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Mathematical Connections
in Art, Music, and Science

Gridfield Geometry

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I am the primitive of the way I have discovered.
Paul Cezanne

Abstract

This paper presents an introduction to a unique new geometry, which I call GridField Geometry. To understand the basic structure of GridField Geometry, the presentation is a step by step construction of a gridfield.

1. Introduction

The foundation of my art [1, 2, 3] is a mathematical procedure, I call GridField Geometry. It is the purpose of this paper to explain the basic structure of GridField Geometry by the construction of a specific gridfield example.

Many geometries are derived from Euclidean Geometry and the Cartesian coordinate system. GridField Geometries are basically curvilinear grid and coordinate systems. They are constructed iteratively starting from the ordinary Cartesian grid with integer coordinates. The parameters selected at each iteration, or generation, are freely chosen by the artist/mathematician in order to explore and experience increasingly complex geometric configurations and shapes. GridField Geometry presents the possibility of creating an infinite number of grid and coordinate systems, which the Cartesian grid coordinate system is but one.

2. The Wave and Wave Field

The basic constituent of a *wave* is a figure element, which is formed by centering the element about an imaginary reference line called the wave *axis*, and then replicating the figure element along the axis of translation, giving it the appearance of oscillation about the axis. The parameters of the figure element are amplitude a, wavelength λ, shape, and orientation. The shapes can be any, or a combination, of three basic shapes. The three shapes are convex, rectilinear, and concave which are identified by A, B, and C respectively. The orientations are X and Y. Figure 1 shows the wave axis as a broken line, and the three types of figure elements oriented in the X direction.

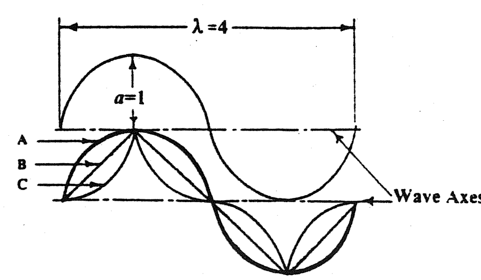

Figure 1: *Figure Element*

A *wave field* is a set of parallel translates of a wave, uniformly separated from each other by a distance of one unit.

3. The GridField

A gridfield is a grid formed by two or more wave fields. There can be any number of wave fields combined to make a gridfield. In this paper, we will be primarily concerned with a resultant gridfield configuration composed of two fields. A simple test for a gridfield is a "checkerboard" pattern of two alternating colors.

GridField Geometries are defined iteratively. The process requires three stages. First, a gridfield is defined on the basis of the current coordinate system. Second, at each iteration a new wave field is drawn into the current gridfield. Third, the gridfield is then modified geometrically through exclusions so that the result may be re-coordinatized. Two kinds of re-coordinatizations will be considered, yielding two different *gridfields*: the *crossfield* grid and the *interfield* grid. At each iteration, the artist/mathematician has the choice of working with either a crossfield or interfield grid. The crossfield grid has the property that its two constituent wave fields intersect in a manner analogous to the horizontal and vertical lines of the Cartesian grid. The interfield grid is more complicated and the resulting interweaving of the two wave fields that define it will have to be discussed later. The details of GridField Geometry construction will be covered by the following examples. The first example will be the construction of a crossfield grid.

4. A GridField Construction Sequence

4. The Crossfield Grid

Step 4.1. To begin the gridfield construction sequence, start with a Cartesian grid and draw a wave starting at the origin (0,0), which is to remain the origin for all subsequent gridfield constructions. The parameters for the wave are a = 1, λ = 12, shape A, and oriented in the Y direction, hereafter shortened to (1, 12, A, Y). Use translation to form a wave field with waves separated by one unit, as shown in Figure 2.

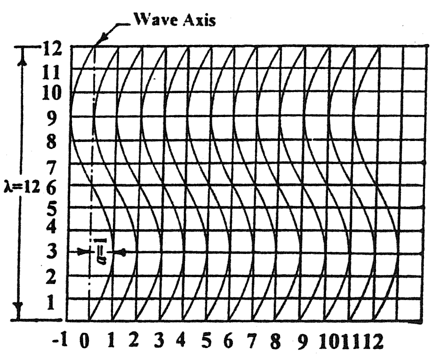

Figure 2: *Translated Figure Element* (1, 12, A, Y)

