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Mathematical Connections
in Art, Music, and Science

PHYSICS: A Search for Simplicity, Beauty and Symmetry

Richard C. Morrison
Department of Physics
University of New Haven
West Haven, CT 06516, U.S.A.
E-mail: morrison@charger.newhaven.edu

Abstract:

From its beginnings in ancient astronomy, the goal of the science of physics has always been to find "the simple theory of everything." As conceited and naive as this goal might seem, it is a very real and current one. Physics or, as it was once called, Natural Philosophy has always been convinced that the universe must have a fundamental simple order and obey a simple, self-consistent set of rules. The history of Natural Philosophy or Physics has been most simply the search for this order and set of rules. It has also always, from the very beginnings of the science, been assumed that simplicity, perfection, order and beauty all go hand-in-hand, and all must be attributes of the underlying laws governing the universe. This paper will attempt to examine some of the ways that this underlying assumption of symmetry and order has led to discoveries of the rules our universe obeys.

The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve.
..Eugene P. Wigner[*1]

Introduction

The idea that the physical universe, its structure and the laws governing its behavior, should obey simple and orderly mathematical relations and symmetries is at least as old as western civilization. Pythagorus, in the sixth century BC, used mathematics to bring order to the universe of observation. In particular, the identification of the degree of symmetry of an object or idea with the degree of perfection of that object or idea is both as old as the ancient Greeks and as new as the current ideas of modern physics. Before continuing, we need to define our terms. In art, symmetry has a broad range of meaning from a simple bilateral symmetry to a highly subjective idea of harmony and balance of composition. In mathematics, symmetry is more precisely defined: *A symmetry operation is a mathematical operation which leaves the final state indistinguishable from the initial state.* For example: the sphere is considered to be the most perfectly symmetric geometric figure because any rotation about any axis or any reflection through any plane will leave the sphere indistinguishable from its original state. A cylinder is less symmetric because it has only one axis about which any rotation will leave it unchanged. The symmetry ideas invoked by physics are primarily based on the mathematics definition, but also often invoke the more subjective artistic ideas. The ancient Greeks believed that the earth was spherical because they believed that it should embody the perfection of the sphere, not because they had any physical evidence of its actual form. As the Ptolemaic picture of the universe was developed to include the observed heavens in the scheme, the natural model was a series of concentric spheres, all perfectly revolving around the earth at the center. The "Copernican revolution" of the 16th century was not an attack on a view deemed as too simple, but rather on a system that Copernicus felt had become much too complicated! By the time of Copernicus, some eighty different spheres were needed to fit the observed motions of the sun, moon, stars and planets into the Ptolemaic system and many details were still left unaccounted for! Copernicus was convinced that the universe had to be much simpler and more elegant than that. It has been exactly that same motivation-- the belief that "the universe must be much simpler and more elegant than that" -- that has led to virtually every major new idea in physics from the time of Copernicus to the present day. Even the seemingly unrelated laws of motion discovered by Isaac Newton in the 17th century have, in the 20th century, been shown to be just the results of fundamental symmetries of our universe, and the future experiments planned and envisioned by today's particle physicists seek still more-perfect symmetries than those already observed.

The balance of this paper addresses the role of symmetry ideas in a sampling of the developments in physics of the last three hundred years and in the topics of greatest interest to physicists of the present time.

Fundamental Symmetries of the Universe and the Basic Laws of Mechanics

Every freshman physics student learns Newton's three laws of motion and the laws of conservation of momentum and conservation of energy. Newton's third law ***If object (a) exerts a force on object (b), then object (b) exerts an equal and opposite force on object (a)*** is a simple symmetry law as stated. Recognition of the other laws of mechanics (and, in fact, all the laws of physics) as fundamental symmetries did not come, however, until the 20th century and this recognition is, in great measure, due to Eugene Wigner who is quoted at the beginning of this paper. Prior to the 20th century, the science of physics was based on a set of fundamental "conservation laws." Students still learn basic physics in this framework with the familiar laws of conservation of energy, conservation of momentum, conservation of electric charge etc. However, Noether's Theorem, a most-important theorem of modern theoretical physics, identifies the source of these conservation laws. The theorem states: *For every continuous symmetry of the laws of physics there exists a corresponding conservation law.*

