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# Polyhedral Designs of Detection Systems for Nuclear Physics Studies

N.G. Nicolis
Department of Physics
University of Ioannina
Ioannina 45110, Greece
E-mail: nnicolis@cc.uoi.gr

## Abstract

Polyhedral symmetries play an important role in mathematics, physics, chemistry, art and architecture. Despite the rich bibliography on this subject, little is known, to the non-expert, on the use of symmetric constructions in specialized scientific applications. We present a short review showing the influence of the geometry of polyhedra in the design of multi-detector systems employed in nuclear physics studies. In detection systems with a small number of detectors, designs based on Platonic and Archimedean solids have been used. On the other hand, in order to accommodate a large number of detector modules, highly segmented shell-like structures have been created. These constructions are based on a geodesic breakdown of an icosahedron and resemble architectural dome structures.

## 1. Introduction

The history of polyhedra has its roots in the works of ancient Greeks. Plato (429 – 347 B.C.) describes in Timaeus the five regular solids (tetrahedron, cube, octahedron, dodecahedron and icosahedron) with specific details on their construction. In the fifth book of the "Collection", the Greek mathematician Pappus of Alexandria (fourth century A.D.) describes thirteen solids, which he attributes to Archimedes. The Archimedean solids are semi-regular, each one of them having regular but not similar faces. These polyhedra have greatly influenced the human thought in many areas. In mathematics, they constitute a subject of research for centuries. Physicists and chemists encounter this geometry in the crystalline structure of solids and of complex molecules. Polyhedra have been the source of inspiration in graphic arts, painting and sculpture. In architecture, significant advances have been made with the construction of geodesic domes, which are based on polyhedral symmetries. Extensive discussions and bibliographies on these topics can be found in print and online [1,2,3].

The purpose of the present article is to point out a less known subject, namely, the influence of the geometry of regular and semi-regular polyhedra in the design of multi-detector arrays employed in nuclear physics studies. This will be demonstrated with several examples from the detector array technology. Our discussion will be limited in representative cases of systems exhibiting a spherical symmetry. In the following, we begin with an outline of the need for using such detection systems and some basic design requirements. We present detector arrays based on Platonic or Archimedean polyhedra, and then, the more sophisticated ones produced with a geodesic breakdown of an icosahedron. A link between geometry, dome architecture and the design of multi-detector systems is realized.

## 2. General considerations

In nuclear physics experiments, the researcher is interested in observing the radiations emitted from a point-like radiation source. This source may be a sample of radioactive material or the interaction region between a projectile and a target nucleus in an accelerator experiment. Depending on the case, the emitted radiation may consist of gamma rays (high-energy photons), neutrons, protons or heavier clusters of nucleons. Furthermore, the radiation may be emitted with different energies and intensities depending on the emission direction. This fact, together with additional experimental requirements necessitated the design and construction of multi-detector systems. These systems consist of arrays of appropriate detectors, which surround the source of radiation with a spherical or axially symmetric arrangement. Thus, they provide coverage by a large percentage (of the complete solid angle) of $4\pi$. The radiation source is located at the center of a usually spherical *reaction*, or *scattering chamber*, held in vacuum. Charged particle detectors are placed inside the scattering chamber, whereas gamma ray detectors are placed outside, since gamma rays can easily penetrate the chamber wall. In the following multi-detector systems for gamma rays, each detector element is a $NaI$ crystal, for the measurement of the gamma ray energies. For charged particles, we have "telescopes" (made of plastic and/or some crystal material) providing particle identification and energy information.

The design of a multi-detector system may be influenced by a number of factors depending on the particular application. Roughly speaking, it is a compromise between the desirable number of detector modules and financial constraints. Detection systems with a small number of detectors are, in general, suitable for the measurement of average quantities, such as the average number of gamma rays emitted in a reaction. On the other hand, a larger number of detectors can lead to a measurement of the complete distribution of gamma rays. Furthermore, the shape of each detector module may be influenced by factors such as the light collection efficiency from the detector crystal.

## 3. Designs based on Platonic and Archimedean solids

In cases of a relatively small number of detectors, designs using Platonic or Archimedean solids have been used. The geometry of the regular dodecahedron has been fully exploited in a gamma-ray and particle detection system, called "the Hedgehog", developed at the University of Rochester [4]. The Hedgehog consisted of an array of 12 closely packed $NaI$ detectors surrounding a spherical (12.7 cm diameter) target chamber. The twelve, cylindrical shape, $3'' \times 3''$ $NaI$ detectors are supported on the pentagonal sides of the dodecahedron frame, shown in Figure 1. The support plates are connected to hollow cylinders made of $Pb$, which provide shielding between adjacent detectors. Twenty 1.9 cm dia-