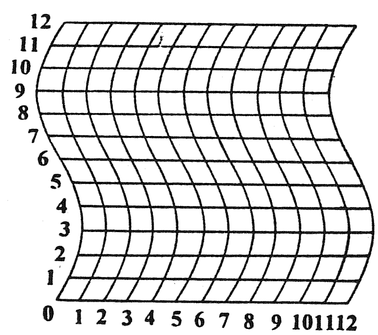

Figure 3: *1st Generation Crossfield Grid*

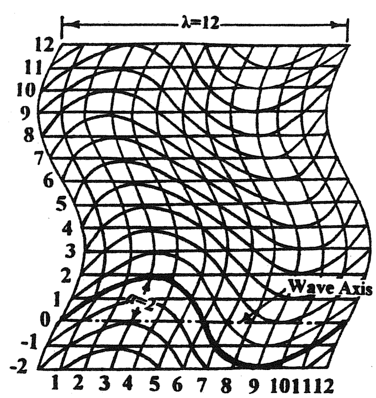

Figure 4: *Translated Figure Element* (2, 12, A, X)

Step 4.2. Working with Figure 2, exclude the vertical lines and coordinatize as shown in Figure 3 to form a crossfield grid, i.e., the crossing of the "horizontal" and "vertical" lines. This is a 1st generation crossfield grid.

Let's continue and consider a 2nd generation crossfield grid.

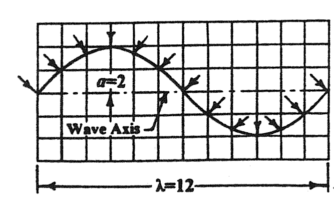

Figure 5: *Key Points on* (2, 12, A, X)

Step 4.3. Using the current gridfield, Figure 3, draw a wave starting at (0,0), whose parameters are (2, 12, A, X). Use translation to form a wave field with waves separated by one unit as shown in Figure 4.

The process for drawing a wave into a gridfield is to identify the key points on the wave from its Cartesian coordinates, in this case, Figure 5, and translate these to the current gridfield, which is the boldface line in Figure 4. This is an effective method for drawing purposes, but, obviously, not mathematically accurate.

Step 4.4. Working from Figure 4, exclude the horizontal lines and re-coordinatize as shown in Figure 6. This is a 2nd generation crossfield grid, bringing us to a totally curvilinear gridfield and coordinate system. Note, in this example, there are 12 different wave translations from our initial wave.

The above procedure can be used indefinitely in constructing subsequent crossfield grids, using any number of parameters chosen by the artist/mathematician.

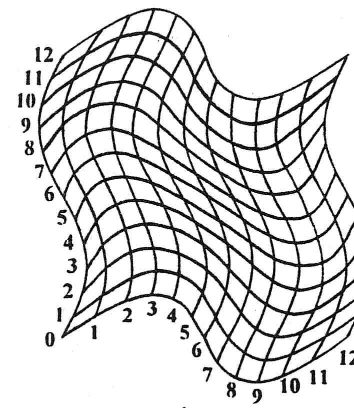

Figure 6: *2nd Generation Crossfield Grid*

5. The Interfield Grid

As stated earlier, the interfield grid is more complicated than the crossfield grid. The interfield grid is formed when a wave field is drawn into the current wave field and oriented in the same direction. Indeed, the current wave field becomes the wave axes for our drawn field.

For a specific example of an interfield grid, continue as follows:

Step 5.1. Starting at (0,0) on our crossfield grid, Figure 6, draw in a wave whose parameters are (2, 8, *A*, *X*) – the boldface line in Figure 7. In this example, the current gridfield has been extended to accommodate the full wavelength, $\lambda = 24$, of our resultant wave. Use translation to form a wave field with waves separated by one unit as shown in Figure 7.The resulting grid is a 3rd generation gridfield.

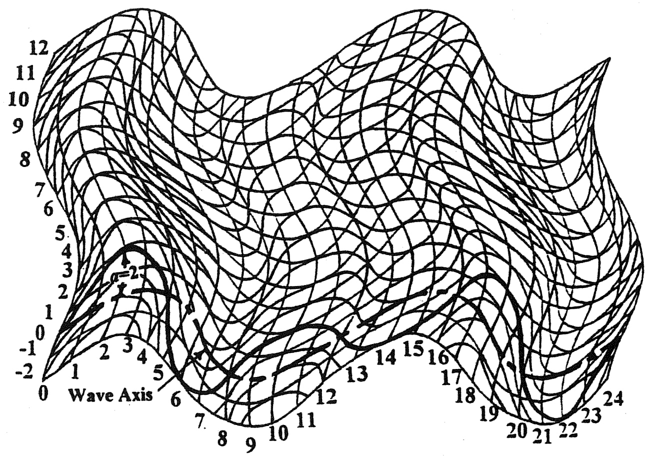

Figure 7: *3rd Generation Gridfield*

Step 5.2. To maintain our two field gridfield configuration, eliminate the current Y field and re-coordinatize, labeling the axes to bound a 24 x 24 quadrant as shown in Figure 8.

