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Mathematical Connections
in Art, Music, and Science

Tessellations: A Tool for Diversity Infusions in the Curriculum

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Abstract

Introducing diversity topics in the natural, mathematical and computer sciences is a hard task, since these disciplines are traditionally labeled as "diversity-unfriendly", due to their primary foci of study. In this paper we illustrate how tessellations can be used as a tool for infusion of multicultural topics, as well as a framework, designed after the Towson University course "Computers and Creativity", where these concepts have been successfully implemented.

1. Introduction

Diversity is a buzzword omnipresent in the academic circles in the continuing efforts to diversify the curriculum. The term "diversity" is hard to define, but everyone seems to understand it as plethora of varieties. We can define it as narrowly or as widely as needed in a given discourse, but the bottom line is that diversity is "being aware of what is there", [5]. Due to the nature of the subject, it seems easy to have whole courses or significant modules on women's issues, gender, national origins, disability etc. in the social sciences and the humanities. However, the natural, mathematical, and computer sciences are believed to be more hostile to these topics. The infusion of diversity in such courses cannot be done as explicitly as in some other disciplines, but many of us do it every day in our classrooms.

In order to illustrate the microinfusion of diversity topics in the teaching in the "diversity-unfriendly" disciplines, here are some tidbits that can be used not only to achieve the educational goal of a lecture, but also go the extra mile. They refer to the computer sciences specifically. Instead of showing the students how to build tables in Excel on a generic example using data on the imaginary "ACME Tomato Company", [13], use data on the Gross National Products of various countries, [2], for example. When discussing the Fibonacci numbers, and the recurrence relations, tell the story of Fibonacci, whose real name is Leonardo, and came from the town of the Leaning Tower of Pisa, and make a side-point on the role of this Italian genius from some 800 years ago in the introduction of the Arabic numerals and the positional numbers system in Europe. In Human Computer Interaction-related topics, spend some time discussing Equal Opportunity Computing, i.e. computing for people with disabilities. When discussing computer security, privacy, and ethical issues, spice it up with the geographical distributions of the domiciles of the virii, while stressing that the different laws that different countries have with respect to the Internet privacy. Many times what is legal in one country is not necessarily legal in others.

Rather than theorizing on how to do the diversity topics integration in the curriculum, this paper showcases the use of the tessellation theory as a tool for infusion of multicultural topics, and presents the Towson University course "Computer and Creativity" (COSC 109), [11], where tessellations suitably fit the educational goals.

The paper is organized as follows. Section 2 discusses the tessellations and tilling from the mathematical perspective, and its foundations in art. Section 3 overviews the course in Computers and Creativity, and discusses the diversity topics introduction in it. Section 4 discusses the technical and other considerations and observations of the use of tessellations in the course. The last section concludes the work presented in this paper.

2. Tessellations in Mathematics and Art

This section will present ways where computers are employed to introduce the idea of tessellation to students. Here we overview briefly the basics of tilings, and their foundations in Mathematics and Art. Studying of designs that are constructed based on repeating patterns provides a rich source for introducing infinite groups to undergraduate students in mathematics, and to others, where these concepts are not introduced as formally.

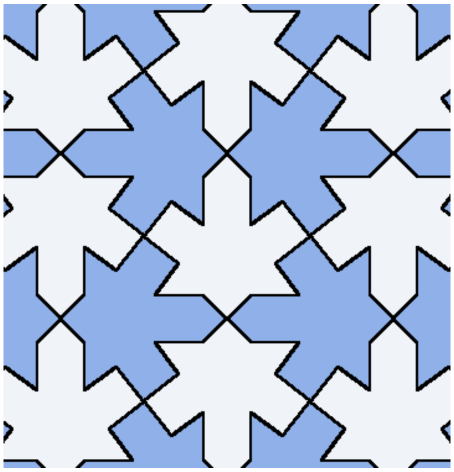
Despite the large number of artwork of repeated patterns in one and two-dimensional spaces over time and cultures, the study of their mathematical properties had begun much later, with the work of Johannes Kepler, [18]. The designers and geometers of the past created friezes and tessellations of all possible classes, but they never discovered that their creations could be classified based on their symmetries.