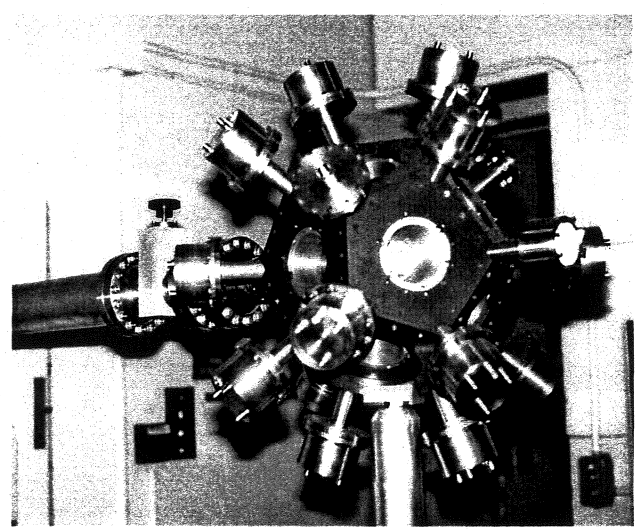


**Figure 1:** *The frame of the "Hedgehog" array developed at the University of Rochester.*

meter pipes expand radially outward of the spherical target chamber. Their locations correspond to the dodecahedron vertices.

A cut-through of the target chamber and detection system is shown in Figure 2. The beam travels along a diameter of the spherical chamber and interacts with a target placed at the center. A target ladder is mounted on an air lock system. This allows target changes without letting the system up to air. Four target frames are shown on the target ladder. The focusing of the beam on target is made with the aid of a small quartz viewer, of the target size, mounted on the last frame and viewed by a closed circuit TV system. The rest of the pipes are connected to cylindrical containers designed for mounting particle detectors. The *NaI* gamma-ray detectors are placed inside the hollow lead cylinders, which end up to truncated conical shields. The end of the conical shields has the shape of a regular pentagon and is in direct contact with the wall of the scattering chamber. The "hedgehog" has proven useful in a study gamma rays emitted in coincidence with charged particles in nuclear reactions [5].

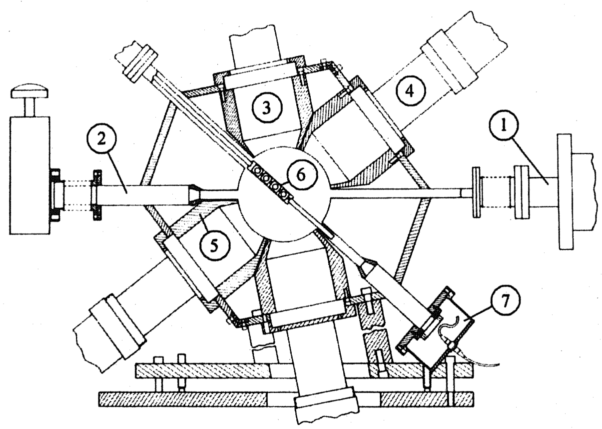



**Figure 2**: *Schematic drawing of an equatorial cut-through of the target chamber and detection system. (1) Beam entrance. (2) Beam exit. (3) NaI crystal. (4) Photomultiplier tube. (5) Pb shielding. (6) Target holding system. (7) Particle detector mount and electrical connection.*

At the University of Pennsylvania, a modular $4\pi$ array of up to 26 charged particle detectors (phoswich scintillator telescopes) was constructed [6]. The geometry of the array was based on the Archimedean rhombicuboctahedron, the solid with 26 faces (18 squares and 8 equilateral triangles). Figure 3 shows the vacuum chamber of this device.

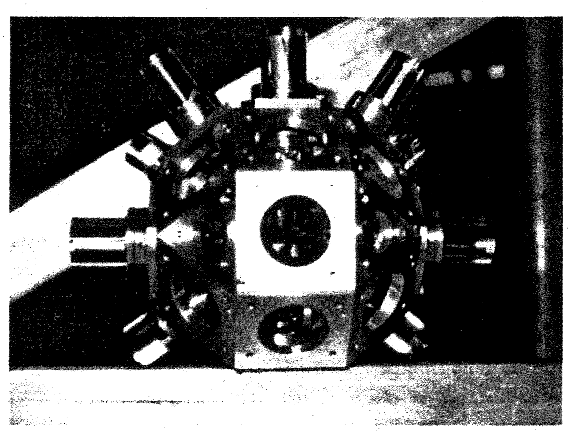


**Figure 3**: *The vacuum chamber of the $4\pi$ phoswich array of the University of Pennsylvania.*

Each detector telescope forms one face of the polyhedron surrounding the target. Two square faces are left open for the beam entrance and target support. The distance from the target to the front face of a square detector is 18.5 mm. Each detector is mounted on light pipe centered in a (square or triangular) vacuum flange. The light pipes expand radially outward, and each flange is bolted to the corresponding face of the aluminum vacuum chamber, whose outer surface forms a similar polygon. The geometry of the rhombicuboctahedron allows a convenient set of angles for detectors, provides a reasonable granularity for the particle detection, and retains a high degree of azimuthal symmetry, while requiring only two simple shapes for the detector telescopes. The same geometry has also been chosen in the design of the detector array EXOGAM, currently under construction [7].