Re-coordinization is determined as follows: As in the previous examples, let the new origin coincide with the old origin. After the elimination of the y-axis in Figure 7, we have just two waves emanating from the origin as can be seen in Figure 8 – the now current y-axis and x-axis waves. Because the X and Y axes are no longer at 90^0 to each other, the y-axis wave is designated to be either to the left or above the x-axis wave, depending on the orientation of the current gridfield. To

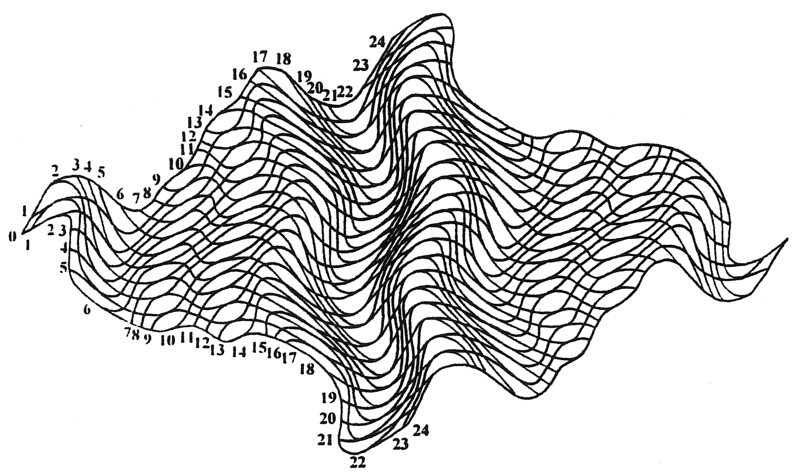

Figure 8: *3rd Generation Gridfield with Interfield Grid*

recognize the y-axis wave, start by following it from the origin and then *alternate* it from wave to wave as local extrema are encountered. Similarly define the new x-axis by following it as it emanates from the origin and alternates from wave to wave as local extrema are encountered. See Figure 9 where the local extrema are indicated by the arrows. Now to see the new "horizontal" and "vertical" wave fields, i.e., the new horizontal and vertical lines, follow the above rules starting from each of the y-axis labels and each of the x-axis labels to produce the results of Figure10.

