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Tangramoids

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Abstract

This paper describes in a rather epistemological way the genesis of a class of three-dimensional objects that feature views of flat figures organized with the seven pieces of the Chinese Tangram puzzle, when looked at from certain privileged directions, and therefore called tangramoids. A subclass is considered here, where the objects are structures assembled from plane, generally opaque, polygons. The objects can either be closed (irregular polyhedra) or open (when not all edges of polygons are adjacent to others). The tangramoids discussed here are further limited to feature plan views of the generic square Tangram figure, and of Tangram figures involving a square made of the five smaller pieces. Finally, mainly instances displaying a characteristically pleasing symmetry are discussed. They are represented in space as sculptures or in the plane as graphical works including interactive computer renderings.

1. The Tangram and the genesis of tangramoids



Figure 1: The Tangram and some of its fabulous silhouettes

The understanding and representation of the threedimensionality of mechanisms, architecture and living creatures have ever been a passion for me as an unrepentant aesthete, so no wonder a calling as an engineer specialised in computer-aided structural mechanics has turned into a calling as an artist active in sculpture, painting and cyber graphics... With this background, always distinctively biased towards harmony and elegance with side interests in games, optical illusions and other brain teasers rather than business and productivity, I have been fascinated for almost twenty years by the Chinese Tangram puzzle that I like to display in its generic square form, oriented as a diamond (fig. 1). You probably know the Tangram is a simple set of seven pieces that a legend attributed to a young boy who once dropped a square tile and couldn't reconstruct it from the broken parts. He found instead [1] numerous other wonderful figures (see examples on fig. 1) ranging from abstract shapes to people to animals to objects of all kinds, silhouette representations, patterns immediately recognizable by the way our brain acutely maps 3-d space into synthetic symbolic forms. For years I have contemplated the generic square, with all the pieces showing, exploded, like in the solutions sheets to the plain black silhouette puzzles offered to the children and adults alike (Napoléon Bonaparte and Lewis Carroll were known adepts of the game !) and found pleasure in this pattern that imbeds such a multitude of striking possibilities, in itself already a wonder that gives this shape a priceless value. So important to me I included it in the entrance floor of our home (fig.2), with an inside/outside metaphor, two large triangles in roughened stone outside, and five coconut mats inside.



Figure 2 : Entrance Tangram

This pattern is not symmetric due to the presence of a square and a parallelogram which break the symmetry of top and bottom of the square as seen as a diamond. Strikingly enough, if one removes the top triangle, one gets a shape that resembles a *diamond stone* in its generally known form as a *brilliant*, seen from the side... which could have been an unconscious hint already for what would be to come... And yet it seemed there was symmetry in that pattern, so I kept looking at it and wondering what could give this impression...



Figure 3: « L'Artriste et ses Modèles »

Yet on another caricature in the same vein, that laughing/biting character with eyes in small triangles again (fig.4). Later, in 1999 (fig. 5), a work on the Golden Section, with fractal pentagrams producing a cube (!) with projections of the Tangram, where Heidegger's pitcher is modelled like in another work/installation with moving parts (fig. 6), and a quest to infinity "à la Escher" in another spiralling leap with fractal Tangrams more in a tessellation approach...

Figures 4-6: Tangrim grin; Fractal Pentagram & Tangram; Heidegger's pitcher

I used it in a playful way in several works, with a distinct 3-d approach in mind. In 1997, committing a rather syncretistic fusion of several alleys of interest, I produced during the full night of my 50th birthday this rather complex image (fig. 3) where I morphed/matched/mapped/fitted a female figure from my academies in this Tangram pattern, with hints to shapes (animals like Kermit the frog, the Egyptian ibis bird, name initials) yet another feminine Picassian pose with navel in left bottom corner, stereogrammic dots, allusions/illusions of literature ("cachez ce sein que je ne saurais voir": stippled circle centred on the design, like in an engineering drawing), etc, but more importantly, in the present context, this face of the old man (me?) with his green eyes being the small triangles of the Tangram...