In order to accommodate a larger number of detectors, modifications based on Archimedean solids have proven useful. Starting with the snub dodecahedron, one may construct its dual, in which faces and polyhedron vertices occupy complementary locations. The resulting solid is the pentagonal hexecontahedron (PHH) consisting of 60 irregular pentagons. By cutting off each of the 12 5-fold vertices and replacing them by regular pentagons, we end up with a truncated PHH. The truncated PHH has a total of 72 faces, 60 hexagonal and 12 pentagonal. The truncation procedure is illustrated on the left of Figure 4. The pentagonal faces align with the faces of a regular dodecahedron [8].

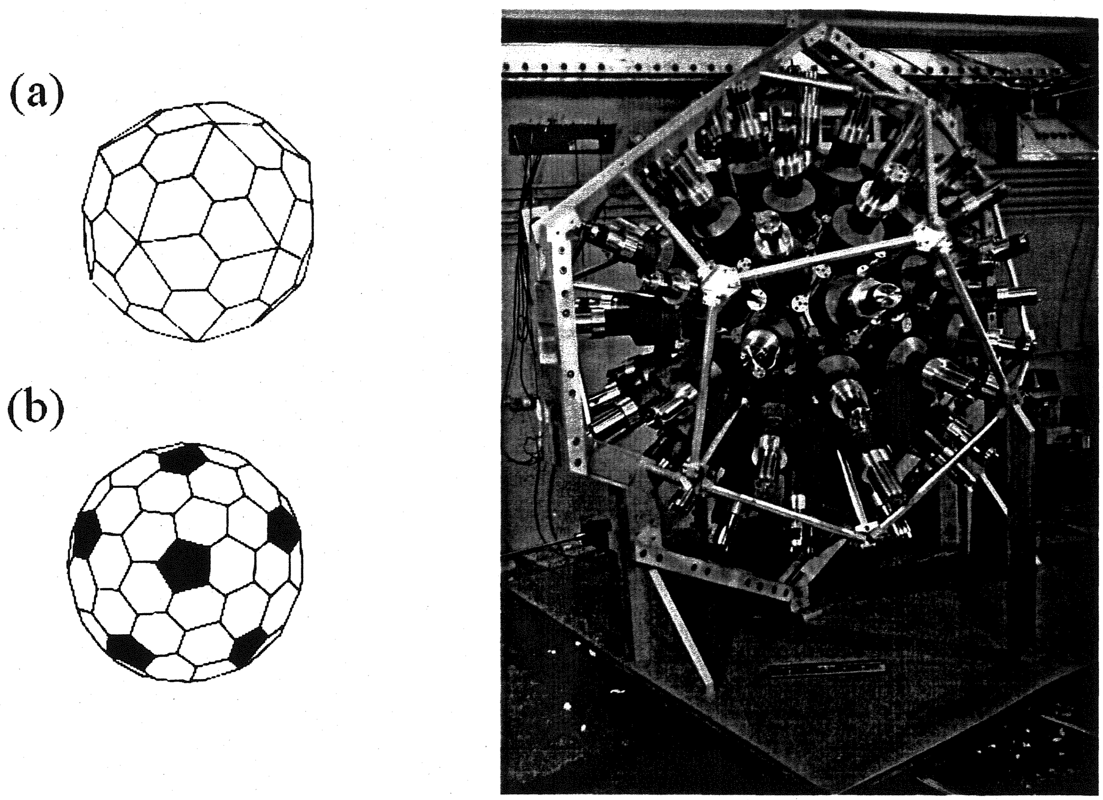



**Figure 4**: *Left: (a) The pentagonal hexecontahedron is the dual of the Archimedean snub dodecahedron. (b) A solid with 72 sides is obtained by truncating the 5-fold vertices of the previous solid. Right: The Spin Spectrometer with both sides closed. Signal cables are not connected.*

The truncated PHH forms the basis of design of the Spin Spectrometer, developed at the Washington University and installed at the at the Holifield Heavy-Ion Research Facility (HHIRF), at the Oak Ridge National Laboratory [9]. Seventy-two $NaI$ detectors approximate a hollow sphere with inner radius 178 mm and shell thickness 178 mm. Each detector is a tapped prism that can be removed radially. The cross section of twelve of the detectors is a regular pentagon. Each pentagonal detector is surrounded by five identical detectors of irregular hexagonal cross section. One pair of pentagonal detectors is removed from the spectrometer to accommodate the beam entry and exit pipes, which are welded to the inner spherical reaction chamber. The supporting frame is made of aluminum rods and has the shape of a dodecahedron, whose centers lie behind the centers of the pentagonal detectors. The frame is split into halves that stand on two independently movable platforms. The two halves may be rolled back on tracks perpendicular to the beam line to give access to the reaction chamber. Figure 4 shows the Spectrometer standing on the platform with the two hemispheres closed.