Figure 1: *The seven one-dimensional repeating patterns, Frieze groups, extracted from artworks of different cultures and periods in time.*

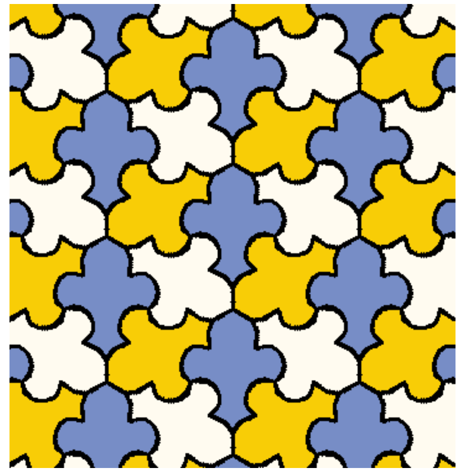
Mathematically, based on the four rigid transformations in a plane (isometries), all one-dimensional one-color patterns (friezes, bands, borders) can be categorized into seven classes. Likewise, two-dimensional patterns (wallpaper patterns, crystallographic groups) of one color are divided into seventeen classes. During the 1920s, the classifications of one and two-dimensional patterns became known, [12, 14]. The seventeen two-dimensional patterns were published in 1891, [7], but did not become widely known until later.

Disciplines other than mathematics benefit by the studies and applications of repeating patterns. Figure 1 presents a set of one-dimensional patterns extracted from artwork of different cultures and periods in time.

In 1924, Polya first illustrated the 17 wallpaper patterns [14]. Escher studied the rules that governed these patterns and made several sketches of them in his copybook. In addition to Alhambra designs, these illustrations were a source of inspiration for Escher.



(a)



(b)

Figure 2: (a) *A P4m Wallpaper Pattern, and* **(b)** *a P31m Wallpaper Pattern*.

Today we have many software packages to use when studying tessellations. *Tessellation Exploration* is a software utility designed to create tilings using basic geometric figures such as triangles and quadrilaterals. The software is able to tessellate with 33 different types of tiles. The software has been developed based on Heech classification of the 28 types of asymmetric tiles that can fill the plane in an isohedral manner without using reflections, [17]. The other five in this software are tiles that utilize reflections. An isohedral tiling is defined as selecting two congruent figures so that there is always a symmetry motion that will move one of the figures exactly onto the other. In addition to creating wallpaper patterns, this utility provides an environment that helps students analyze isometries used in a tiling.

Figure 2 represents two renditions of Polya illustrations which are performed in Tessellation Exploration, [15]. Figure 2 (a) is identified as D°4 in [14]. The basic shape to construct this pattern in Tessellation Exploration is a triangle. The isometries employed are a reflection and a quarter rotation. The mathematical notation for this pattern is P4m. It belongs to the square lattice of wallpaper patterns and its highest order of rotation is 4. It can be generated by 1/8 of its square unit. Figure 2 (b) is the rendition of D°3 Polya's illustration, which has been created based on a triangle and two isometries of a reflection and a rotation of 120°. The mathematical notation for this pattern is P31m. It is in the hexagonal lattice with the highest order of rotation 3. It can be generated by 1/6 of a hexagon unit.

Tessellation Explorations creates with ease Escher-like tilings and thus is an excellent environment in illustrating how mathematics can help in the production of art. On the main menu of this software click on "Create a New Tessellation". Then "Choose a Base Shape" screen appears along with a triangle, quadrilateral, pentagon, and hexagon. Select one of them, for example, the quadrilateral, and click "Quadrilateral" and then "Next" to continue. Then the "Choose the Moves" screen appears. Now we select the transformations that we would like to use in our tessellation. The translations that we choose determine the following:

1. How the sides of the polygon are transformed to create the base tile.

2. How the base tile will be transformed to create the tessellation.

On "Choose the Move" you have the choice of selecting a single transformation or a certain combination of transformations. We choose, for instance, "Slides" tab and click "Create My Own". A tile appears on the "Tessellation Creator", ready for us to shape it and add our artwork.

