ISAMA
The International Society of the
Arts, Mathematics, and Architecture

BRIDGES
Mathematical Connections
in Art, Music, and Science

Mathematics and Esthetics —Science and Art in the Bay Area

Benjamin Wells
Mathematics, Computer Science
University of San Francisco
2130 Fulton Street
San Francisco CA 94117 USA
Email: Wells@usfca.edu

Abstract

First taught at the University of San Francisco in fall 2002 and scheduled again for fall 2003, *Mathematics and Esthetics—Science and Art in the Bay Area* (MESABA).is a unique freshman seminar that provides instruction in both math and art as well as academic support for new college students. It celebrates and is enriched by the art, nature, science, and people of San Francisco and the Bay Area. But its structure and content can prove useful for many quite different courses and settings. So this is an argument for both unicity and catholicity.

1. Introduction.

A quote from my proposal for the freshman seminar *Mathematics and Esthetics—Science and Art in the Bay Area* (MESABA) appeared in [11]: "My personal perspective is that it is desirable not to divorce truth and beauty; indeed, it is impossible. Trying to, limits the teacher and the students. Being open to beauty in mathematics stimulates love for it, and that, among other things, makes better mathematics."

The application was accepted, but the course given differed from the proposal in significant ways. Instead of the planned science course with a required experimental computer-based laboratory section, a quantitative course with the following outcomes was mandated: *students will be able to determine whether a problem lends itself to a mathematical solution, and, if so, design such a solution, implement the design or identify and correct problems with it, and evaluate the validity of the solution and its relevance to the original problem—using reasoned discourse as the norm for decision making.* Here, "mathematical" can mean one or more of algebraic, algorithmic, statistical, symbolic, numerical, or computational.

The mathematical content did not meet this burden well during the first offering in fall 2002. One purpose of this paper is to outline the emerging structure of an otherwise satisfactory course that in its fall 2003 incarnation will meet these expanded goals more completely:

A. Be a freshman seminar, providing a map, tutorial, and practicum for college success;
B. Be a Core Curriculum science/quantitative course, fulfilling the outcomes listed above;
C. Include a lab component affording computer explorations and built-in time for field trips
D. Celebrate San Francisco and the Bay Area;
E. Help balance the head and heart through a felicitous conjunction of math and art.

Along the way, the reader may come to see that while many courses in art and math can be given equally successfully in metropolises or rural college towns, MESABA is a product of special conditions present in the Bay Area and at the University of San Francisco. On the other hand, many of the structural features and most of the multimedia content of MESABA are applicable and available for diverse courses emphasizing different blends of science and art. These need not be limited to first-year experiences, but I would be particularly interested in any past or future resonances with other introductory courses.

2. Background of this course.

When I was a graduate TA in a liberal-arts mathematics course at UC Berkeley, at friend showed me a book on Escher; I used it in class the same week. From then on I have tried to bring art into the math classroom. Association with a private arts-oriented secondary school deepened this commitment, so that when I moved to the University of San Francisco, I experimented with arts-based projects and demonstrations for the students.[1] Next was a turn at conducting the Davies Forum, endowed by Louise M. Davies to pursue *the search for values in contemporary America* at USF. My spring 1989 semester topic was "Approaching Infinity—Mathematics and the Mystic Quest"; our featured speaker was Douglas Hofstadter, author of *Gödel, Escher, Bach—An Eternal Golden Braid* [3]. One stream of this course flowed on as nine offerings of a freshman seminar entitled "Chaos and Order," and Benoit Mandelbrot was a guest speaker.[2] Some of the same themes appeared in my Templeton Science & Religion award course called "Infinity, Chaos, and Mysticism in Science and Religion," offered in 2000. All eleven of these courses had major arts-based instructional components. Giving talks at Art & Math 98 and Bridges 2002 and hanging science-based art on and off campus put me in contact with a wider community of science-art participants and projects that will continue to enrich these courses. Although a fresh flowering, MESABA blooms in an established rose garden.

3. Freshman seminars at USF.

The National Resource Center for The First-Year Experience [13] at the University of South Carolina is a nexus for hundreds, perhaps thousands, of freshman academic programs. These come in many flavors. SC itself offers University 101, an optional course most incoming students take; it teaches college skills only. UC Berkeley offers 70 one-unit freshman seminars, content-driven, taught by faculty members, mostly pass/no pass, and not intended as college skill workshops. Harvard lists similar courses, but fewer and worth twice the credit. Washington State University has two-unit add-on seminars linked to regular dedicated General Education courses.