All this just to illustrate wanderings of the mind that conducted gradually to the tangramoid. Some five years ago the clue revealed, and it worked as a discovery that didn't stop inspiring me. All in a sudden indeed, the square and the parallelogram which normally kept breaking the symmetry of the figure, started bulging out, in 3-d space, and becoming identical diamonds made of two equilateral triangles. The two large triangles down there bulged out too as the equilateral triangles of the under half of an octahedron. Through a set of very mean *transgressions* that are described in detail in the application document for a sculpture competition in Brussels [2], I figured out a 3-d object that looked like the Tangram in its square form from several viewing directions and called it a "tangramoid". This new object could be what the square shape exploded with a little space in between (fig.1) represents...

2. Variety of the class of tangramoids sharing same views of the Tangram

The tangramoid so far came to life due to these square and parallelogram which are projections of diamonds in space, four of them, to be precise, succeeding each other like blades of a turbine. This induced the first model made from a single sheet of cardboard cut out and folded into a tangramoid that displayed two Tangrams per revolution. Two lower triangles were held together with a central vertical square with an opening for getting the views right, the upper part being the triangular top (fig. 7).





This first true tangramoid looked still unsatisfactory because I expected it should be possible to crown the turbine with "something" that would permit to view a Tangram from *all four* sides. A cone would have been a possibility, but would spoil the object with a curved surface, so I left it in my cabinet for better times.

Three years later, deciding to participate in that sculpture competition, I took the model out and started thinking again, and true ! the triangular top could be represented by a pyramid tilted 45 degrees versus the under pyramid ! And there came the first satisfactory tangramoid. The Polydron game [3] wouldn't permit building a model, but the by then new "green struts" of the Zometool game [4] combined with a few blue and yellow struts for the top allowed to build one, as the 34 belong to an octahedron, and the upper ¹/₄ was that tilted pyramid. Glued "sails" of Japanese paper provided buckets so mv blowing could nicely spin the mill (fig. 8).



Figure 8: Open sailed tangramoid



Figure 9: Characteristic views of closed sailed tangramoid

Much to my surprised disappointment, when the model was proudly presented against a strong wind in the garden, it didn't spin at all but merely oscillated ! Indeed, unlike good old windmills of past times or present airplane bladed ones that spin stand-alone, my rotor needed a stator to have the wind engulfing with the right incident angle into the buckets, like I did without realizing when blowing in them! So this tangramoid would need extra wings secured close by to generate those swirling gusts needed to power the mill ! For reasons of strength the structure was closed into a non-convex irregular polyhedron with axial symmetry. Additional unexpected miracle : the shapes between the Tangram views look like *Chinese lanterns*, a quite fortunate outcome for a Chinese puzzle ! (See fig.9, upper left, with JavaView[6]).

The project was now ready for the competition for which I envisioned a large 5 meter tall structure in aluminum pipes fitted with sails that would spin under the wind on a pedestrians bridge in Brussels on a 3 meter tall pole, as a ludical "signal" to attract people up there from a busy city neighborhood into an unexpected wonderful nature walk that replaces early railway tracks (fig. 10-12; left pic uses JGV [5]).



Figures 10-12: Large tangramoid (no rotor sails) in situ on bridge and promenade, Tangram characters burned on railing

As there was room for several variants, a stripped-down open also tangramoid was developed, with only seven "minimal" the pieces, tangramoid, perhaps the most impressive of all, where the upper triangle is again modeled with a single triangle, but this sitting across two time diamonds to allow for four Tangrams to show up per cycle (fig. 13, with JGV [5]).