To shape the tile you need to use the following tools: , , and . The first tool, , is the "Resize the Tile" tool. Click and drag a corner handle, which appears around the tile. As the tile is resized, the tessellation on the left side of the screen changes. The next tool, , is the "Shape the Tile" tool. There are two ways to shape the tile using : You can move a vertex point, and you can add points to the tile's outline. The last tool, , is the "Delete Points" tool. To delete points that you have added click on and then click on an unwanted point. The point is removed and the side returns to its previous form.

Geometer's Sketchpad, [1], is another package that can be used in exploring tessellations. It is a visual geometry software program, based on the rules of constructions using compass and straightedge. Using the buttons to the left of the screen (, , , ,) this utility draws objects such as points, line segments, rays, lines, and circles. In this chapter we introduce this utility and use it in order to construct geometric objects.

Figure 3 is a tessellation that is created using the Geometer's Sketchpad. Careful examination of this figure reveals that one, base tile is being replicated via duplication by translations horizontally and vertically, with a perfect fit to the adjacent tiles. The magnitudes of the horizontal and vertical translations are the same. This suggests that a square is somehow involved in designing our base tile. We also observe that the left part of the tile's outline is identical with its right. Moreover, the top part of the tile's outline is the same as its bottom.

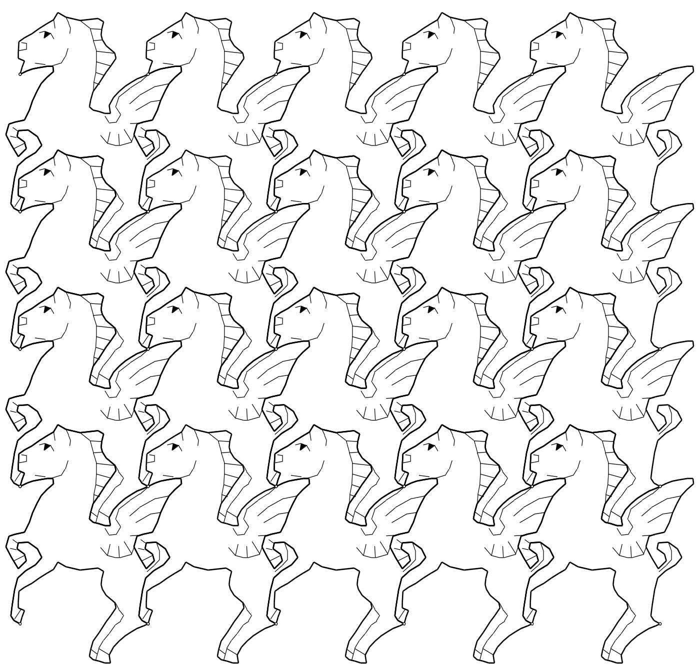


Figure 3: *Pegasus: Escher-inspired tiling*, [15].

Figure 4.a shows a square with four sides of *a*, *b*, *c*, and *d*, and it also shows that side *a* has been modified. Figure 4.b shows a translation that takes this modification to the opposite side, *c*. Figure 4.c

presents a modification on side *b*. The final image shows how a translation takes this modification to side *d* and completes the creation of our base tile.

