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Mathematical Connections in Art, Music, and Science

Geometric Sculpture For K-12: Geos, Hyperseeing, and Hypersculptures

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Abstract

Just as children are taught the 3R's, they should also be taught the S (for seeing). Our purpose is to teach children how to see they way a sculptor sees as well as the way a mathematician sees. Children will first make their own geometric sculpture (geos). This direct hands-on experience will facilitate learning to see the way a sculptor sees. Once they have constructed their geo, they can then construct a copy twice as large. They can also view mirror images as well as construct mirror images. These exercises will facilitate learning to see the way a mathematician sees. The children may also construct hypersculptures which will enable them to hypersee.

1. Construction of Geos

1.1 Geos. Geos are geometric sculptures constructed by connecting discrete planar shapes such as triangles, quadrilaterals, and arcs, as well as more general shapes. Just as musical notes appear on a page to form a composition, the planar shapes are placed in space to form a composition. I have been influenced by poses from Balanchine ballets as well as the photographs of Lois Greenfield of dancers suspended in space.

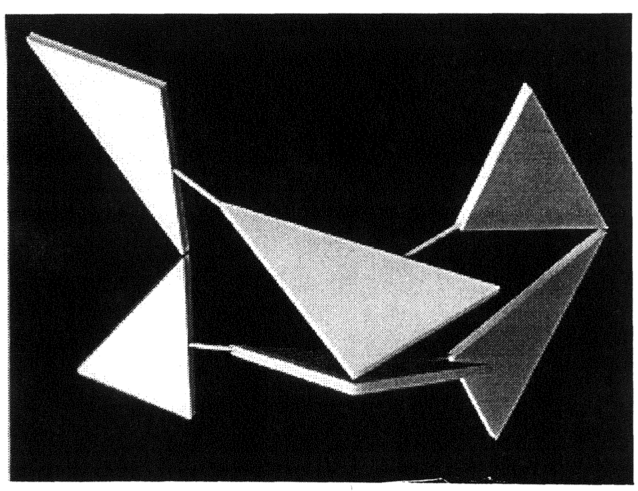

Figure 1: *Trio*

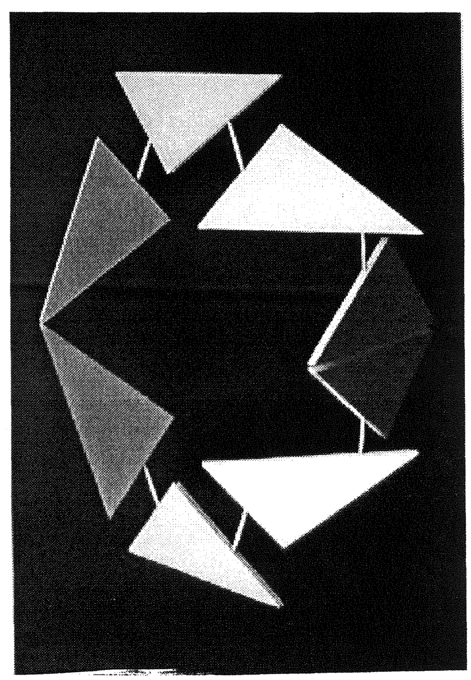

Figure 2: Quartet

The method of connecting the planar shapes is unique and is best explained by considering small geos where the planar shapes are cut from colored foam board. The shapes are connected by inserting round pointed toothpicks in the styrofoam edges of the foam board. Examples of geos are shown in Figures 1-8. The geos were photographed on a mirror base to obtain the reflected images. In Figures 1-6 the geos were constructed using three or more identical right triangles of dimensions 3"x4"x5". In Figures 1 and 2 the toothpicks are visible and the triangles are separated, whereas in Figures 3-6 the triangles touch. Connections are either edge to edge, point to edge, or point to point.