The USF model differs from most others in at least one its characteristics: each USF seminar is a specialized-content course combined with college success training that also satisfies a general education (Core Curriculum) requirement with full 4-unit credit and grade; it is open only to 15 first-term freshmen, no transfers; instructors are faculty members, usually fulltime and tenured. Each seminar features field trips, mentoring, introduction to campus services, and surveys of research methods and facilities. What guarantees the model's uniqueness is the requirement that every seminar be linked to San Francisco and the wider Bay Area, directly implementing the last sentence in the USF mission statement[3] linking the curriculum to the locale.

---

[1] Puppet shows described in [11] have been in episodic development since 1970.

[2] Escher, Mandelbrot, and Hofstadter have all argued for the visualization of mathematics. In fall 2003, Hofstadter led a graduate seminar in Bloomington on visualizing group theory [4]. USF Associate Dean Tristan Needham authored the award-winning *Visual Complex Analysis* [7]; he also gave the administrative approval for MESABA.

[3] *Educating minds and hearts to change the world.* The core mission of the University is to promote learning in the Jesuit Catholic tradition. The University offers undergraduate, graduate and professional students the knowledge and skills needed to succeed as persons and professionals, and the values and sensitivity necessary to be men and women for others.

The University will distinguish itself as a diverse, socially responsible learning community of high quality scholarship and academic rigor sustained by a faith that does justice. The University will draw from the cultural, intellectual and economic resources of the San Francisco Bay Area and its location on the Pacific Rim to enrich and strengthen its educational programs. —*USF Mission Statement* (http://www.usfca.edu/mission/index.html)

4. Core Curriculum requirements.

In spring 2003, the USF Core Curriculum replaced the decade-old General Education Curriculum and reduced the total units while adding flexibility. CC embraces standard courses specified through learning outcomes rather than the GEC-mandated tailored and rigid courses. In the context of MESABA, the "problem/solution" of the Core-required outcomes in §1 takes on an additional meaning in the arts: the problems can include artistic goals and methods and esthetic criteria, and the solutions may feature informal mathematical description and mechanical or computational tools. Thus, the controversy over whether Renaissance painters achieved perspective by the optical projection of a camera obscura, by the mechanical projection of sighting linkages, or by several competing geometric projections of a drafting layout is not only a question for art history, but also for mathematics.

In addition to Core goals, students are told that they can expect to appreciate and evaluate:

- the origin, evolution, and efficacy of the scientific method;
- popular science & art writings and their impacts and claims;
- broader issues of esthetics and artistic creativity, spanning sciences, arts, and cultures;
- mathematics & physics as infrastructure (and the computer as tool and medium) for art & artists;
- the interdependency of the sciences and the arts as a paradigm for the interconnection and reciprocal responsibility of civilization and nature.

5. Location^3.

How does the seminar benefit from being in San Francisco? Our region is at once a hotbed of both refined and vibrantly novel art, a fountain of scientific research, and a volcano of computer science and technology. These environmental forces combine to make the San Francisco Bay Area a whirling center for the collaboration of science and art. For us, two local resources predominate: museums and artist-scientists. Besides viewing on-campus art in 2002, we visited the following six exhibition spaces off campus [13].

The major US collector and dealer of Escher works, **Vorpal Gallery** is owned by Muldoon Elder, an artist and mathematician; the first Escher retrospective in a decade is scheduled for 2003.

The **San Francisco Museum of Modern Art** holds the first painting identified as modern art (Matisse's *Femme au chapeau*) and always shows works by Magritte, Calder, Pollock, LeWitt, Kelly, Richter, and others mentioned in class. Later this year it will be the only US venue for the first Chagall retrospective since 1985.

Carlo Séquin's office at UC Berkeley showcases constructions, flat and solid images, sculptures, and computer displays (Figure 1).

Helaman Ferguson's *Eightfold Way* [6] is installed on the veranda of the **Mathematical Sciences Research Institute** in Berkeley (Figure 2).

Land's End, part of the Golden Gate National Recreation Area, displays natural fractals in beach, sea rock, cliff, chaparral, coniferous, and cavern environments, plus terrific vistas (Figure 3).

The Exploratorium,[4] the first, biggest, and still best of the hands-on science museums, displays over 25 science-art works (many by Bay Area artists) and twice that number of additional exhibits that are related to science and art.