Figure 13: cross-eye stereo view of minimal tangramoid



Figure 14: Corten steel tangramoid sculpture



Figure 15: *Tangramoid building with JavaView*[6] *and JGV*[5]

Scott Vorthmann, who implemented the Zome3D [8] Zometool simulation software package, pointed out recently his belief that the half struts of Zometool do not really make part of "the system". They in fact introduce alternate systems for which one could use, as Scott suggested, nodes of another colour, with the rules "1) any half strut connects the "white universe" and the "black universe", to pick two colours arbitrarily, and 2) any normal strut lives entirely within one universe or the other - both endpoints have the same colour". The tangramoid both provides a fine example of this and stresses that indeed half struts are essential, for their very existence makes the representation of special forms like the tangramoid possible (6 dashed struts on figure; grey = green struts, white = yellow struts) !

Figure 17: Zome3D rendering + coloured nodes

This form lends itself to be presented as a sculpture lying on a side, that could be made from Corten rusted steel sheets 10mm thick (fig 14, with JavaView [6]). Later I thought of a building with a triangular door, a large glasshouse triangular window, a zenithal triangular window adjacent to a diamond horizontal roof, etc to ideally be located between tall buildings, some windows of which would enjoy a dramatic Tangram view (fig.15). Such a structure was readily set up from wooden struts and ropes in the garden (fig. 16), to eventually become a studio for my art work. Helicopters that fly around the Genval lake area will spot Tangrams, as would occasional hot air ballooners...



Figure 16: Tangramoid structure



The same shape was also turned into a necklace jewel (fig. 18), and later was laser-etched inside of a crystal block (fig. 19). Finally, using the KnotPlot Program [7], there was much room to fool happily around with "tangramoidoid" shapes, i.e. shapes looking like tangramoids... (fig. 20).



Figure 18: Tangamoid necklace

Figure 19: Crystal

Figure 20: Tangramoidoids for fun

To conclude this part on tangramoids of the exploded square Tangram silhouette, let's mention that a model tangramoid was presented recently during a personal exhibition at La Pommerage, Belgium, in an installation :"Where damn is that Tangram" featuring a terra cotta sculpture and a mirror for the public to read hints written reverse, to find what the girl is looking for (evidently not her contact lenses) (fig. 21).



Figure 21: Tangram installation with terracotta and mirror

3. Tangramoid representing Tangram figures involving 5 piece square

Another most inspiring Tangram construct is the square made up of the five smaller pieces, with the large triangles floating around as they like. Sure enough, an infinity of 3-d objects open or closed can be developed (fig.22). In these examples the faces need not be opaque.



Figure 22: Three variations of open non-symmetrical tangramoids with Tangram figure where the five smallest pieces are in square, with JavaView [6]

Fig. 23 shows, again with JavaView, a closed axisymmetrical core without the triangular "wings", where most of the faces must be opaque to work.



Figure 23: axisymmetrical (order2) tangramoid with Tanram figure with only 5 smallest pieces in square

One could use a roof having the form of such an open tangramoid to cover a building. The large triangles could be triangular lawns organized in such a way that they complete a Tangram shape with the five smallest parts forming a square. Fig. 24 presents a Zometool model with yellow and green struts.



Figure 24: Zometool model for tangramoid roof

4. Room for further research

So far only subclasses of tangramoids were discussed, those featuring orthogonal views of only two families of shapes, the unique seven piece square, and a variety of five-piece squares with trianglular wings floating around. One could extrapolate to objects showing views of all kinds of Tangram figures, and not only in orthogonal views but also in perspective, and one might think there could be many more having nice characteristics appealing to aesthetic criteria. So let's leave the subject open for further interesting findings which no doubt will show up for the glory of that delightful little game: the Tangram.

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References

[1] Joost Elffers, Tangram, the ancient Chinese shapes game, Penguin Books, 1976.

[2] S. Verbiese, *The tangramoid, transgression for the Tangram,* Sculpture Competition 2001, IBGE-BIM/AEA, Brussels.

[3] http://www.polydron.com

[4] http://www.zometool.com

[5] http://www.geomview.org

[6] http://www.javaview.de

[7] http://www.pims.math.ca/knotplot/

[8] http://www.vorthmann.org/zome/

I did not mention theoretical references, as I am not a mathematician, leaving to specialists, if interested, to dig out possible mathematical nuggets these structures might conceal.