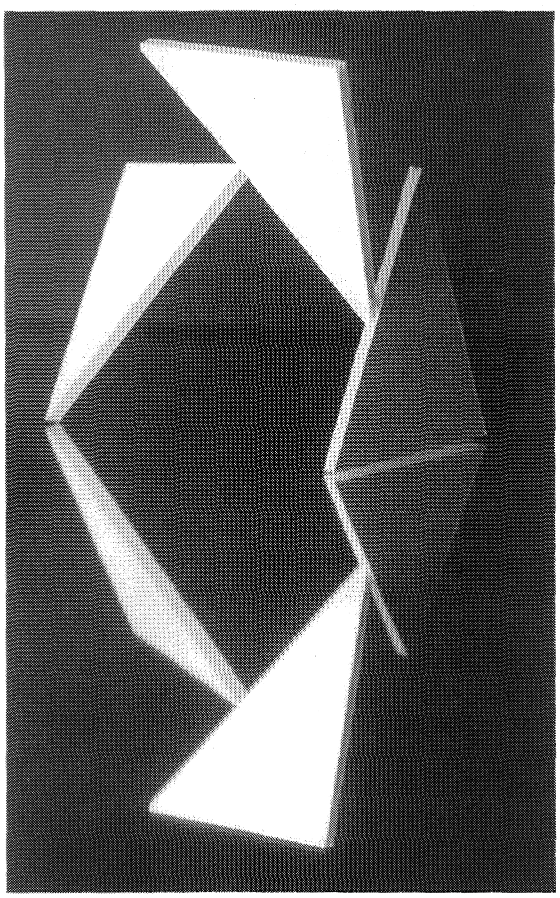

Figure 3: *Arch I*

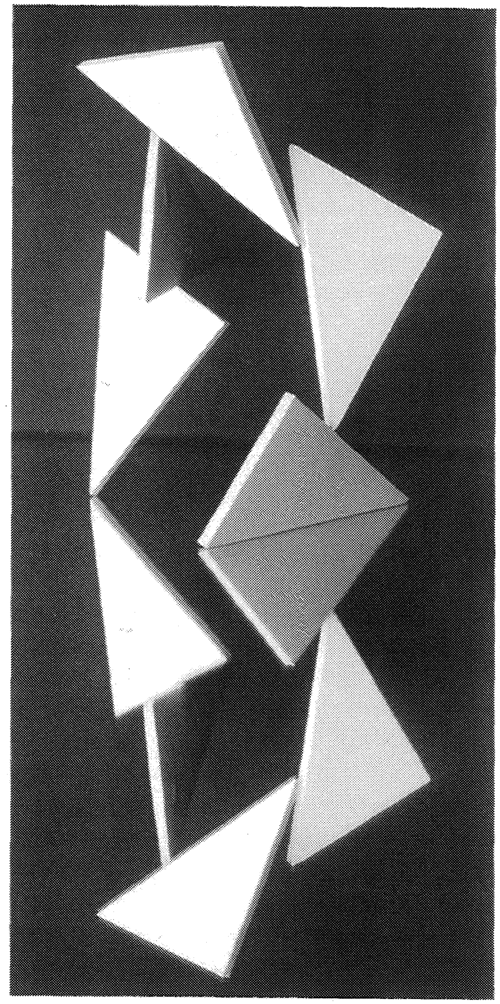

Figure 4: *Arch II*

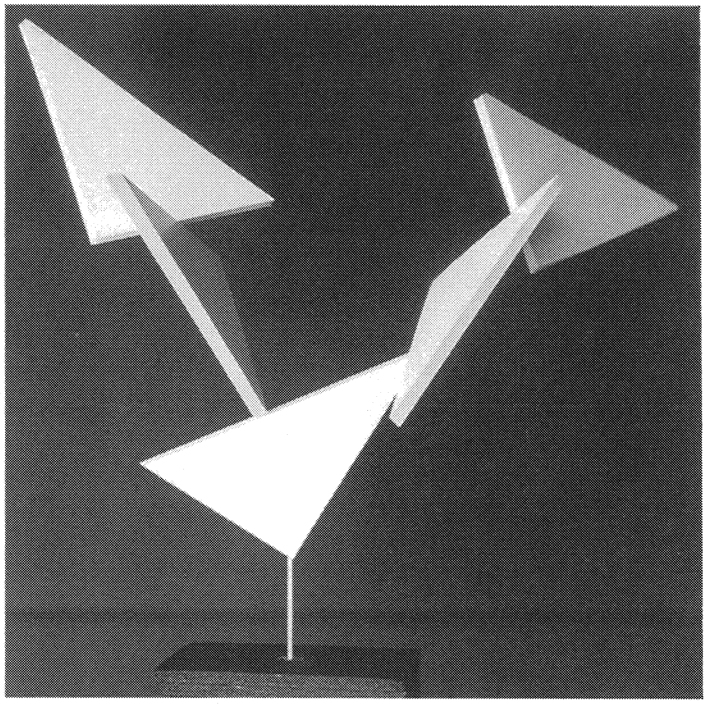

Figure 6 : *Flight*

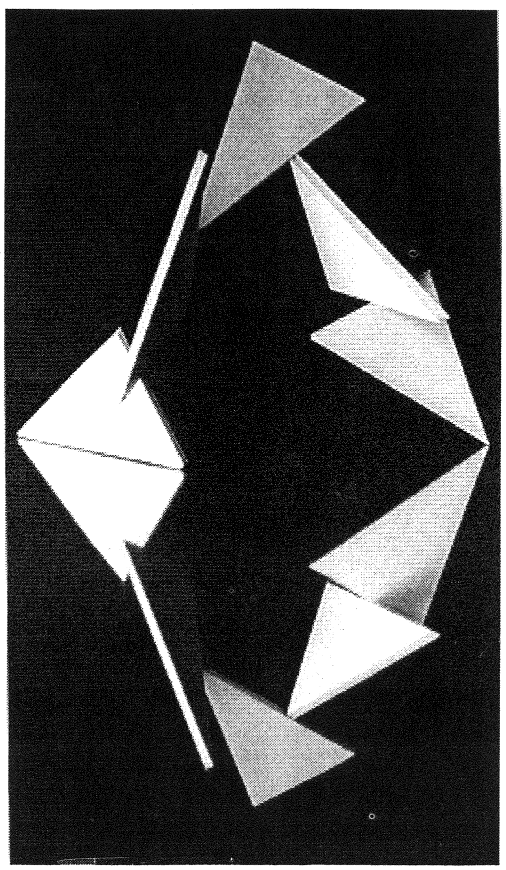

Figure 5: *Arch III*

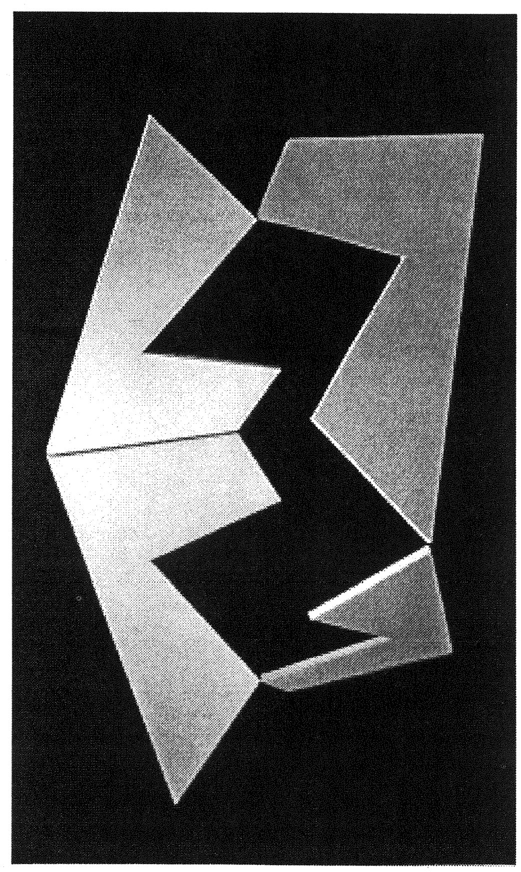

Figure 7: *Duet*

The geo Duet in Figure 7 is constructed from two copies of the same shape touching point to point. The geo Reclining Figure in Figure 8 is constructed from four different quadrilaterals.

1.2. Orientations. Geos generally have more than one orientation. For example, Flight in Figure 6 is the same object as Arch III in Figure 5. The object in Reclining Figure in Figure 8 is shown tipped forward in Duality in Figure 9.

