamic behavior of geometry programs such as Geometer's Sketchpad, Cabri and Cinderella. That capability has significantly changed the quality and impact of geometry experiences. Furthermore, it has provided broad opportunities for learners to experiment with, explore and visualize objects in qualitatively new ways. One construction in one of these media, unlike on a piece of paper or a chalkboard, provides a source of experimentation with a range of examples, making available a collection of representations for further study. This learning to visualize differently requires learners to think differently. Change in the instructional strategies is an imperative, especially if teachers' education was traditional and opportunities have not been provided to bring teachers to the "speed" in a dynamic geometry classroom.

Unlike the static images constructed by hand using straightedge and compass, dynamic geometry figures can be manipulated, having the variant properties changed by dragging. One can consequently observe invariants, producing large amounts of data to analyze [13]. Dynamic geometry tools are engaging students in active learning in geometry. Solving problems, posing questions, creating conjectures, searching for connections, considering counter-examples or formal deductive proofs are enriched with our ability to visualize and reason using diagrams. In each of these formats, dynamic geometry is instigating a significant change. We are able to make more visible our internal representations of geometric figures and geometric transformations, and to refine them further when necessary.

A kaleidoscope can be visualized as two mirrors at an angle of $\pi/3$ or $\pi/4$ to each other. When an object is placed between the mirrors, it is reflected 6 or 8 times (depending on the angle). A dynamic geometric representation of this in Geometer's Sketchpad can produce interesting designs along with a better understanding of how the kaleidoscope's fascinating pictures come about.

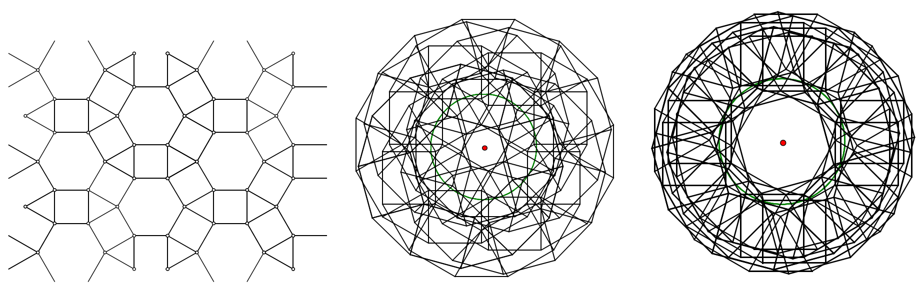

Figure 3: *Geometric transformations: Tiling and kaleidoscope(s)*

Pedagogical Implications

Technological tools available for visualization broaden both curricular and pedagogical opportunities. Geometry explored through geometric art is both motivating and real-life connected. Dynamic geometry provides for students' development of both internal and external representations in both geometry and artistic expression. It has already been noted that as technology develops, its uses in the classroom provide both opportunities and major challenges. There are many new pitfalls and misconceptions, along with new ways of motivating students and providing a broader range of experiences in a shorter timeframe ([10], [13], [1]). Not only do teachers need to be continually aware of what their students are really seeing in the dynamic geometry representation, which may be quite different from what the teacher expected, but also the pedagogical instructions and questions themselves need to change [3]. For example, after constructing the three medians of a triangle, a paper-and-pencil student might be asked to describe what he/she observed and to repeat the exercise with a different triangle, while a dynamic-geometry student could be asked to manipulate the original triangle to see what will happen to his/her construction as the shape of the triangle changes. And the construction may or may not "hold together" to verify the teacher's expected result, depending on the steps the student took in the construction. Also, new possibilities arise for investigation, such as the "split points" command, with which a more advanced student might experiment, to explore how many different ways a single point in the construction has been used.

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