A few years later, the same collaboration completed the construction of the "Dwarf Ball" (Figure 5); a miniature system consisting of 72 plastic phoswich detectors, with the same geometry as the Spin Spectrometer [10]. The small size of the Dwarf Ball allowed its placement inside the reaction chamber of the Spin Spectrometer. The combined setup made possible the simultaneous detection, in a 4π geometry, of the gamma rays and light charged particles emitted in nuclear reactions.

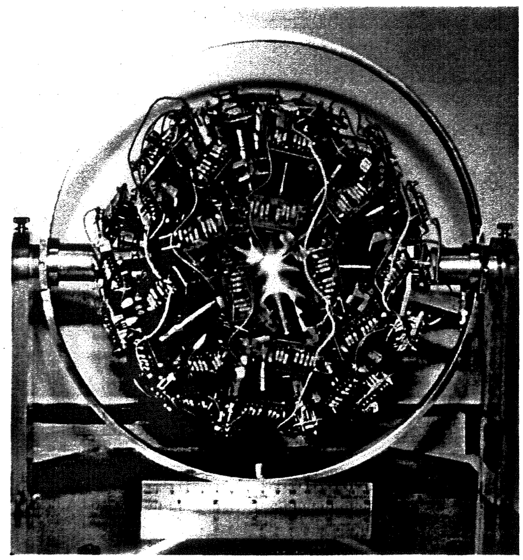


**Figure 5**: *Photograph of the Dwarf Ball detection system.*

## 4. Geodesic subdivisions

Systems with a high segmentation may result efficiently from an icosahedron, due to the high degree of symmetry it possesses and the close approximation it provides to its circumscribed sphere. The icosahedron has 31 symmetry axes and a corresponding number of *great circles* or *geodesics*. The great circles fall into three classes of six, fifteen and ten, corresponding to symmetry axes defined by the pairs of opposite vertices, the pairs of opposite edges, and the pairs of opposite face centers, respectively. The great circles can be envisioned by assuming the icosahedron rotating about its symmetry axes.

A subdivision can be introduced on a single face of the icosahedron, or some set of faces constituting a tile unit, provided the subdivision is symmetrical with respect to the face or tile unit [11]. Beginning with the equilateral triangular side, a simple choice is to divide it into $n^2$ smaller triangles, by breaking each edge into $n$ pieces and connecting the break points with lines parallel to the triangle sides. In the literature of geodesic domes, this is referred to as n-frequency Class I breakdown. An example of a 2-

frequency Class I breakdown is given in Figure 6a. Upon successive rotations and reflections, the complete solid angle can be covered. In this case, the resulting object will have 80 faces.

Another possibility is to draw lines perpendicular to the triangle edges to form a new triangular grid. This is a Class II breakdown. Alternatively, one may use as a tile unit, the triangle that straddles the common edge of two polyhedron faces and its base joins their centroids together with its symmetric with respect to the plane defined by the two centroids and the polyhedron center. Figure 6b illustrates a 10-frequency subdivision induced on the tile unit. With a recombination of the resulting triangles, we obtain a pattern containing hexagons and pentagons, shown in Figure 6c. In this figure, since the point O is an icosahedron edge, five consecutive rotations by 72° will produce a complete pentagon, five hexagon A, five hexagon B, 5 hexagon C, and 10 halves of hexagon D. The full hexagon D is obtained from mirror symmetry, in which case we get 2 pentagons, 10 hexagon A, 10 hexagon B, 10 hexagon C, and 10 hexagon D. When the whole sphere is covered, one has 12 pentagons, 60 hexagons of type A, B, C and D, i.e. a total of 252 faces.

Once the planar surface of the icosahedron is subdivided, a similar topology has to be created on the circumscribed sphere. A reasonable choice is to perform the subdivision directly on the spherical surface, using great circle arcs corresponding to the lines in the plane tile unit [11]. Details for the calculation of lengths and angles of the tile unit have been discussed in the literature [11,12,13].