San Francisco is home to three more world-class art museums: Palace of the Legion of Honor, Asian Art Museum, and the M.H. de Young Museum (reopening in 2005).

---

[4] Founded by Frank Oppenheimer in 1969, *Exploratorium: the Museum of Science, Art, and Human Perception* [13] has often hosted art events, including the 2nd Wednesdays Art Series and the Artist-in-Residence program.

As far as people go: Leonard Shlain, author of Art and Physics [10], visited the seminar; Karl Schaffer of Dr. Schaffer and Mr. Stern Dance Company demonstrated their work; John Stillwell, noted Australian geometer now at USF, spoke and toured with us; Keith Devlin, mathematician and popular author, talked about visual mathematics; Brenda Laurel, author of Computers as Theatre [5], and Rob Tow visited the class; Ed Catmull, Pixar Animation Studio's president and cofounder, conversed with seminar members; Carlo Séquin hosted our brief tour of UC Berkeley; Muldoon Elder showed us Escher works from our own wish list; and art historian Peter Brooks taught us the difference between Renaissance and Baroque space. Countless other art/science practitioners work nearby. Some have already talked at USF, including Alan Rath, Michael McGuire, Eben Ostby, John Chowning, Cliff Stoll, Scott Kim, Rudy Rucker, Paul Debevec, Pete Docter, Abbe Don, Michael Naimark, Jaron Lanier, and Roberto Ziche. The Consortium of the Arts, who performed for my Templeton and Davies courses, is not only my chief source of training in fine and performing arts and art appreciation, but also provides audio and video materials used in many of my courses besides this one.

More than impressing the reader with the richness of resources in the Bay Area, these enumerations are meant to inspire other teachers at urban institutions to explore their own neighborhoods, and persuade those in remote areas to consult websites mentioned—and websites undiscovered.

6. Bridges.

6.1. Student introduction to MESABA. Students are told:

"The general purpose of the Freshman Seminars (and this one goes beyond!) is to provide a support system for new students that is embedded in a participatory course on an exciting and integrating topic closely related to the San Francisco Bay Area. These seminars, and we the instructors, are here to help you. As far as *this* course is concerned, our purposes may be briefly stated: to explore, even create, bridges between math, science, and art, and the transcultural experience of truth and beauty. Many times, we shall find that it is hard to distinguish the last two.

"We'll learn how mathematics supports science that enlivens art. We'll hear from local artists who are inspired by science, who incorporate technology into their creative process or product. We'll also hear from local scientists who find that art is an inevitable collateral activity for their science.

"Readings in popular scientific books are supplemented by audio/visual and printed materials; easy to use software lets you explore art forms, physics-based dynamics, chaos, evolution, and fractal geometry in laboratory experiments. Field trips and a service learning component further connect the course with the local community. There is no science, math, or computing prerequisite for the course."

6.2. Textbooks. In 2002, we used as textbooks *The Colossal Book of Mathematics* by Martin Gardner [1], *A Beginner's Guide to Constructing the Universe* by Michael S. Schneider [9], and *Style: Toward Clarity and Grace* by Joseph M. Williams [12]. In addition, students were encouraged to get one of *What Shape is a Snowflake?* by Ian Stewart or *Fragments of Infinity* by Ivars Peterson. The Gardner book provided a mathematical springboard for many of the topics in both art and math. Schneider offered an unusually supportive introduction to geometry and numerical and geometrical musings that many found nonthreatening compared to math they had studied before; this book was less popular among my colleagues. Williams suggests algorithms for revising one's writing, going beyond the Strunk and White injunction to "use good style." Although popular in tone, Gardner is too obscure for neophytes. For 2003, it will be replaced by Burger and Starbird's *The Heart of Mathematics* [1], a liberal-arts math text that incorporates artistic sensibilities. Over 60% of the book is directly applicable to the course. There will be additional readings but no optional texts Videos, slides, and software supplement the readings.

6.3. Organization. Although the mechanics of the course, its projects and papers, journaling, team learning, computer explorations, and group exams are of interest, let us ignore these and the college success training in favor of content. What exactly are the bridges between math and art that MESABA features? We can capture some by considering three metaphorical axes that correspond to qualities of space, energy, and mind. The math and art and the analysis have no specificity to the Bay Area. The art is mostly 2D and still, but the course also considers 3D, animation, music, poetry, and dance (please see [14] for images of art mentioned).