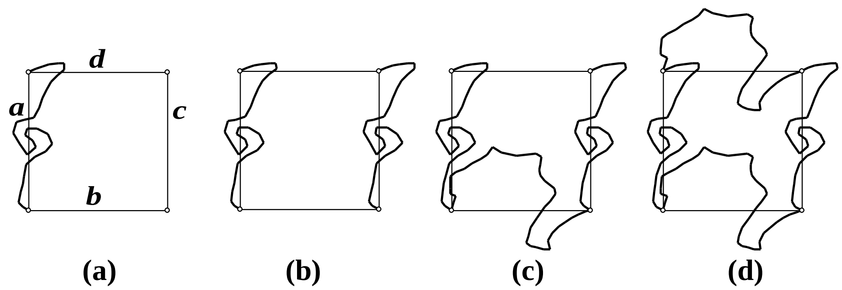


Figure 4: *Creating the base tile of Figure 3, in Tessellation Explorations.*

David Hilbert wrote "In mathematics, as in any scientific research, we find two tendencies present. On the one hand, the tendency toward abstraction seeks to crystallize the logical relations inherent in the maze of material that is being studied, and to correlate the material in a systematic and orderly manner. On the other hand, the tendency toward intuitive understanding fosters a more immediate grasp of the objects one studies, a live rapport with them, so to speak, which stresses the concrete meaning of their relations. As to geometry, in particular, the abstract tendency has here led to the magnificent systematic theories of Algebraic Geometry, of Riemannian Geometry, and of a sense of algebra. Notwithstanding this, it is still as true today as it ever was that intuitive understanding plays a major role in geometry. And such concrete intuition is of great value not only for the researcher, but also for anyone who wishes to study and appreciate the results of research in geometry", [9]. What we may be able to add to the above statement is the use of technology and computers has extensively enhanced our abilities to intuitively comprehend various subjects. However, the long history of these objects still keeps on telling the stories of time, cultures and people.

3. Computers and Creativity: Course Framework

In this section we discuss a framework for a course in computers and creativity that serves as environment for the use of the tessellations in our efforts to enhance diversity.

Computers and creativity are often considered a difficult combination, and many computer science departments stay away from offering creative computer courses in their curriculum. Computer art is often taught in art departments for upper level students, and producing interesting results. However, much of the material taught in those upper level courses concentrates on sophisticated software that takes time and skill to master. At Towson University, we decided to develop a course model for computer novices and introduce the beginning computer user to creativity and computer science. The course "Computers and Creativity" introduces students to computer science using a different approach by introducing the computer to experience creativity.

Computing sciences often concentrate on teaching sciences and mathematics first, and then introduce students to computer applications. The majority of computer science curricula contain few course offerings of computing in arts and humanities. Computer science students are dominantly practical, and their orientation is often towards job-oriented skills, [8]. Although skills in graphic arts are highly valued in industry, many computer science students are intimidated by computers in arts, [6]. They often possess the technical ability to master the software, but have little or no experience in applying creative thinking in their products.

Technology has become present in all aspects of living, and as a result, people's work and leisure are merging. Engineers and artists become collaborators. Engineers are noticing how people really work and incorporate many of the observed uses into new tools and artists are infusing themselves into the computer communications industry, [4]. In an attempt to incorporate these changes into our computer science curriculum, we created a course to serve as a connector between the two disciplines of computer science and fine arts. Our goal at Towson was to create a computer course for beginners centering on experiencing creativity using computers, with emphasis on experience, and to enable students in this course to have the possibility to experience the computer as a creative instrument.

We built an entry-level computer science course, titled: "Computers and Creativity". It is classified as a general education course in the category: "Creativity and Creative Development". The course is a 15-week, three-credit course and consists of a lecture and a laboratory component. The course's main objective is to provide the environment and tools so that the computer can be used to express and enhance creativity. Students experience traditional creative forms such as drawing, painting, photography, and writing as well as creative forms unique to computer technology such as computer graphics, multimedia, dynamic presentations, computer-based animation, world-wide-web publishing, digital photography and movies, virtual reality and the use of computers for music publishing and performance. Computer concepts which are relevant to the use of the computer as a creative tool are studied.

The course consists of four units, each unit lasting 3-4 weeks. The first unit is an introduction unit and introduces the student to basic ideas of creativity and techniques to improve creative thinking. It also introduces basic computer concepts. Unit 2 covers computer based drawing, computer based painting and computer based animation. Several tools are used to illustrate concepts and express creative ideas. Students are encouraged to implement any idea that they have and there are very few restrictions on suitable projects. The third unit introduces the web design and development process using application development tools and considering basic design rules. The final unit, unit 4, works with sound manipulation and also contains the final project. This project is a showcase for the student and they can create any project they are interested in as long as they use the tools that they have used during the semester.

