# Tessellation Techniques

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## Abstract

Tessellations are continuous repetitions of the same geometric or organic shape, a tile or tessera in Latin . Inspired by the abstract tessellations of the Alhambra, aided by a correspondence with Coxeter, Escher developed a method of transforming polygonal tessellations into representational tessellations of organic shapes.

## Mathematical Presentation

Tessellations of polygons can be edge-to-edge and vertex-to-vertex (Fig. 1) or translated to be edge-to-edge but not vertex-to-vertex (Fig. 2).

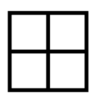

**Figure 1:** *Quadrilaterals, edge-to-edge and vertex-to-vertex*

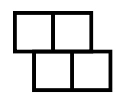

**Figure 2:** *Quadrilaterals, translated*

Escher first determined which polygons could tessellate by themselves, e.g. triangles, quadrilaterals, regular hexagons (Figs. 3 and 4 ), and which could not, e.g. regular octagons and regular pentagons (Fig. 5). (A polygon is regular if all of its sides and interior angles are congruent.)

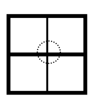

**Figure 3:** *Quadrilaterals*

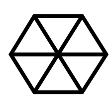

**Figure 4:** *Triangles*

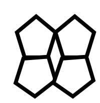

**Figure 5:** *Regular Pentagons*

At the vertex of a tessellation, the sum of the angles of the adjacent polygons is 360 degrees. So, for example, a regular hexagon can tessellate by itself because at the vertex of the tessellation the sum of the angles is 360 degrees, each of the three adjoining angles being 120 degrees (Fig. 6). Conversely, regular octagons cannot tessellate by themselves because each angle of a regular octagon is 135 degrees and 135 is not a factor of 360 (Fig. 7). 2(135)=270. 360-270=90. Therefore, the other polygon needed to complete the tessellation is a square.

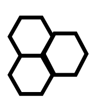

**Figure 6:** *Hexagons*

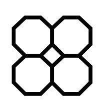

**Figure 7:** *Octagons*

The formula for determining the number of degrees in each angle of a regular polygon is 180(n-2)/n, where n = the number of sides. Alternatively, the number of degrees in each angle can be found by dividing the regular polygon into triangles which share the same vertex. The sum of the angles in each triangle = 180 degrees. For example, in an octagon, there are six triangles. 6(180)=1080. Therefore, the sum of the angles in the octagon is 1080. There are eight angles in the octagon. 1080/8=135

## Curricular Connections

There are numerous ways that a lesson about tessellations can be integrated into the general curriculum:

- Mathematics: The connections to mathematics are obvious: mathematical terminology such as midpoint, polygon, rotation, translation, reflection; the derivation of the formula mentioned above; information about Coxeter and his work.
- Art: M. C. Escher, Victor Vasarely and Bridget Riley are all artists who have employed tessellations in their work. Escher developed a method of transforming polygonal tessellations into representational tessellations of organic shapes such as fish, birds, angels and devils.
- Architecture/World Cultures: An investigation of the Alhambra can move from the observation of architectural and interior design to an important discussion of the positive and negative interactions between different cultures and religions.
- Literature: Washington Irving wrote *Tales of the Alhambra* by imagining what might be happening outside his window during a prolonged stay at the Alhambra. Students could emulate Irving and create stories or poetry based on what they imagine to be happening outside their own windows.
- Music: A simple round such as *Row Row Row Your Boat* involves (mathematical) translations of the entire song. The music of composers such as Philip Glass utilizes a more complex form of aural tessellation.

  Row row row your boat, gently down the stream...

  Row row row your boat, gently down the stream...

  Row row row your boat, gently down the stream...

## Hands-on Activity

This hands-on presentation will focus on a few polygons which tessellate by themselves and various methods of transformation such as translation, rotation about the midpoint of a side, rotation about a vertex, and glide reflection. Examples of Escher's tessellations will be shown to illustrate some of these methods. After transforming a polygon into an organic shape (Figs. 8 and 9), participants will create their own tessellations. They can develop them as abstract or representational objects. Suggestions will be given for various ways of adding color and otherwise finishing the tessellation as a work of art.

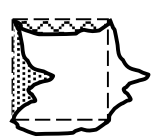

**Figure 8**: *Square, translation*

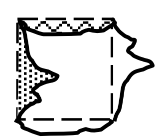

**Figure 9**: *Square, translation and glide reflection*

## References

[1] *ARThematics Plus: Integrated Projects in Math, Art and Beyond,* Stefanie Mandelbaum and Jacqueline S. Guttman, 1st Books, 2003

[2] *Introduction to Tessellations*, Jill Britton, Dale-Seymour Publications, 1989