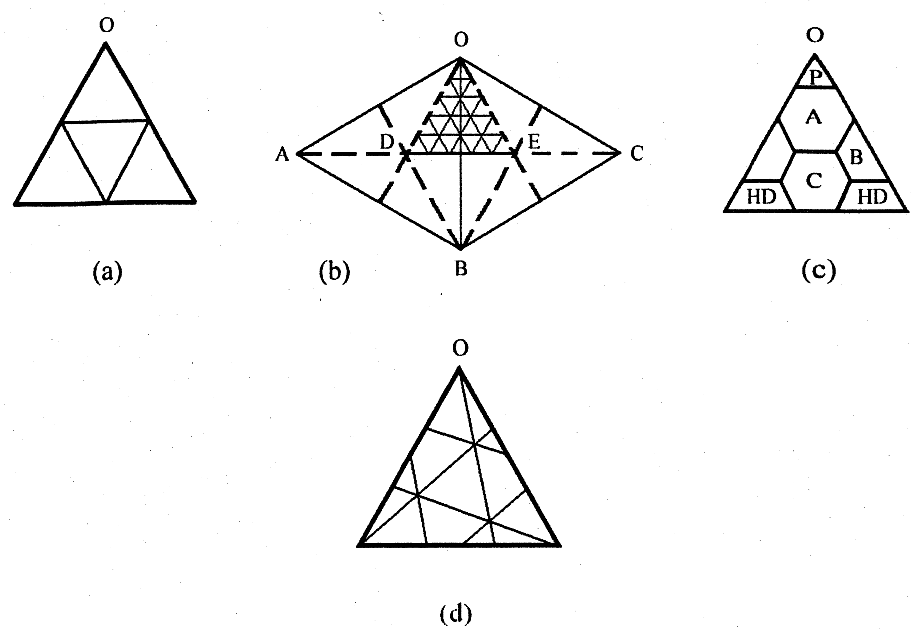



**Figure 6**: *(a) A 2-frequency Class I subdivision of an icosahedron face. (b) An example of a Class II breakdown. The Class II triangle ODE straddles the common edge OB of the icosahedron faces OAB and OBC. The 10-frequency breakdown refers to the tile unit ODBE. (c) Pattern created in the Class II triangle. (d) Example of a 3-frequency Class III subdivision.*

It is also possible to have subdivisions of Class III, in which the lines of segmentation on the icosahedron face run oblique to the edges. An example of a 3-frequency Class III subdivision is shown in Figure 4(d). Such a subdivision may lead to solids topologically equivalent to those derived from the snub dodecahedron [12]. The design of the Spin Spectrometer, already discussed, can be derived from such a subdivision.

## 5. Designs based on a Class I breakdown

Icosahedral subdivisions of Class I can easily produce systems with a high segmentation. As shown on the left of Figure 7, each of the 20 major triangles of an icosahedron can be subdivided into four smaller minor triangles to form a more spherical object with 80 faces. With a further subdivision of each minor triangle into nine, we obtain an object with 720 faces.

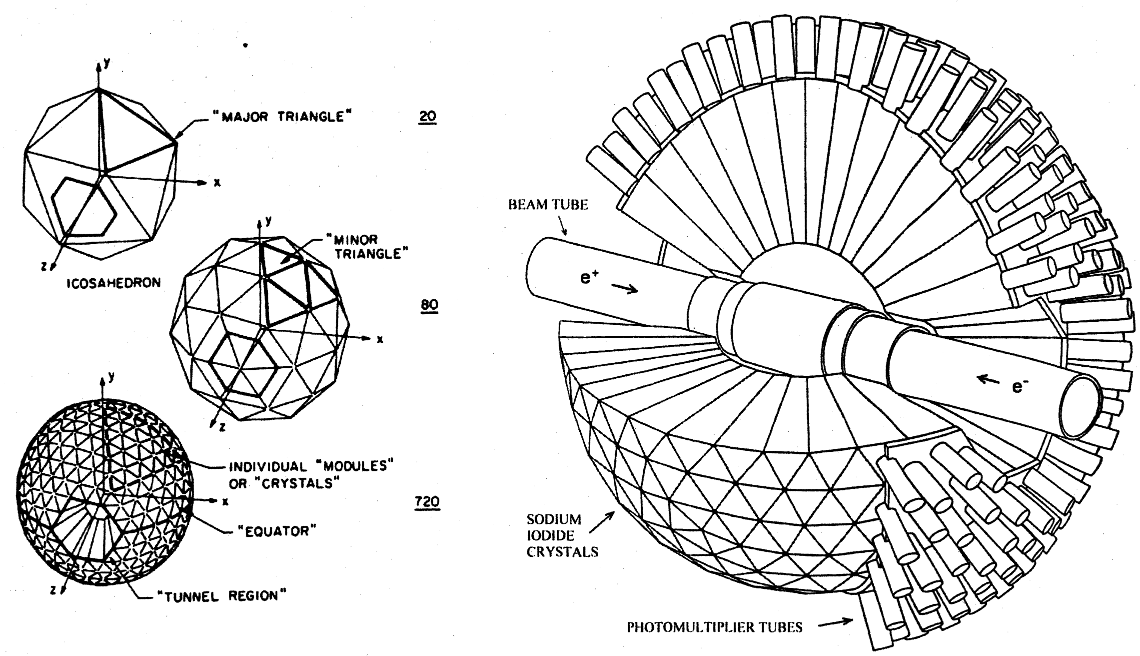
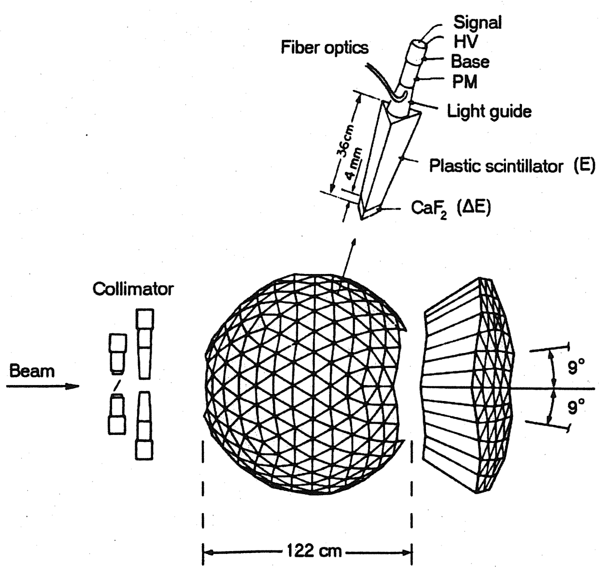