For example: any fundamental experiment that we do in our laboratory will have exactly the same result if we move our laboratory to Brazil, or Ethiopia, or even Mars. In other words, the laws of physics are independent of where we are in the universe. (symmetry under transformation of the space coordinates) This fact can be shown to be equivalent to the law of *conservation of momentum.* If we repeat experiments recorded 100 years ago, we will find the same rules apply today or, if we look at light emitted by a distant star a billion years ago we observe that its wavelength is exactly the same as would be produced in the same atomic process today. In other words, the laws of physics are independent of the passage of time. (symmetry under transformation of the time coordinate) This can be shown to be equivalent to the law of *conservation of energy.* The fact that experiments done on a ship at sea or on a 747 flying at 500 miles per hour will give the same results as those done in our lab show that the laws of physics are independent of motion - the equivalent of Newton's first law. To state all of this from the other direction: if we accept the notion that -- changing the location or orientation of our laboratory, the speed or direction at which it is moving through space, or the time at which we do our experiment -- will not affect the results we obtain, then we can derive all the laws of mechanics from these simple assumptions. Or, in symmetry language, the workings of the universe are symmetric with respect to position, orientation, time, and velocity.

Symmetry and Maxwell's Equations

The time period of 1800 to 1850 saw major activity in the study of electric and magnetic phenomena and, in particular, the realization by scientists that electricity and magnetism were closely related. Prior to 1820, they had no reason to believe there was any connection between the two. In that year, however, a Danish scientist, Hans Christian Oersted made the accidental discovery that an electric current caused a nearby compass needle to turn. A fundamental symmetry law of physics, Newton's third law of motion, says that if an electric current produces a force on a magnet then the magnet must produce an equal and opposite force on the current. This was quickly verified by experimenters and, soon after, Andre Ampere formulated the law that now bears his name relating an electric current to the magnetic field it produces. In modern notation, this law is usually written:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

where B is the magnetic field vector and the integral is around any closed path encircling the current, (i) ; μ_0 is simply a constant dependent on the system of units used.

Once again, symmetry was invoked to say that if an electrical current can produce a magnetic field then a magnetic field should be able to produce an electrical current. This connection was a bit more elusive, however, and it was not until 1831 that Michael Faraday made the discovery that while a fixed magnetic field would not produce a current, a field that was *varying in time* would indeed induce a current in a coil of wire. In its modern form, Faraday's Law is usually written:

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$$

where Φ_B is the *flux* of the magnetic field through the area enclosed by the integration loop and E is the electric field vector - the field responsible for the flow of current in a wire

In 1864, English scientist James Clerk Maxwell combined these two laws with similar formulations of statements that

(1) Electric field lines begin and end on electric charges, (q), and
(2) Magnetic field lines close on themselves with no beginning or end
(i.e. there are no magnetic charges)

These four equations pulled together in a simple form everything that was known about electric and magnetic fields.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \qquad \oiint \vec{B} \cdot d\vec{A} = 0 \qquad \oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 i$$

Maxwell again recognized that if the laws of physics were to be orderly and symmetric, the last of the above equations needed another term: if a changing magnetic field induces an electric field then a changing electric field should induce a magnetic field! Using arguments that vector fields must be continuous, Maxwell added a term to the last equation to make it:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

where ε_0, like μ_0, is simply an artifact of the system of units used.

This set of the four Maxwell Equations represent a complete theory of the Electromagnetic force and, to this day, it remains the only one of the basic forces in nature that is completely described by theory.

Even after 140 years, however, there are still physicists who are unhappy that there is still one possible symmetry missing from Maxwell's Equations: if there were such a thing as a magnetic charge, m, also called a *magnetic monopole*, and we represent a current of these charges as "p," then the above equations would be perfectly symmetric with respect to interchanging electric and magnetic charges and currents:

$$\oiint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \qquad \oint \vec{E} \cdot d\vec{s} = \frac{p}{\varepsilon_0} - \frac{d\Phi_B}{dt}$$

$$\oiint \vec{B} \cdot d\vec{A} = \mu_0 m \qquad \oint \vec{B} \cdot d\vec{s} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$$

The fact that this set of four simple and elegant equations completely describes all electromagnetic phenomena, predicts the existence of electromagnetic waves and gives a precise value for the speed of light is one of the greatest triumphs of physics. One can safely say that the major goal of physics is to describe *all* physical forces and interactions with a comparable set of equations.