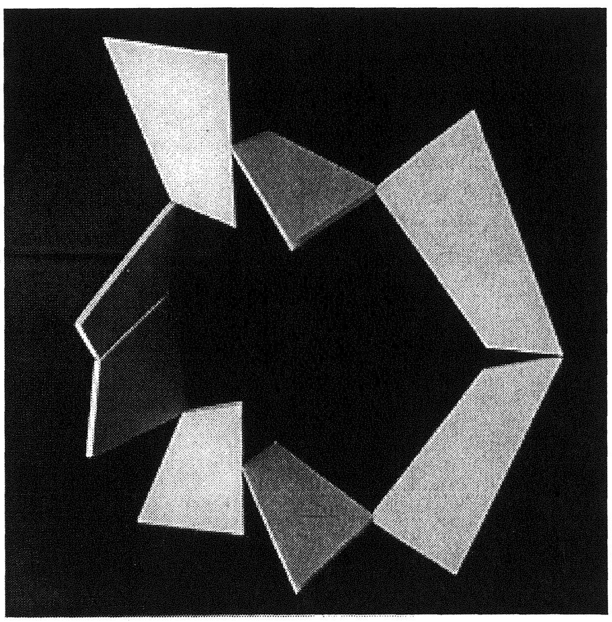

Figure 8: *Reclining Figure*

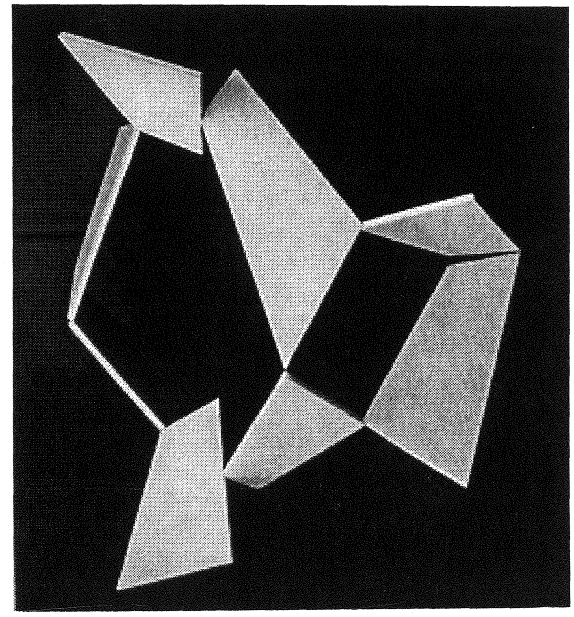

Figure 9: *Duality*

Seeing that an object may have a variety of interesting orientations leads to a much more complete appreciation of the three-dimensional content of an object. This serves as an introduction to hyperseeing and hypersculptures which are discussed below.

2. Hyperseeing

2.1. All-Around Seeing. In order to see a two-dimensional painting on a wall, we step back in the third dimension. We then see the shape of the painting (generally rectangular) as well as every point in the painting. Thus we see the painting completely from one viewpoint. Now theoretically, to see a three-dimensional object completely from one viewpoint, we would need to step back in a fourth dimension. From one viewpoint, we could then (theoretically) see every point <u>on</u> the object, as well as see every point <u>within</u> the object. This type of all-around seeing, as well as a type of x-ray seeing, was known to the cubists such as Picasso and Duchamp. In particular, cubists were led to showing multiple views of an object in the same painting.

2.2. Hyperspace and Hyperseeing. In mathematics four-dimensional space is referred to as **hyperspace** and I refer to (theoretical) seeing in hyperspace as **hyperseeing**. Thus in hyperspace one could hypersee a three-dimensional object completely from one viewpoint. Although we do not live in hyperspace, it is still desirable to attain a <u>type of hyperseeing</u> in our own three-dimensional world. This is possible by viewing a hypersculpture, as defined below.

3. Hypersculptures

3.1. Hypersculptures. There are abstract three-dimensional objects which have no predetermined top, bottom, front, or back. This type of object can look quite different when placed in different orientations, such as horizontal or vertical. An observer may not even realize that it is the same object. Thus to more completely appreciate the diverse sculptural content of an object, it is natural to present it in different orientations.

A sculpture will be defined as an object in a given orientation relative to a fixed horizontal plane (the base). Different sculptures are said to be related if they consist of the same object in different orientations. A hypersculpture is a set of related sculptures. Thus to more completely appreciate the diverse sculptural content of an abstract object it is natural to present it as a hypersculpture. Furthermore, the experience of viewing a hypersculpture allows one to see multiple views from one viewpoint which therefore helps to develop a type of hyperseeing in our three-dimensional world. Viewing a hypersculpture is really a new visual experience. It is a more complete way of seeing a three-dimensional object. Hypersculptures were introduced in [1] and discussed further in [2]. An introduction with figures is given in [3].