**Figure 7**: *Left: Polyhedral structures resulting from subdivisions of an icosahedron. Right: Schematic layout of the Crystal ball detector at SLAC.*

The geometry of the "80-hedron" has been exploited in the multi-detector system of Ref. [14], designed to study the emitted particles in relativistic nuclear collisions. The 720-sided polyhedron of Figure 7 has also been used with the modification that faces within the region marked as "tunnel region", as well as its symmetric ones were omitted. The remaining object of 672 sides was used in the design of the Crystal Ball detector [15] operated initially at the Stanford Linear Accelerator Center (SLAC). The Crystal Ball, shown schematically on the right of Figure 7, is a segmented spherical shell with an inner radius of 25.5 cm and an outer radius of 66 cm consisting of 672 optically isolated *NaI* crystals. Each crystal has a shape of a triangular prism. It is about 40.6 cm long with a small end dimension of about 5 cm and a large end of about 12.7 cm; it is viewed by a $2''$ phototube. The crystals were stacked to form two mechanically separate hemispheres, one upper and one lower. The Crystal Ball is one of the instruments involved in exploring the structure of quarkonium, the state of matter made up of a quark and an antiquark of the same kind. The detector is mounted at a particle-storage ring; beams of high-energy electrons ($e^-$) and positrons (or anti-electrons, $e^+$) collide at the center of the detector and give rise to new particles, including quarkonium.

In nuclear collisions at relativistic energies, a large number of particles are emitted with preference in the forward direction. For the purpose of such a study, a $4\pi$ detection system with particle identification was built at the Lawrence Berkeley Laboratory [16]. It consisted of the Plastic Ball, involving 815 particle telescopes and a Plastic Wall of 176 telescopes covering 97% of $4\pi$. The design of the Berkeley Plastic Ball followed the one of the SLAC Crystal Ball, with modifications made in details of the entrance and exit ports. Starting with the 720-sided polyhedron, modules in the backward cone between 160° and 180° and in the forward cone between 0° and 10° were omitted. Furthermore, in the region between 10° and 30°, modules were subdivided into 160, instead of the 40 of the original design, and positioned at a larger radius. A schematic drawing and a top view picture of the Plastic Ball are shown in Figure 8.



