

Exploring Art with Mathematics and Computer Programming

Alberto López-Santoyo
Fac. de Filosofía y Letras
UNAM
Arquitectura 19
04360 México, D.F., MEXICO
E-mail: fylgeo@servidor.unam.mx

Abstract

Even with simple concepts of pre-university mathematics and the aid of technology using computer programming one can explore the power of mathematics in art. Mainly distance between two points, the parabola, line segments, interpolation and random numbers were employed here. The images shown were obtained with very limited facilities using a very modest computer and a dot matrix printer. Perhaps teachers should encourage students to explore art using their mathematical knowledge and the aid of modern technology.

1. Introduction

Pre-university mathematics provides a wide range of material to explore from the point of view of art. One can start with simple concepts such as the distance between two points and then go to more complex ones. The aid of a computer is of course of great importance, but no matter how modest a computer is it is powerful enough to allow any one with some knowledge in mathematics and some experience in computer programming to get complex and artistic images. Students are able to explore art using some mathematics. Teachers may encourage them to do this since most students like to play with computers and produce works of art.

GW BASIC was used as programming language in the examples illustrated here. The outputs were obtained with a very modest computer and a dot matrix printer.

Duality is present in the four images made by computer due to the influence from real life experience.

The main mathematical items employed were as follows:

- a) Distance between two points.
- b) The parabola.
- c) Line segments, interpolation and random numbers.

In order to get a satisfactory result it was necessary to program the printer in text mode. The characters were the smallest available (upper captions) and the distance between characters and lines were reduced from the standard settings in order to get a compact set of lines and columns.

Some characters had to be designed to get the expected appearance according to what they had to represent and also because square characters were needed. Standard ones are larger in height than in width. This makes a big problem if one tries to obtain circles. Using the printer manual it was possible to program in GW BASIC some characters used. Of course this demanded patience and time in order to get the desired results.

With the aid of algebra it was possible to get complete flexibility for the size of the graphs, although it is not shown in the examples presented here. All the images can be printed in any size starting from a very small one to the biggest that can be obtained in a standard printer.

The four images made by computer are reduced 80 per cent. This means that 80 units of length in the reduced image correspond to 100 units in the original one.

2. Concentric Circles

For representing two sets of concentric circles (fig. 1) the concepts of distance between two points and the circle were used but only the equation for the distance between two points was needed:

$$D = [(X1-X2)^2 + (Y2-Y1)^2]^{1/2}$$
 where D is the distance between the two points. X1, Y1 are the coordinates of one point and X2, Y2 are the coordinates of the other point. The obtained distance was used as the radius of a certain circle.

In the example illustrated the circles meet in the central part and give a sense of struggle between equal forces. The center at the lower left corner is white and the other on the upper right corner is black. From each center concentric circles radiate in a grading sequence from white to black or black to white.