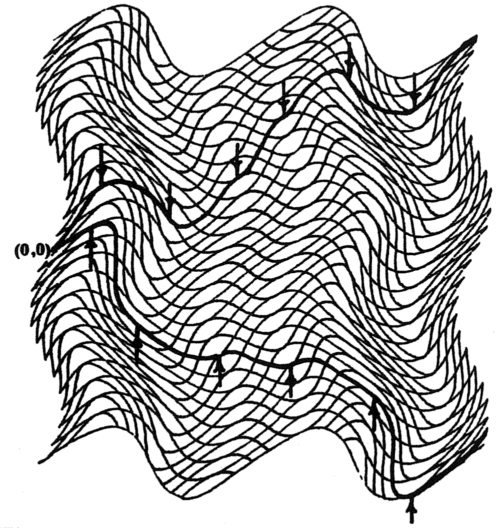

Figure 9: *Local Extrema of Current Interfield Grid*

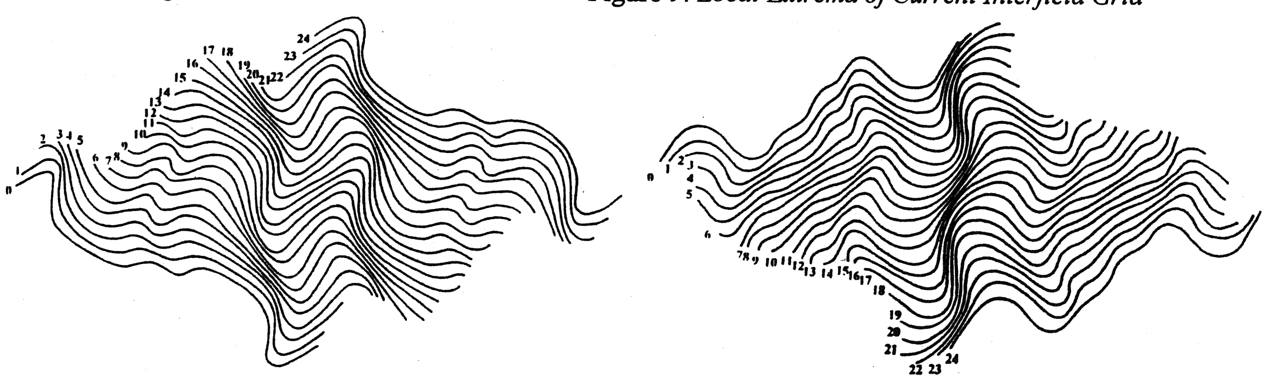

Figure 10: *"Horizontal" and "Vertical" Wave Fields*

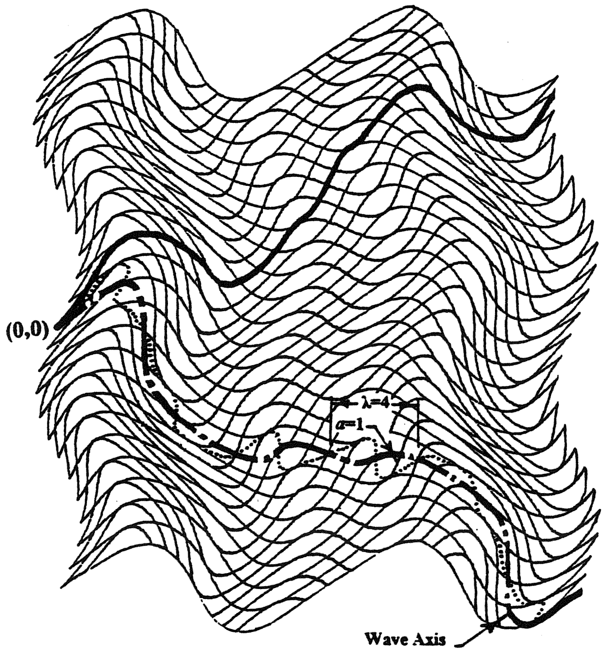

Figure 11: *Replication of Figure Element on X-Axis*

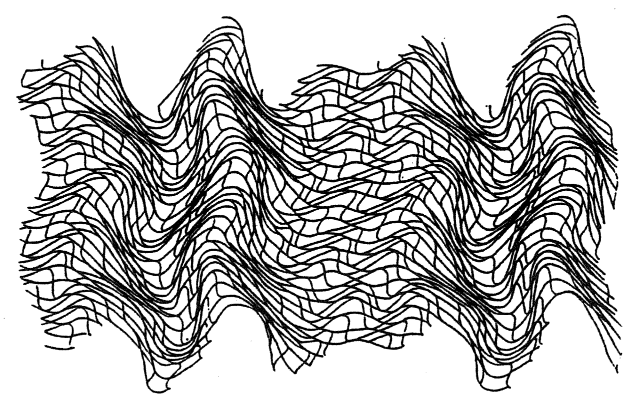

Figure 12: 4^{th} *Generation Gridfield with Interfield Grid*

Step 5.3. For a 4^{th} generation gridfield, assume a figure element whose parameters are $(1, 4, A, X)$. This is the same figure element of Figure 1. Starting at (0,0) on our current gridfield, draw in the figure element and replicate it along the axis of translation as shown in Figure 11. Use translation to form a wave field with waves separated by one unit. Eliminate the y-axis field. The result is a 4th generation gridfield, Figure 12, with an interfield grid.

Let us stop at this point as further iterations/generations would develop complexities unnecessary for the purpose of this paper. However, there is one more parameter that must be considered when constructing an interfield grid, and this I call, phase positioning.

5. Wave/Field Phase Positioning

The placement of a figure element in a gridfield is crucial when creating an interfield grid. It sets a phase relationship between the current gridfield and the translated wave field, which in turn determines the rhythm and shape of the resulting geometry. In the above examples, the starting position of our figure element on its wave axis was the origin. However, this need not be the case. On any wave, axis of translation, there are an infinite number of starting positions. For this paper, we are assuming just λ possible starting positions on a wave of wavelength λ. Therefore, in the example, Figure 6, we have 12 (λ = 12) possible starting positions in the X direction. To demonstrate phase positioning, use the crossfield grid of Figure 6 and assume a figure element with parameters $(2, 12, A, X)$. Draw in the figure element and replicate it at four different positions as shown in Figure 13: Figure 13a shows the starting position of the figure element (indicated by λ = 12) at the origin, (0, 0). Figure 13b shows the starting position at (4, 0), Figure 13c at (5, 0), and Figure 13d at (6, 0).