6.4. The real and imaginary axis. Shlain [10] argues that changes in our views of space correspond to changes in our art, so that the Renaissance artists' mastery of perspective correlates with the mechanical advances of Newton and the mathematical physicists, just as Cubism corresponds to the Lorenz transformations of relativity. For us, this just raises the curtain on a grand involvement of space and art. Here, "real" means Euclidean, classical, two- and three-dimensional space, while "imaginary" suggests higher dimensions, Minkowski space, tori, knotted space, and perspective based on platonic solids other than the cube. After celebrating the mastery of the real in the perspective technique of Brunelleschi, Uccello, and Leonardo, this leads to distant comparisons, such as suggestions of infinite space in Persian miniatures, Raphael, and Dali. But it also invites closer consideration as well, such as the distinction between two Baroque artists: the controlled distortions of space detectable in Vermeer's *The Music Lesson* compared with the blatant, almost violent, breaking of the picture plane by Caravaggio in *The Supper at Emmaus* and his other immersive, in-your-face compositions.

6.5. The linear and nonlinear axis. Linear differential equations govern the mechanistic universe Newton showed us. Nonlinear science is the home of chaos, fractals, and discontinuity. But in the frequency domain, it flips: smooth symmetry in space may have a discrete power spectrum, while fractals, such as music of every genre, may have continuous $1/f^d$ power distributions. Chaotic dynamical systems, even those moving smoothly such as Calder's mobiles, have fractal attractors in phase space. Symmetry and chaos are combined in Mike Field's art and Eric Heller's visualizations of physics. Escher shows us symmetry, fractals, and chaos, sometimes in the same work, such as *Order and Chaos*, which he discusses on video. Seurat's *Grand Jatte* demonstrated the illusion of continuity from discreteness. The complementary energy of order and chaos is striking in physics and in art.

6.6. The rational and irrational axis. Think also in terms of natural and artificial, simple and self-referential, tangible and virtual, smart and wise, mental and emotional, certain and ambiguous, computerless vs. computer-aided vs. computer-generated vs. computer-created. "Rational" suggests the same anchoring as the left end of the other two axes; "irrational" serves not as a polar opposite, but as the fuzzy complement. Of course, the axes are orthogonal, and the octants they determine can exemplify many blends. Here are three examples of contrasts along this axis: Galen Rowell's nature photography and Ken Musgrave's procedural landscapes, Brent Collins's freely sculpted minimal surfaces and Séquin's generated plans that Collins follows for collaborative works; Bernini's direct *David* frozen at the point of action, and Magritte's *Personal Values* frozen in abstraction. Pollock's drip paintings such as *#1 Lavender Mist* might evoke a high value of nonlinearity, and on this axis, they also fall far to the right. For by his account of his process, he is not randomly splatting but allowing his feelings to govern his motions. Both Escher and Magritte exemplify one more pair of labels for this axis: consistent and paradoxical. A final personal observation: aside from the general space-bending of the Baroque, Bernini lies fairly on the left side of the axes, but in my view, *L' estasi di Santa Teresa* goes beyond the rational to the Real. As Meher Baba said, "The curl of the Beloved is the twist of thought."

6.7 Summary. These axial musings do little to explicate the mathematical content of the seminar, but the reader can see where descriptive, algebraic, geometric, numerical, and symbolic mathematics do enter. The goal is not to create space for the mathematics, but to show more generally (and more specifically)

that mathematical considerations such as the very structure of space have implications for art beyond metaphor and casual similarity.

7. Conclusion.

MESABA focuses on the interplay of mathematics and esthetics, why math is beautiful and how math aids in creating art. It ignores a metaquestion of whether there is an attractive mathematical account of esthetics, in parallel with the Paulos's analysis of math and humor [8]. There may be, but it would be a hard sell to freshmen.

Two final bridges lead students to deeper connection with the community. Service learning projects have been featured in all my freshman seminars. In 2002, we joined a hundred other USF students to help clean up Golden Gate Park (Figure 4). Past projects have included work at two homeless shelters, Habitat construction, and Outreach Ministry's childcare-space reorganization. In early December, the class always visits an exhibition on campus of NAMES Project quilt squares.[5] Not only are quilts often mathematical art, but the celebration of lives through the AIDS Memorial quilts provides another example of the twin lessons that I paraphrase from chaos theory: no one does anything alone, and you yourself can make a difference. Every locale offers service learning opportunities, and the quilt squares tour widely and are all available online.