Maxwell's equations can be written in an even more elegant form using the notation of vector calculus:

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \tau} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial \tau} + \vec{j}$$

(where ρ is charge density, $\vec{j}$ is vector current density, $\vec{D} = \varepsilon_0 \vec{E}$ and $\vec{B} = \mu_0 \vec{H}$)

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Should magnetic charge of density φ and vector current density $\vec{z}$ exist, these equations become:

$$\nabla \cdot \vec{D} = \rho \qquad \nabla \cdot \vec{H} = \varphi \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial \tau} + \vec{z} \qquad \nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial \tau} + \vec{j}$$

Symmetry and Einstein's Special and General Reletivity

The theories of Special Relativity and General Relativity formulated by Einstein in the early years of the 20th century are fundamentally extensions of symmetry ideas. Einstein's bold and unprecedented step was to put symmetry ahead of hitherto unchallenged dogmas of the constancy of meter sticks and clocks. In Special Relativity, Einstein simply postulated that the invariance of physical laws with respect to motion at constant velocity, embodied in Newton's first law, should also apply to the laws put forth in Maxwell's equations of electricity and magnetism. This straightforward assumption leads directly to the famous results of length contraction and time dilation, and the fact that space and time are intermingled, that seem so revolutionary in Special Relativity. Even the famous $E = mc^2$ comes directly from this simple postulate with only the aid of Newtonian physics and first-year calculus.

General Relativity is built on a more subtle observed symmetry of our universe: the fact that no experiment can distinguish between the effects of a gravitational force and the effects of an accelerated frame of reference. This idea leads immediately to the prediction that light must "fall" in a gravitational field just as a stone does, and to the even more startling prediction of what we have come to call "black holes." Albert Einstein rocketed from obscure physicist to world-wide fame in 1921 when the first experiment confirmed that light is indeed bent as it passes by a massive object like the sun. More recently, the Hubble telescope has provided far more dramatic proof with photographs of distant galaxies seen through "gravitational lenses" that result from extremely massive objects in the intervening space.