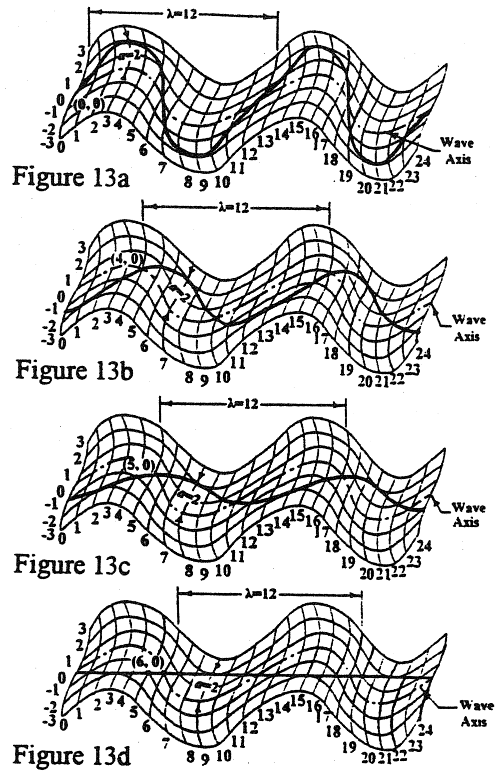

Figure 13: *Phase Positions*

6. Conclusion

This paper has explored 4 generations of wave fields, using modest parameters. At each iteration, a choice was made to demonstrate a crossfield or interfield grid geometry. Indeed, at the very beginning, in Step 4.2, we could have chosen to have an interfield grid rather than a crossfield grid, by excluding the horizontal lines instead of the vertical lines, as seen in Figure 14.

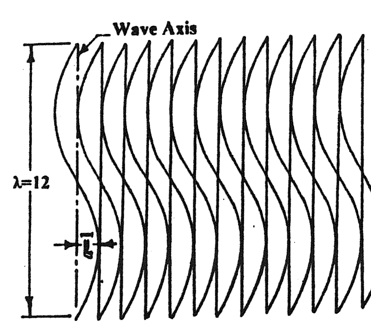

Figure 14: *1st Generation Interfield Grid*

The majority of illustrations in this paper were hand drawn. The physical act of drawing these geometries is laborious and increasingly difficult because of the complexity. This is where the mathematician and computer programmer become important. Some work has been done [4], but as yet no one has gone beyond a 1st generation interfield grid.

GridField Geometry being new, its practicality is unproven, accept in the area of art and design as shown in my own work. Examples can be seen in my paintings [1], one of which is shown below. Its application to other areas, such as music, physics, and science in general, I hope to explore in future papers. Some possibilities have been mentioned in previous papers [2, 6, 7], with some rather fanciful comparisons and speculations in the sciences [7].

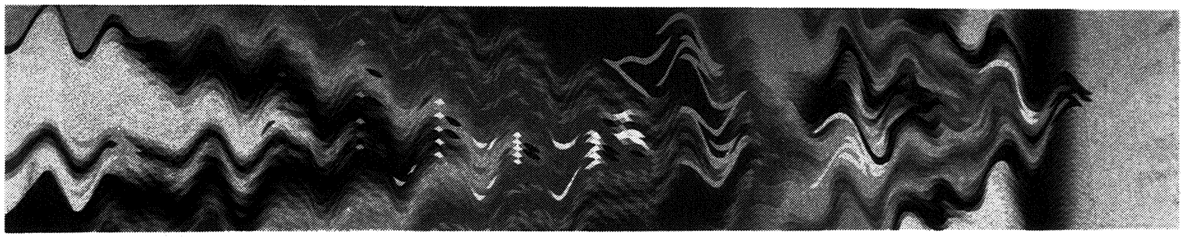

Figure14: *Painting by Douglas Peden, "Come Together".*

References

[1] "17 Paintings by Douglas Peden." **http://www.mi.sanu.ac.yu/vismath/pedens/index.html.**

[2] Peden, Douglas D. " The Influence and Use of Music and Mathematics in My Art." In Bridges: *Mathematical Connections in Art, Music, and Science Conference Proceedings*, pp.223-229. Sarhangi, Reza, ed. 2001.

[3] Peterson, Ivars. "Grid Fields." In *Fragments of Infinity*, Chapter 5.(Canada: John Wiley & Sons, Inc., 2001). Also, "Art of the Grid." **http://www.sciencenews.org/20000812/mathtrek.asp.**

[4] Bob Brill, a computer programmer from Ann Arbor, Michigan and myself have found a computer procedure for gridfields to the 1st generation interfield grid, using his book, *Algorithmic Imagery Using E.* (Zenagraf Ann Arbor). John Sharp, a mathematician from Watford, England has also managed a solution to the 1st generation interfield grid.

[5] Sharp, John. "Doug Peden's GridFields." **http://www.counton.org/explorer/gridwarping/peden.shtml.**

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