We can do no great things, only small things with great love. —*Mother Teresa*

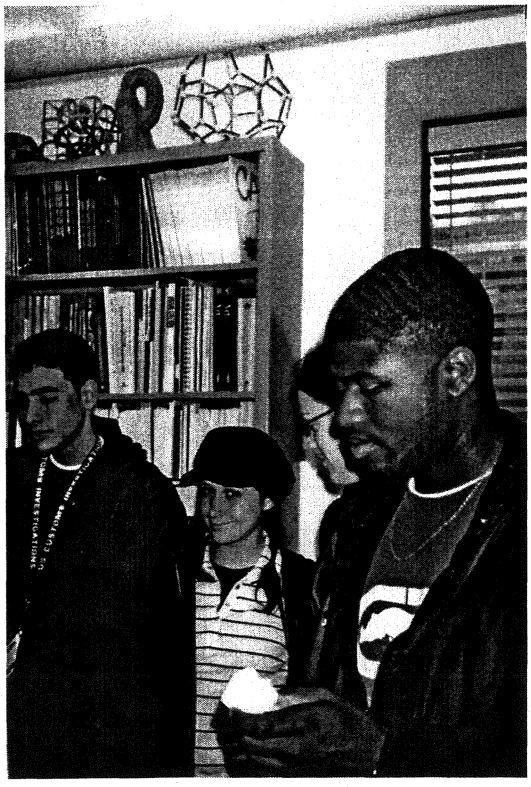

Figure 1. *Field trip to Carlo Séquin's office.*

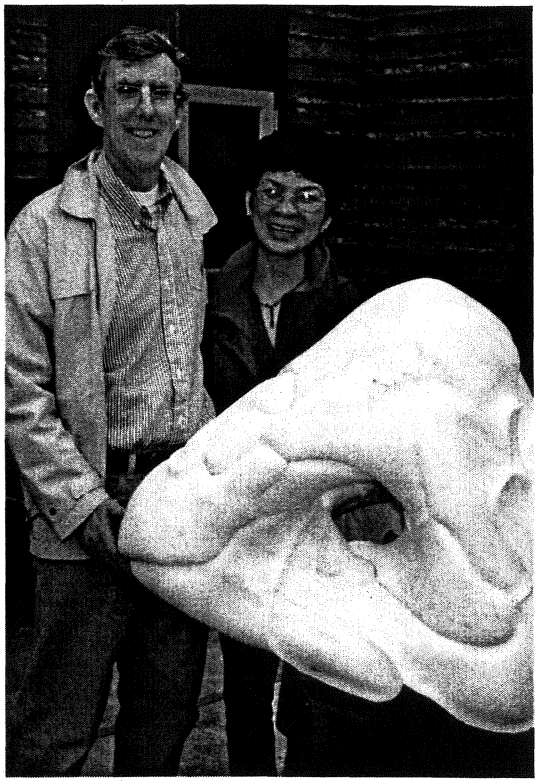

Figure 2. *Stillwells and Eightfold Way at MSRI.*

[5] The NAMES Project Foundation, sponsor of the AIDS Memorial Quilt (http://www.aidsquilt.org/), began in San Francisco, and has now moved to Atlanta.

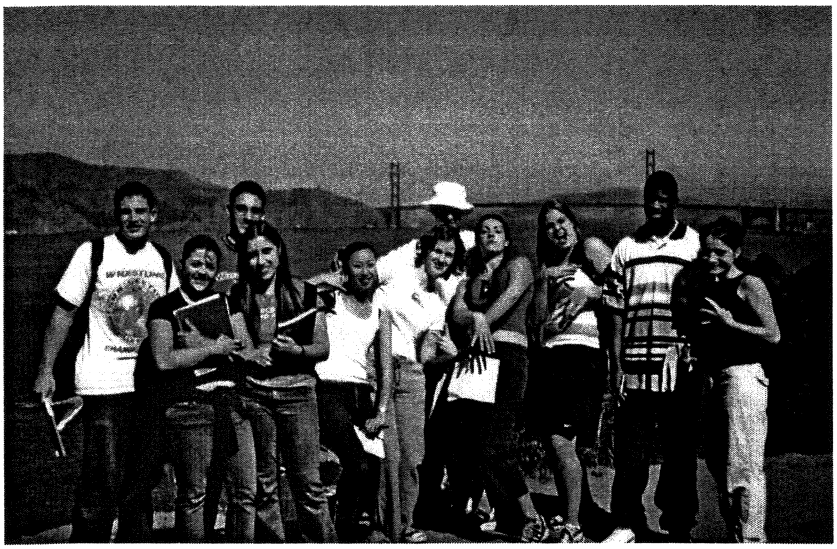

Figure 3. *Fractal field hunt at Land's End..*

Figure 4. *Service Learning in Golden Gate Park*

References

[1] E.B. Burger and M. Starbird, *The Heart of Mathematics—An Invitation to Effective Thinking.* Emeryville CA: Key College Publishing. 2000.