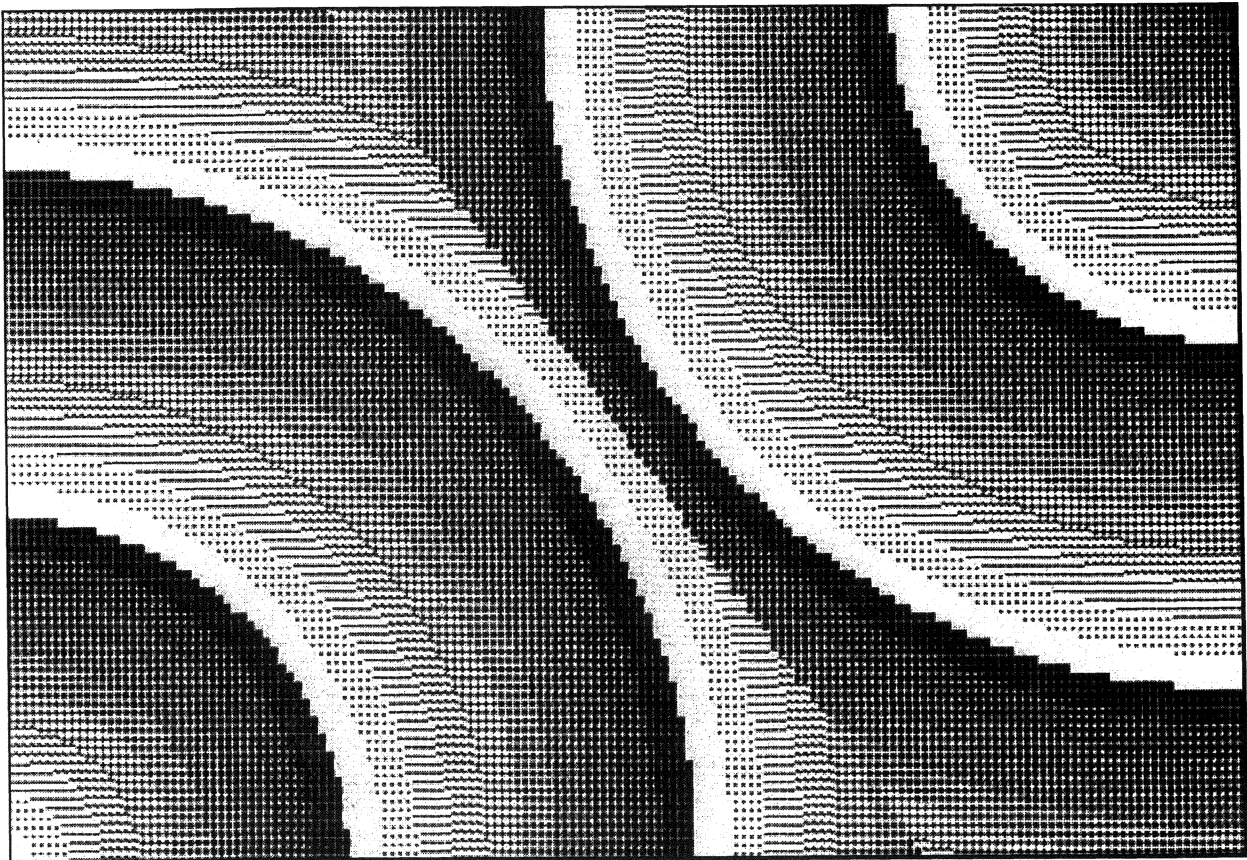


Figure 1: Concentric circles created using the distance between two points as radius

A matrix was chosen with the number of characters (columns) to be printed along a sheet of paper and the number of rows necessary for the desired graph. In the matrix the position of two points were given as centers for the circles and the coordinates of those points were determined in relation to the established matrix. The centers were chosen at the upper right and lower left corners of the rectangle.

To find the way any point or pixel had to be represented or what character had to be used, it was necessary for each pixel in the matrix to compute its distance to the nearest center and use that distance as radius. Then select automatically the corresponding character chosen beforehand for that radius.

3. Parabola and Concentric Circles

A parabola was drawn over a set of concentric circles that grade from black to almost white, from the center outwards. The axis of the parabola was defined horizontally at the middle part of the rectangle. The center of the circles was chosen on the same line on the left edge of the rectangle.

The equation of the parabola was used together with the distance between two points in the case represented in figure 2. The parameters for the parabola were established in such a way as to get a satisfactory appearance for it. The center for the concentric circles was chosen outside the parabola on its axis. The axis of the parabola is here horizontal and therefore its equation is:

$$Y^2 = 4aX \quad \text{where "a" is the parameter that gives the distance from the vertex to the focus.}$$

The parabola had preference over the concentric circles when printed. Therefore the circles act as a background for the parabola.



Figure 2: Parabola over a set of concentric circles

Here again as it was done on the concentric circles of figure 1, the rectangle of the chosen size was scanned pixel by pixel or point by point to allow the computer define or choose the corresponding character to be used. For each point or pixel there are three possibilities:

- a) The point is outside the parabola.
- b) The point is on the parabola.
- c) The point is inside the parabola.

If the point is outside the parabola then by using the equation of the distance between two points (the pixel to be printed and the center of the concentric circles), the corresponding character for a concentric circle was chosen.

If the point is on the parabola the character chosen beforehand for that locus was printed.