The course introduces the student to many creative areas in computing and not just one particular product or a few application areas. Students learn a variety of tools, and spend approximately 50% of their total class time in labs. Assignments are only minimally structured to allow students to bring in their individual idea of creativity. Considering that creativity is highly individual, this course is strongly based on a constructivist learning approach. Constructivist learning environments focus on what a person knows the interpretation and construction of individual knowledge representation, and the promotion of higher order thinking. Social context is increasingly important in the meaning making process and is incorporated as much as possible. Learning in a constructivist environment is seen as an active process, where individuals take responsibility for their learning processes and experience. This theory promotes a holistic view of learning and supports the idea that there is no exactly "right" or "wrong" way of learning and seems to be a natural fit to teach computers and creativity, [10].

For example, when working with computer based drawing, students are given the objective that they must show mastery of using several tools (freeform shapes, geometrical shapes, text tool, etc.) and they can choose the topic of their drawing. Many objects that are produced over the length of the course are also being re-used, showing connections between various tools and techniques. In an early assignment, students take digital photographs of each other and themselves and learn to edit this photograph with photo editing software. This digital photograph is then used in several other assignments, for example, in computer based animation to produce special effects.

Constructivism is a paradigm honoring the situated nature of knowledge and enables students to experience an individual knowledge-building process in an authentic, collaborative environment. Knowledge grows when multiple perspectives are shared in collaboration with other learners, [3]. Learning is seen as a process of working toward a more complete and coherent understanding. The kind of discourse that supports such learning is not a discourse in which students display or reproduce what they have learned. It is the kind of discourse that advances knowledge in sciences and disciplines, [16].

Since students are actively engaged in the knowledge construction process, students take ownership for their own learning process.

To foster a constructivist learning process, this course encourages collaboration. Students can work in teams, but they are not required to work in teams. To encourage collaboration and display of the artwork that students have produced, the course uses an online learning environment, Blackboard. All assignments in this course contain personal elements (like a personal digital photograph) or are in file formats that are copy protected. This allows us to share all of the artwork in a discussion board online. This encourages communication and collaboration as students are inquiring other students on certain techniques or making general comments about their posted artwork.

4. Tying the Ropes

This section discusses the use of the tessellations in the "Computer and Creativity" course at Towson University.

The Tessellation Exploration Software Package is extremely helpful in the early stages of the "Computer and Creativity" course, and fits perfectly to the instruction goals. Being a tool with a friendly interface, it makes the general student population comfortable using it. Many of the students in an introductory course are still uneasy about using computers, so this software package helps them overcome the technological anxiety, and focus on the application, rather than the hardware issues.

After learning the basic mathematics and art background of tessellations, the students do their first project using this application. The project is to design a tiling motivated by patterns found in architectural or archeological items, and then post their work on their web site, with a description of the history of the motivating object. The students vote on the most successful project, after having read all the data on their classmates' web sites.

In an in-class discussion and presentation of the projects, the students have been proven eager to learn about the history of many of the motivating pieces, and they do not hide their astonishments by the buildings, rugs, mosaics, etc.

As a variation of the project, in one of the class sections, the students were asked to investigate the archeological floorings that are displayed at the Baltimore Museum of Art, which is rich in antique excavations of tilings. This museum also hosts the richest collection of works of Henri Matisse, whose works often serve as motivations to students to study visual art.

While challenging their creativity and experimental spirits, the students learn about different cultures, people, habits, and ways of life. Many of the objects where we see repeating patterns tell exquisite stories. The atmosphere in the class is nurturing, motivated, heated while students are competing to come up with their best ideas and designs of both the patterns and web sites, and ultimately educating in the multicultural sense.