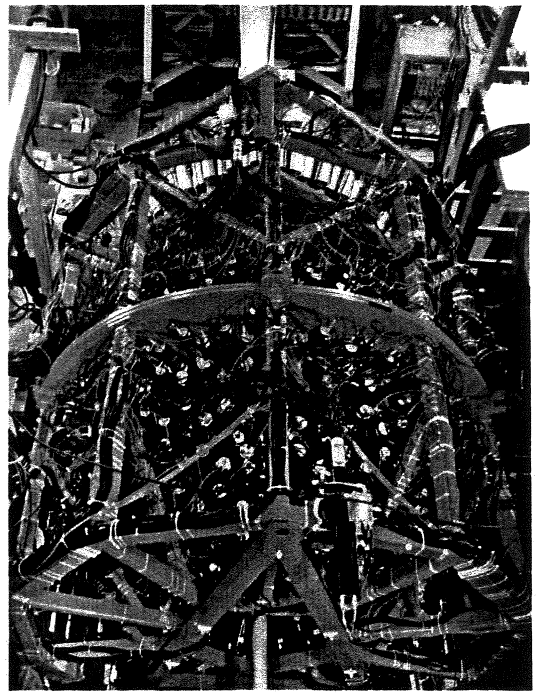



**Figure 8:** *Schematic drawing of the Plastic Ball detection system (left), and top view (right).*

With a 3-frequency Class I subdivision of an icosahedron, one may recombine the resulting triangles to obtain a polyhedron with 32 sides, consisting of 20 hexagons and 12 pentagons. The detection array CACTUS (Figure 9) of the Cyclotron Laboratory, University of Oslo, Norway is based on such geometry [17]. Alternatively, this design can be derived from the Archimedean truncated icosahedron.

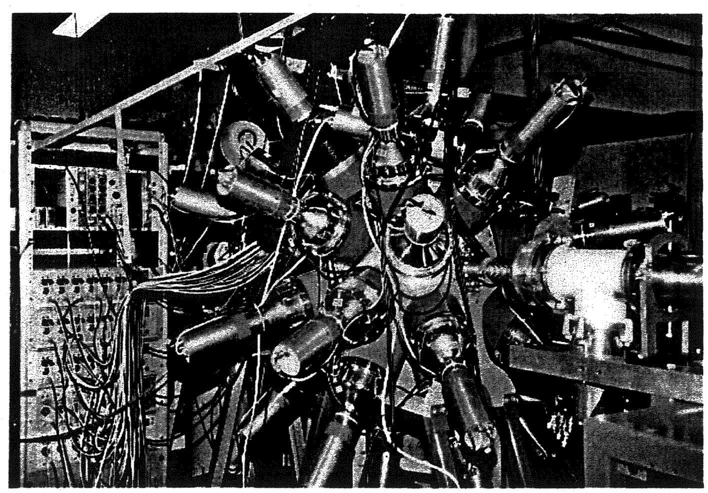


**Figure 9:** *The detection array CACTUS of the Cyclotron Laboratory, University of Oslo, Norway.*

With a 6-frequency Class I subdivision and triangle recombination we obtain a polyhedron with 122 sides, consisting of 110 hexagons and 12 pentagons. A multi-detector system consistent with such a polyhedron is the under construction GRETA array [18].

## 6. Designs based on a Class II breakdown

An 8-frequency Class II breakdown results in a polyhedron with 162 sides, 150 hexagons and 12 pentagons. The Heidelberg Crystal Ball detection system [19] consists of 162 *NaI* detectors can be classified in this category. The authors of Ref. [19] refer to a procedure based on a triangulation of a dodecahedron, which produces exactly the same type and arrangement of polygon sides.

A finer subdivision is obtained with the 10-frequency breakdown described in connection with Figure 6b and 6c. The resulting polyhedron with 252 faces forms the basis for the design of the PIBETA calorimeter for cosmic ray studies [20]. After modifications made along a diameter, to allow for beam entry as well as "edge" detectors, the resulting system appears with 240 modules. A schematic drawing of this device is shown in Figure 10.

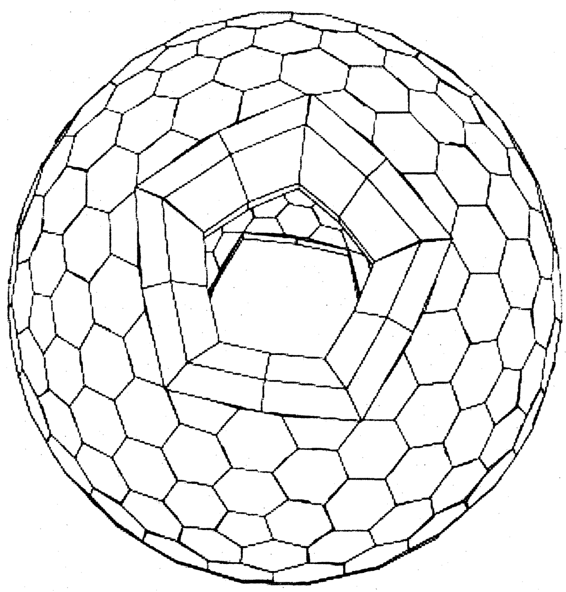


**Figure 10**: *Schematic drawing of the PIBETA calorimeter.*

## 7. Summary

From our account, based on representative cases, we realize the great impact of the geometry of polyhedra in the design of multi-detector systems for nuclear physics studies. Platonic and Archimedean designs have been used effectively, due to their functionality and high degree of symmetry. Furthermore, the icosahedral symmetry has manifested in the design of multi-detector systems with a high segmentation. In this respect, geodesic dome construction has influenced scientific design. As a result, we have a kind of "scientific art" which, besides many advances in science, has produced interesting structures from the aesthetic point of view.