If the point is inside the parabola its distance to the axes was calculated. The computer chose the corresponding character in order to get straight lines, parallel to the axes, in a sequence from white to black.

4. Line Segments and Random Numbers

Line segments were drawn between 14 selected points, using linear interpolation, in order to get a terrain profile representing the two main volcanoes near Mexico City (Ixtaccíhuatl and Popocatepetl). The points were selected from photos and pictures of the two mountains. Some cosmetics were applied when doing this. For example it is very common to see in some paintings that the two mountains appear nearer together than they actually are.

The photo illustrated can give an idea of what they look from a field just outside Mexico City.



Photo 1: *View of Ixtaccíhuatl and Popocatepetl volcanoes*

Mexico City was originally founded on a lake. This lake has been drying out with time and only some small parts are left. It is now a bit difficult to find a place where one can see what the view was like in older times. Therefore what is depicted in these images is more a memory of a past landscape that people used to see.

The altitude of these mountains is enough for them to have permanent snow. For most of the year they look as it is shown in the photograph and in the images made by computer.

4.1. Landscape at night. In a first try, the image as shown in figure 3 intends to represent a night scene of the volcanoes and their reflection on a lake.

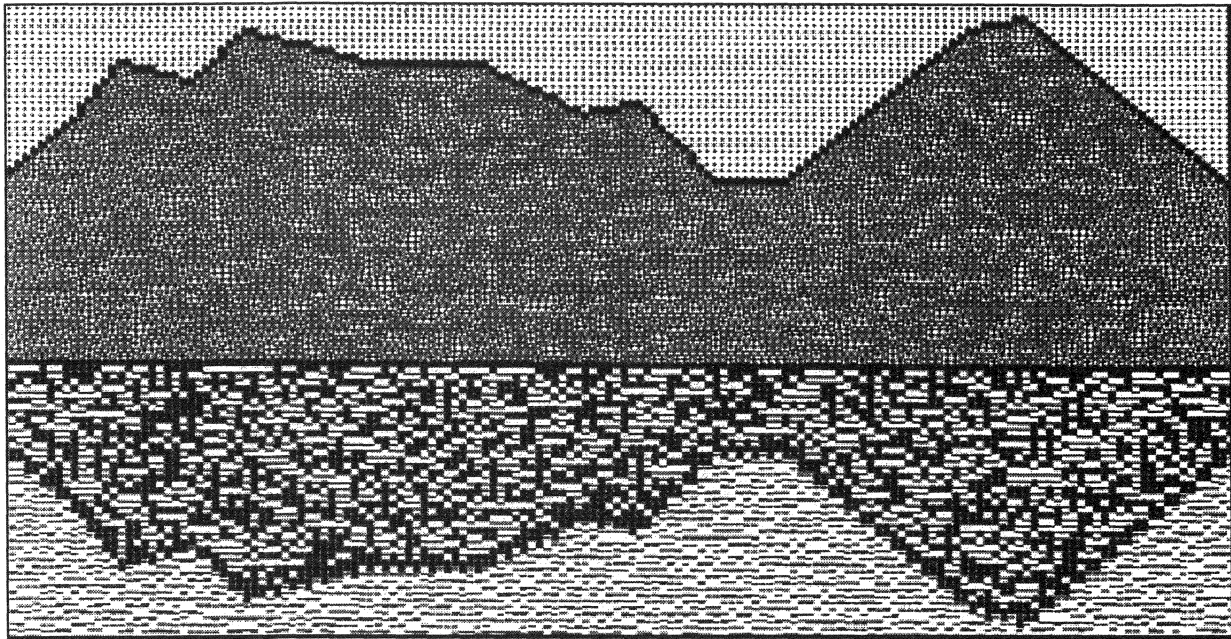


Figure 3: *Simulation of a view of the two volcanoes and its reflection on a lake at night*

Several layers had to be defined for this image. The main one is the profile of the terrain. Above it a character was chosen to fill the space representing the sky. Then a black character depicts the profile. Below the profile two gray characters printed at random simulate the mountains. Then a straight black line marks the boundary between land and water.

