## BRIDGES Mathematical Connections in Art, Music, and Science

## Making Sense of "Tensegrities"

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How to see and understand tensile-integrity structures: A physical and visual demonstration of what they are, how they work, and the underlying "push-pull" geometry that determines their shapes.

Tensile-integrity structures, commonly called tensegrities and commonly associated with the names of R. Buckminster Fuller and Kenneth Snelson, are stable three dimensional structures in which compression members, sticks, float without touching each other within a continuous surrounding network of strings in tension. Using the most elementary three stick tensegrity possible, we will show how its form is determined by pairs of intersecting planes which lock the sticks in place at lines of intersection of the planes, and we will indicate how this principle can be applied to the analysis of other tensegrities.

Since tensegrities are embodiments of incidence geometry, essentially projective geometry, we will indicate how to "compute" a tensegrity graphically by making a "perspective" drawing of it as well as by using standard methods of engineering descriptive geometry. This will lead us into elementary Euclidean geometry (which used to be taught in high schools) where we will find that the very first diagram in Book 1 of Euclid's Elements is well on its way to being the plan view of a three stick tensegrity. However, we will discover that Euclid's system of geometry is incomplete if we wish to deal easily and intuitively with tensile-integrity push-pull geometry. Euclid has only one kind of straight line segment and two kinds are needed, namely, sticks and strings. Also, he has no way of dealing with wholes which are greater than the sums of their parts, and which loop around and "feed back" on to themselves. In fact, in his common notions Euclid assumes that the whole is equal -- always -- to the sum of its parts, never greater or less than. Tensegrities are examples of emergent structures where operations on physical things of one kind, sticks and strings, yield physical things of a new kind, tensegrities. They are not simply additive. They are not simply groupings of sticks and strings. They are opposed and intersecting planes of force. Some new axioms and notions for a possible post-Euclidean geometry and logic based on these observations and on the physical experience of tensegrities and their components will be suggested.

Why not until now? Back in the 1960s Kenneth Snelson remarked that the early Egyptians of 5,000 years ago could have built tensegrities. They had the requisite materials, sticks and strings. So why didn't they? Or had all the evidence of such structures vanished due to the perishable nature of the materials? And surely, the Greek Archimedes with his profound mechanical, as well as geometrical, intuition was capable of comprehending such structures. It is worth noting that in the 1860s James Clerk Maxwell, among others, developed much of the underlying geometry of tensegrities in works such as "On Reciprocal Figures and Diagrams of Forces". This was in the field of graphical statics and concerned the number of bars or lines needed to construct a stable figure joining a given number of points in space. The three stick tensegrity mentioned above conforms to a formula Maxwell gave. Others further developed this work, most of which has now been forgotten, but no tensile-integrity structures were ever built as a consequence. We will suggest some possible reasons why such structures have been quite literally "unthinkable" until about 50 years ago and why they are only just now beginning to be appreciated and understood.