## References


[1] Jay Kappraff, *Connections: The Geometric Bridge Between Art and Science*, McGraw Hill, 1990.
[2] H.S.M. Coxeter, *Virus Macromolecules and Geodesic Domes*, in A Spectrum of Mathematics, J.C. Butcher (Editor), Aukland, 1971.
[3] George W. Hart, Virtual Polyhedra, http://www.georgehart.com/virtual-polyhedra/vp.html
[4] D. Cline and R. Prechter, Nuclear Structure Research Laboratory. The University of Rochester. Annual Report (1977); N.G. Nicolis, Ph.D. Thesis, University of Rochester, 1986.
[5] N.G. Nicolis, T.M. Cormier, P.M. Stwertka and M.G. Herman, Phys. Lett. **178B**, 339(1986).
[6] T. Chapuran, D.P. Balamuth, J.W. Arrison, and J. Görres, Nucl. Instr. Meth. A**272**, 767(1988).
[7] F. Azaiez and W. Korten, Nucl. Phys. News, Vol. 7, No. 4, p. 21, 1997.
[8] D. Wells, *The Penguin Dictionary of Curius and Interesting Geometry*, Penguin, London, 1991.
[9] M. Jääskeläinen, D.G. Sarantites, R. Woodward, F.A. Dilmanian, J.T. Hood, R. Jääskeläinen, D.C. Hensley, M.L. Halbert, and J.H. Barker, Nucl. Instr. Meth. A**204**, 385(1983).
[10] D.G. Sarantites, L.G. Sobotka, T.M. Semkow, V. Abenante, J. Elson, J. Hood, Z. Li, N.G. Nicolis, D.W. Stracener, J. Valdes and D.C. Hensley, Nucl. Instr. Meth. **A264**, 319(1987).
[11] Hugh Kenner, *Geodesic Math and How to Use It*, University of California Press, 1976.
[12] Magnus J. Wenninger, Spherical Models, Cambridge, 1979 (1999 Dover reprint).
[13] G.W. Hart, *Reticulated Geodesic Constructions*, Computers and Graphics, 24:6, Dec. 2000, pp.907-910.
[14] W.G. Meyer, H.H. Gutbrod, Ch. Lukner, and A. Sandoval, Phys. Rev. C**22**, 179(1980).
[15] M. Oreglia, E. Bloom, F. Bulos, R. Chestnut, J. Gaiser, G. Godfrey, C.Kiesling, W. Lockman, D.L. Scharre, R. Partridge, C. Peck, F.C. Porter, D. Antreasyan, Yi-Fan Gu, W. Kollmann, M. Richardson, K. Strauch, K. Wacker, D. Aschman, T. Burnett, C. Newman, M. Cavalli-Sforza, D. Coyne, H. Sadrozinski, R. Hofstadter, I. Kirkbride, H. Kolanoski, K. Königsmann, A. Liberman, J. O'Reilly, B. Pollock, and J. Tompkins, Phys. Rev. D**25**, 2254(1982).
[16] A. Baden, H.H. Gutbrod, H. Löhner, M.R. Maier, A.M. Poskanzer, T. Renner, H. Riedesel, H.G. Ritter, H. Spieler, A. Warwick, and H. Wieman, Nucl. Instr. Meth. **203**, 189(1982).
[17] Magne Guttormsen, Nuclear Physics News, Vol. 9(4), 33(1999).
[18] M.A. Deleplanque, I.Y. Lee, K. Vetter, G.J. Schmid, F.S. Stephens, R.M. Clark, R.M. Diamond, P. Fallon, and A.O. Macchiavelli, Nucl. Instr. Meth. Phys. Res. A**430**, 292(1999).
[19] V. Metag, D. Habs, K. Helmer, U. v. Helmolt, H.W. Heyng, B. Kolb, D. Pelte, D. Schwalm, W. Hennerici, H.J. Hennrich, G. Himmele, E. Jaeschke, R. Repnow, W. Wahl, R.S. Simon and R. Albrecht, Lecture Notes in Physics **178**, 163(1983).
[20] E. Frlež, I. Supek, K.A. Assamagan, Ch. Brönnimann, Th. Flügel, B. Krause, D.W. Lawrence, D. Mzavia, D. Počanić, D. Renker, S. Ritt, P.L. Slocum, N. Soić, Nucl. Instr. Meth. Phys. Res. A**440**, 57(2000).