3.2. Examples. The hypersculpture shown in Figure 10 consists of a six related sculptures where the object is a rectangle and a right triangle joined at an angle.

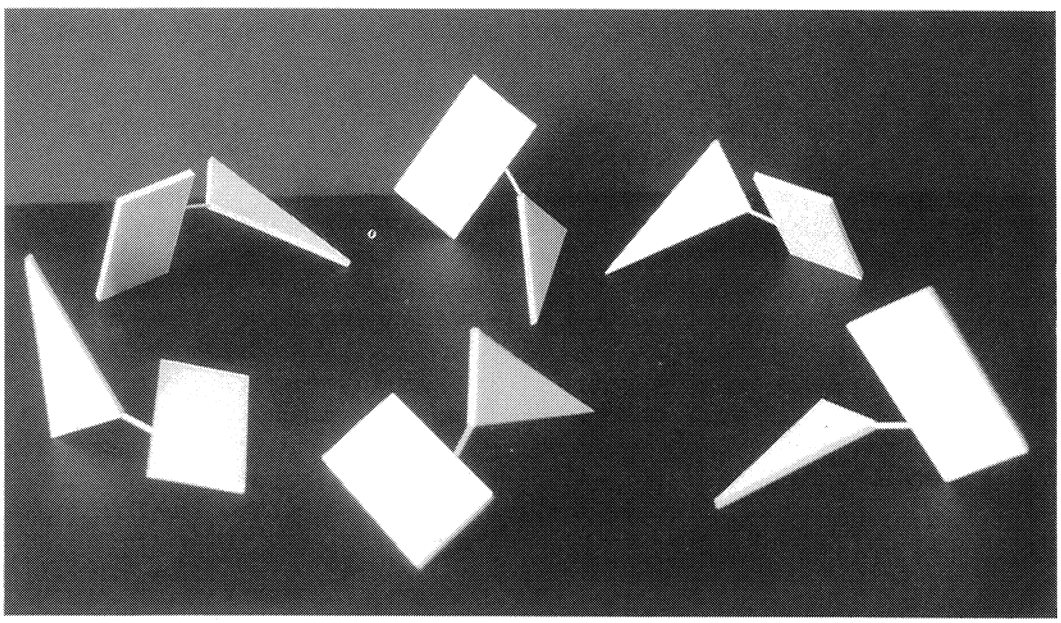

Figure 10 : *Hypersculpture*

The hypersculpture shown in Figure 11 consists of two related sculptures, where the object is a geo constructed from four identical 3"x4"x5" right triangles. The color of the foam board is different in the two related sculptures. The sculpture on the left in Figure 11 rests on three points, whereas the sculpture on the right in Figure 11 rests on an edge and a point. The object in Figure 11 actually has several other possible orientations that are stable on a horizontal surface. The orientations as in Flight in Figure 6 are infinite.

