

Abstraction, Reflection, and Learning Musing of a Mathematician/Educator

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*"We can know objects directly through sensation
and indirectly through contemplation of abstraction."*

Pulliam, Van Patten
History and Philosophy of Education (1999)

Prelude: Reflections on Music, Art and Three (yes, as in the number 3):
How different are my Escher-like stairs from yesterday than once of today?

J. S. Bach's canon per tonus from his Musical Offering is a very famous example of an endlessly rising melody. The melody rises two halftones each time the canon is repeated ("As the keys ascend so may the glory of the king also ascend" - this should illustrate the rising glory of Frederick the Great to whom the Musical Offering was dedicated).

The uppermost voice sings a variant of the Royal Theme, while underneath it two voices provide a canonic harmonization based on a second theme. The lower of this sings its theme in C-minor (which is the key of the common as a whole), and the upper of the pair sings the same theme displaced upwards in pitch by an interval of fifth.

What makes this canon different from any other? When it seems to conclude - it is no longer in the key of C-minor, but now is in D-minor. And it is so constructed that this ending ties smoothly onto the beginning again; thus one can repeat the process and return in the key of E, only to join again at the beginning.

A smooth Escher-like roller coaster?

Jean Piaget, a Swiss philosopher-epistemologist, conducted extensive research for more than sixty years (1920 to 1980) on the development of children's cognitive abilities. Piaget's theory involves the child learning through an active role, "In the area of logico-mathematical structures, children have real understanding only of that which they invent themselves, and each time that we try to teach them something too quickly, we keep them from reinventing it themselves."

In the concept development process from concrete to pictorial, from pictorial to abstract, and stages in between, every stage seems to require action on the environment.

When students enter the formal operational stage, Piagetians believe that mental transformations can take place without the need of concrete materials.

When and where does the merging of these experiences happen?

Is there a starting point of understanding and discovery?