5. Conclusion

In the course that we used to illustrate as a suitable place to introduce multicultural issues via tessellations, students have produced an amazing showcase of work, and many students have developed the ability to use the computer to express their creativity. Many products are far from being perfect pieces of art and most definitely would not win a prize at an exhibition. However, the goal of this course is not to produce high-level artistic work. The goal is to merge the ideas of art and computing sciences and to provide the environment for the student to experience this fusion. We find that the encouragement to experience art in an individual context is highly motivating for our students, and many students are relieved to find out that they have the freedom to produce artwork that is allowed to be imperfect. It frees the student from art anxiety, and we achieve astonishing results once a student frees him- or herself from this stage fright and performance anxiety.

References

[1] Key Curriculum Press: *Geometer's Sketchpad: Dynamic Geometry® Software for Exploring Mathematics*, retrieved at http://www.keypress.com/catalog/products/software/Prod_GSP.html on February 26, 2003.

[2] *The World Bank Group*, retrieved at http://www.worldbank.org/ on February 1, 2003.

[3] Brown, J., Collins, A., & Duguid, P. (2001). Situated Cognition and the Culture of Learning. In D. Ely & T. Plomp, *Classic Writings on Instructional Technology*. Englewood, Colorado: Libraries Unlimited, Inc. (Original Work published 1989).

[4] DeKoven, J., Binkley, T., Entis, G., Maxwell, D., & Smith, A.. (1994) Computer Technology and the Artistic Process: How the Computer Industry Changes the Form and Function of Art. *Proceedings of the 21st annual conference on Computer graphics and interactive techniques*. ACM Press New York, NY, USA, Pages: 494 – 495.

[5] Dudley-Sponaugle, A., Goode, E., Schroeder-Thomas, C. & Trajkovski, G (2003). New Faculty Contributions toward Enchancing Diversity. *Proc. CCSCNE 2003,* Providence, RI, in print.

[6] Eber, D., & Wolfe, R.. (2000). *Teaching Computer Graphic Visual Literacy to Art and Computer Science Students*. Computer Graphics, ACM Press New York, NY, USA, Pages: 24-26.

[7] Fedorov, E.S. (1891). Simmetriya pravil'nyh sistem figur (Symmetry of Regular Systems of Figures) [In Russian]. *Zap. Mineral. Obch*. **28, 2,** 1-146.

[8] Hertlein, G., (1977). Computer Art for Computer People, *Proceedings of the 4th annual conference on Computer graphics and interactive technique,* ACM Press New York, NY, USA, Pages: 249 – 254.

[9] Hilbert, D (1996) *The Emergence of Logical Empiricism*. Garland Publishing Inc.(Original work published in 1927).

[10] Jonassen, D. & Land, S. (2000). *Theoretical Foundations of Learning Environments.* Mahwah, New Jersey: Lawrence Erlbaum Associates, Inc.

[11] Meiselwitz, G. (2003). *Courses*, retrieved at http://pages.towson.edu/meisel/ on February 1, 2003.

[12] Niggli, P.(1924). Die Flächensymmetrien homogener Diskontinuen. *Z. Kristall*. **60**, 283-298.

[13] Parsons, J., Oja, D., June, Parsons, D. (2001). *New Perspectives on MS Office 2000 – Brief*, Boston, MS: Course Technology.

[14] Polya G. (1924). Über die Analogie der Kristalsymmetrie in der Ebene. *Z. Kristall*. **60**, 278-282.

[15] Sarhangi, R (2003). *Elements of Geometry for Teachers*, Boston, MA: Addison-Wesley.

[16] Scardamalia, M., & Bereiter, C. (1996). Student Communities for the Advancement of Knowledge. *Communications of the ACM, 39(4),* 36–37.

[17] Schattschneider, D. & Dolbilin, N (1998) Catalog of Isohedral Tilings by Symmetric Polygonal Tiles. *Math Forum*, retrieved at http://www.mathforum.org/dynamic/one-corona/ on February 1, 2003.

[18] White, C. & Brewda, S (2003). *Translations of Works by Johannes Kepler*. Shiller Institute, Washigton, DC, retrieved at http://www.schillerinstitute.org/transl/trans_kepler.html on February 1, 2003 (Original work published 1619).