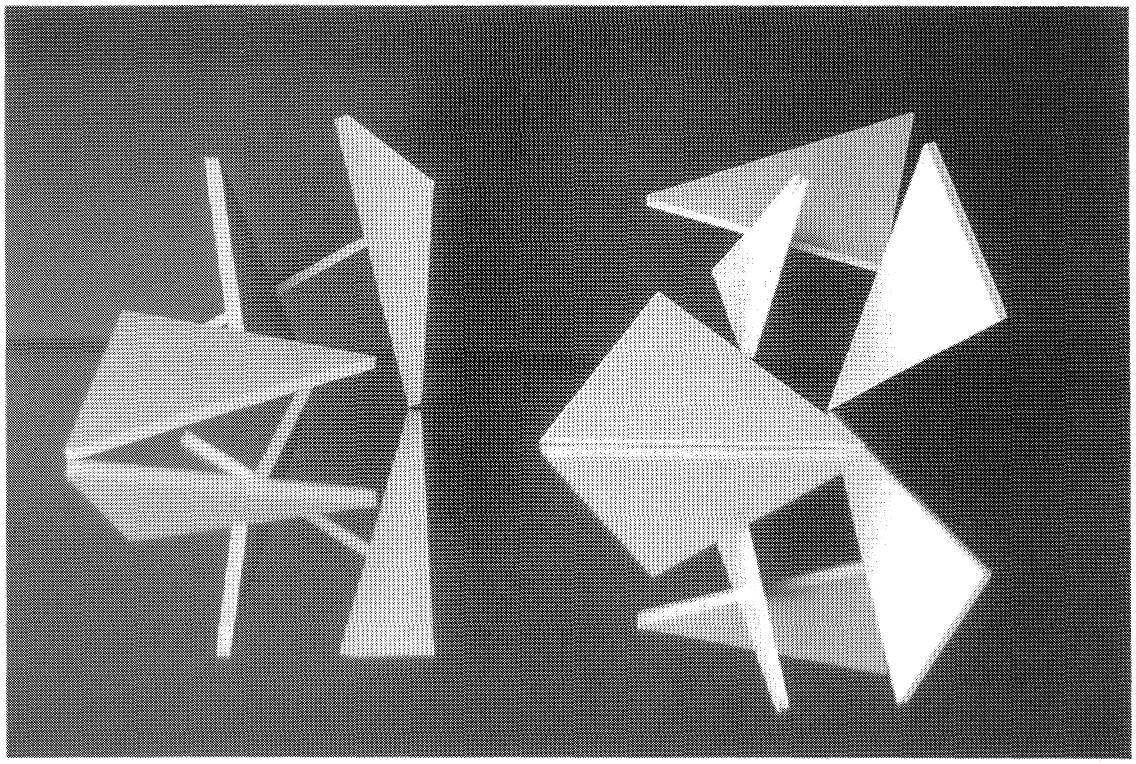

Figure 11: *Hypersculpture*

As mentioned above, a hypersculpture allows one to more completely appreciate the diverse sculptural content of an object. Furthermore, when viewing a hypersculpture one sees multiple views from one viewpoint which helps to develop a type of hyperseeing, which is all-around seeing. Thus one learns to see the way a sculptor sees.

4. Mathematical Exercises

4.1. Doubling the Size. Once a child makes a small geo, it is then a very interesting geometric exercise to make a copy twice as large. In this case the length of sides double but angles remain the same. First they have to double the size of each planar shape. There is a simple way to do this described in the extended notes in [4]. Second, they have to insert the toothpicks at the proper points at the same angles as in the small version. The small version is used as a template to do this. Third, they have to connect the planar shapes at the same angles as in the smaller version. Again the small version is used as a template to do this. This exercise teaches children a lot about lengths and angles in three-dimensions.

4.2. Mirror Symmetry. A second exercise is to construct a mirror image of a geo. In this case one can flip over the planar shapes to obtain the mirror image of each shape. The lengths and angles between shapes will remain the same but left and right (or above and below) will be switched. Details are discussed in [4]. This exercise teaches children a lot about mirror symmetry in space.

4.3. Rotational Symmetry. Children could also be asked to construct geos with rotational symmetry. Examples are discussed in [4]. They could also make multiple copies of a geo and arrange them in a circle to form a hypersculpture with rotational symmetry when viewed from above. For example, two copies would be needed for 180° symmetry. In this case one would be seeing the sculptures from the front and the back from one viewpoint.

The above mathematical exercises motivate the children to concentrate on the geometry of lengths, angles, and symmetry inherent in their geo. Thus the children are learning to see as a mathematician sees.

5. Conclusion

In general, hyperseeing is learning to see in multiple ways. For example, learning to see a geo as both an artist and as a mathematician is another level of hyperseeing the geo. Lastly, these experiences can teach children to see situations from viewpoints other than their own and thus learn tolerance for the viewpoints of others. So this project has applications to life experiences.

References

[1] N.A. Friedman, *Hyperspace, Hyperseeing, Hypersculptures*, Conference Proceedings, Mathematics and Design 98, Javier Barrallo, Editor, San Sebastian, Spain.

[2] N.A. Friedman, *Hyperseeing, Hypersculptures, and Space Curves*, Conference Proceedings, 1998 Bridges: Mathematical Connections in Art, Music, and Science, Reza Sarhangi, Editor, Winfield, Kansas, USA.

[3] N.A. Friedman, *Hyperspace, Hyperseeing, Hypersculptures (with figures), Hyperspace*, volume 7, 1998, Japan Institute of Hyperspace Science, Kyoto, Japan.

[4] N.A. Friedman, Constructing Geometric Sculptures for K-12: Geos, Hyperseeing, Hypersculptures, and Space Curves.