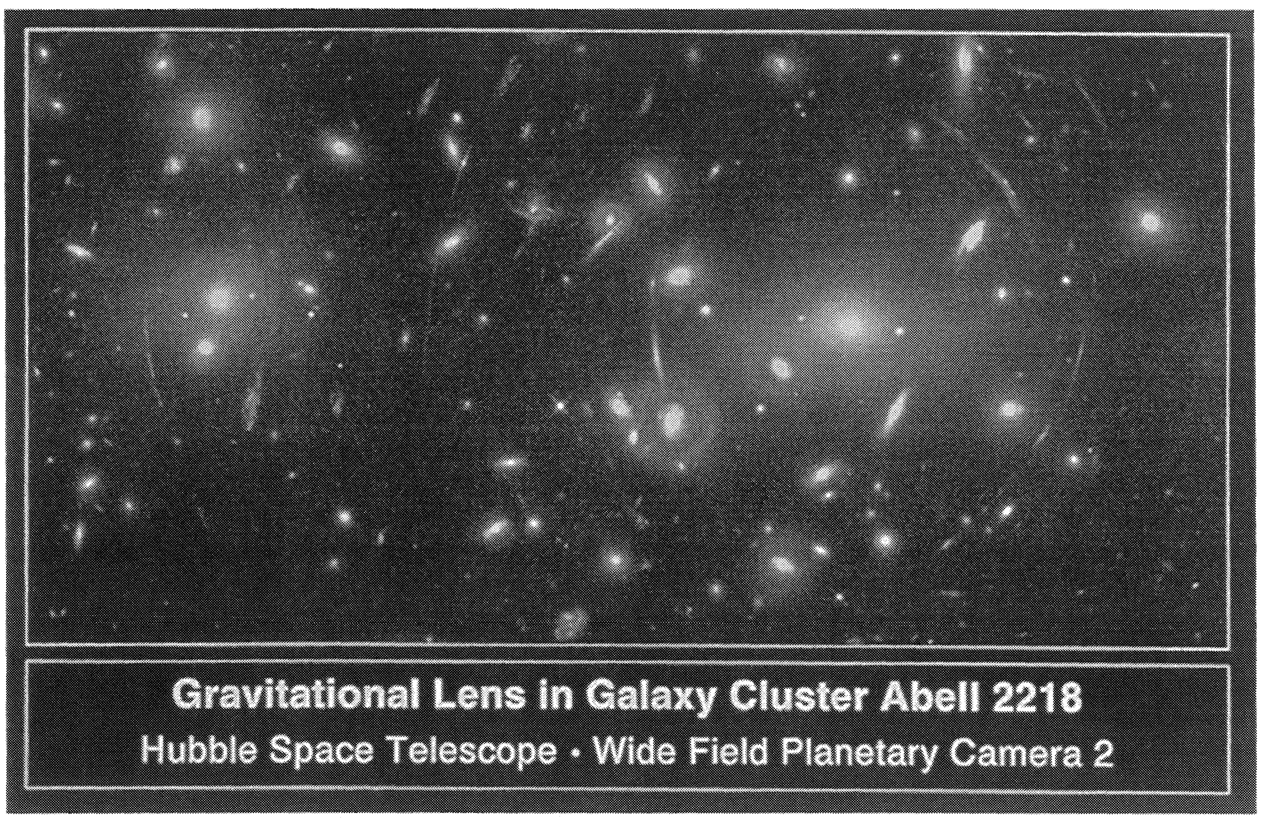

Figure 1: *A Gravitational Lens Produces Multiple Images of a Distant Galaxy*
The Distortion of Space Predicted by General Relativity

Symmetry and Modern Physics

Even after Einstein's striking success with giving symmetry top priority, physicists in other fields were slow to pick up this way of thinking. The new era really began in 1931 when Eugene Wigner published his *Group Theory and its Application to the Quantum Mechanics of Atomic Spectra.*(2) Following Wigner's introduction of symmetry groups into the language of Quantum Mechanics, symmetry ideas slowly but surely moved to the forefront of nearly all thinking in modern physics. In 1963, Wigner was awarded the Nobel Prize in physics for his introduction of symmetry ideas to elementary particle theory. Interestingly enough, by the 1950's, symmetry ideas had become so imbedded in elementary particle theory that two Nobel Prizes (Lee and Yang in 1957 and Fitch and Cronin in 1980) were awarded to people who showed that certain symmetries do <u>not</u> hold in some sub-atomic interactions. Until the work of Lee and Yang in 1956, for example, physicists believed that on a fundamental level the laws of physics made no distinction between right-handed and left-handed systems. In other words, if you could watch some atomic-level process both directly, and in a mirror, (a reflection transformation) you would have no way of knowing which picture was the mirror reflection. In processes governed by the "weak interaction," beta decay for example, this turns out not to be true. Mirror symmetry is one that nature does not possess in every case.

One of the most striking examples of the use of symmetry ideas in the advancement of modern physics is in the development of what is now known as "The Standard Model" of elementary particles. By the 1930's, our knowledge of the fundamental structure of matter had reached a very happy state. All atoms were composed of just three different entities: protons, neutrons and electrons. The physicists had reduced Mendeleyev's table of 92 basic elements to a more elementary set of just three. This is the kind of thing that always makes physicists happy. By the late 1950's, however, "atom smashers" (cyclotrons, synchrotrons etc.) had managed to produce a veritable zoo of more than thirty new "elementary particles," with no end in sight – always the kind of thing that makes physicists think "the universe must be much simpler and more elegant than that." Then, in 1963, Murray Gell-Mann discovered that all of the known particles governed by the strong nuclear force could be placed into a particular mathematical symmetry group called SU(3). Each of these "elementary particles" has a definite mass, electric charge, and a set of characteristics (quantum numbers) named, for historic reasons, "spin," "isospin," "hypercharge," and "baryon number." What Gell-Mann found was that if he plotted a graph of hypercharge vs. isospin for all the known particles with a given spin and baryon number, the resulting patterns were identical to the projections of the SU(3) symmetry group. Just as Mendeleyev's Periodic Table of the previous century pointed the way to the fact that the chemical "elements" are, in fact, built of simpler objects (protons, neutrons and electrons), Gell-Mann's ordered groups of "elementary particles" pointed the way to a still deeper underlying layer of structure.