For the reflection of the mountains and for the reflection of the sky a pair of characters were selected for each one and printed at random using random numbers. These characters were designed to simulate a slight movement of the water surface. In order to represent the reflection of the terrain profile it was printed head down and with a double line so that it also simulates the slight movement of the water surface.

4.2. Landscape at daylight. When the image shown in figure 3 was finished, something a bit more complex was planned. The same landscape had to be represented but as it would be seen in daylight. This required the addition of two features: the snow layer on the upper part of the mountains, and the vegetation cover below it. The snow layer should appear on the profile of the volcanoes as well as on their reflection on the water.

Figure 4 shows the result of this attempt. The layer of snow is above a certain height. One character printed at random and having a low density simulates the snow cover on the highest parts of the volcanoes.

In order to represent the vegetation cover on the rocky surface below the snow layer, two characters were employed. The darker one simulates the vegetation cover. This character was printed at random and gives the idea of being upon the other one. It grades in density from low to medium as the vegetation cover increases with lower altitude.

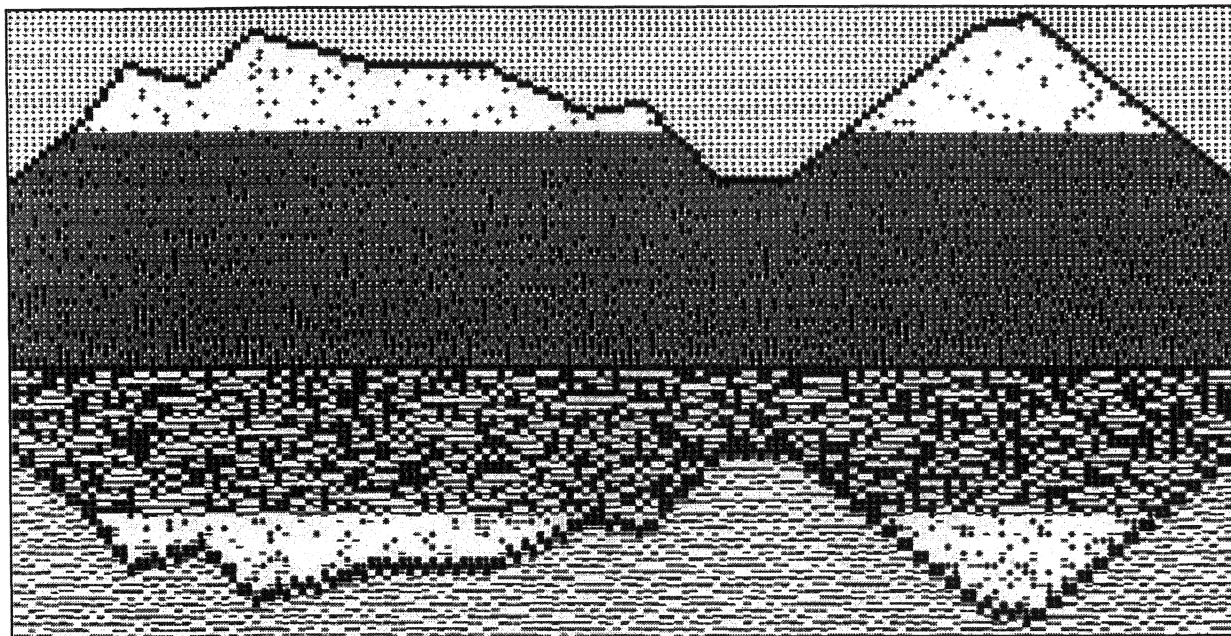


Figure 4: *Simulation of a view of the two volcanoes and its reflection on a lake at daylight*

Conclusions

It is possible to take advantage of mathematics in the field of art. Even pre-university students should be able to develop their artistic abilities or reinforce them by creating images with the aid of mathematics and computer programming. Teachers should try to encourage their students to explore art with the aid of mathematics.

Modern technology give us each time more options and software support to be used in several fields. This paper is an attempt in exploring art with mathematics and computer programming. Something similar might also be done with other tools of modern technology –software– and mathematics which students can handle.