[2] M. Gardner, *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems.* New York: Norton. 2001.

[3] D.R. Hofstadter, *Gödel, Escher, Bach: an Eternal Golden Braid.* New York: Basic Books. 1979.

[4] _____, personal communication. Also see: http://neville.math.indiana.edu/~nathan/m590f02/

[5] B. Laurel, *Computers as Theatre.* Reading MA: Addison-Wesley. 1991.

[6] S. Levy (ed.), *The Eightfold Way,* New York: Cambridge University Press. 1999.

[7] T. Needham, *Visual Complex Analysis*. New York: Oxford University Press. 1997.

[8] J.A. Paulos, *Mathematics and Humor: a Study of the Logic of Humor*. Chicago: University of Chicago Press. 1980.

[9] M.S. Schneider, *A Beginner's Guide to Constructing the Universe: the Mathematical Archetypes of Nature, Art, and Science*. New York: HarperCollins. 1994.

[10] L. Shlain, *Art & Physics*. New York: Morrow. 1991.

[11] B. Wells, *The Rootsellers—Retelling the Galois Group of a Quartic Polynomial*, Bridges 2002 Conference Proceedings, pp. 235-246.

[12] J.M. Williams, *Style: Toward Clarity and Grace*. Chicago: University of Chicago Press. 1995,

[13] WWW resources (consulted 2/4/03)

Exhibitions visited:

Vorpal Gallery http://www.vorpalgallery.com/
San Francisco Museum of Modern Art http://www.sfmoma.org/
Carlo Séquin http://www.cs.berkeley.edu/~sequin/
Mathematical Sciences Research Institute http://www.msri.org
Land's End GGNRA http://www.nps.gov/goga/sites.htm
Exploratorium http://www.exploratorium.org/

Freshman seminar programs mentioned:

The National Resource Center for The First-Year Experience http://www.sc.edu/fye/
University of South Carolina http://www.sc.edu/univ101/
University of California, Berkeley http://fsp.berkeley.edu/
Harvard University http://www.fas.harvard.edu/~seminars/fs/
Washington State University http://salc.wsu.edu/Freshman/ (visited 2/2/03)

[14] Pointers to artists and works cited (consulted 2/4/03)

Matisse	http://www.sfmoma.org/exhibitions/exhib_detail.asp?id=11
Brunelleschi	http://www.usm.maine.edu/~schiferl/ayBrun2002.html
Uccello	http://www.abstract-art.com/abstract_illusionism/ai_03_put_into_persp.html
Leonardo	http://www.mos.org/sln/Leonardo/LeonardosPerspective.html
Persian	http://www.mirror.org/greg.roberts/
Raphael	http://www.ladseb.pd.cnr.it/infor/ontology/lascuola.html
Dali	http://www.worldventurefunds.com/suissefinancialgroup/dali_crucifixion.html
Vermeer	http://www.cacr.caltech.edu/~roy/vermeer/ych.html
Caravaggio	http://www.kfki.hu/~arthp/html/c/caravagg/06/35emmau.html
Calder	http://www.visitingdc.com/s_nga_calder.htm
Field	http://nothung.math.uh.edu/~mike/ag/art.html
Heller	http://www.ericjhellergallery.com/
Escher	http://www.nga.gov/cgi-bin/pimage?62996+0+0+ggescher
Seurat	http://www.mystudios.com/treasure/seurat/seurat-grand-jatte.jpg
Rowell	http://www.mountainlight.com/gallery.html
Musgrave	http://www.kenmusgrave.com/art_gallery.html
Collins	http://www.cs.berkeley.edu/~sequin/SCULPTS/collins.html
Bernini	http://www.thais.it/scultura/image/sch00006.htm, /sch00349.htm
Magritte	http://www.sfmoma.org/collections/recent_acquisitions/ma_coll_magritte.html
Pollock	http://www.nga.gov/feature/pollock/lm1024.jpg
Bernini	http://www.kfki.hu/~arthp/html/b/bernini/gianlore/sculptur/1640/therese1.html