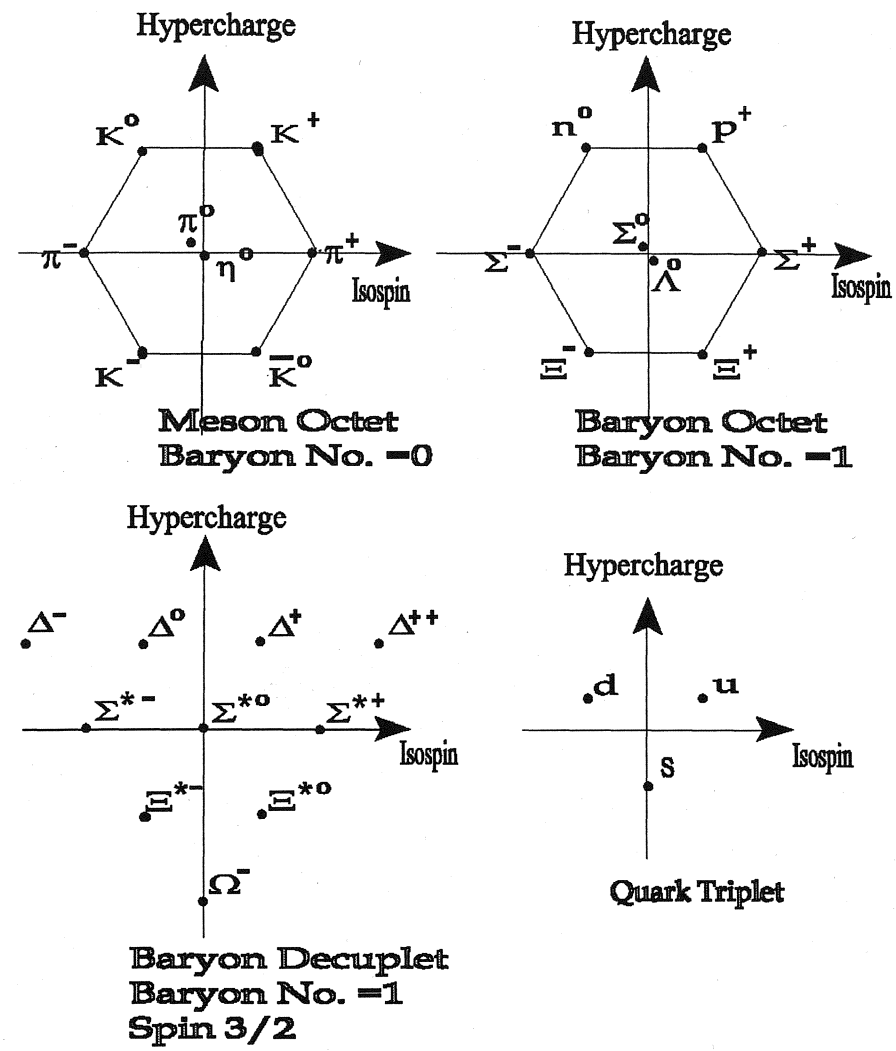

Figure 2: Plots of *Hypercharge* vs. *Isospin* for the subatomic particles[3] known in 1960

At the time, the Ω^- particle, shown at the tip of the decuplet triangle above, had not yet been discovered. Its subsequent discovery with the properties predicted by Gell-Mann's model was an immediate success of the theory. In this model, the members of the triplet projection are, in fact, the elementary components which combine to produce all the other particles in the picture. Gell-Mann gave these three building block the generic name of "quark," borrowed from James Joyce's *Finnigan's Wake*, and gave them the individual names of "up," "down," and "strange." A completely new feature of Gell-Mann's model was that the postulated quarks carry fractional electric charge: $+{}^{2}{}_{3}$ on the *up* quark and $-{}^{1}{}_{3}$ on both the *down* and *strange* quarks. The *up* and *down* quarks are both stable and are the building blocks of the proton and neutron while the *strange* quark decays to a *down* quark, and all particles that include a *strange* quark are radioactive. Following the initial success of the model, symmetry was immediately invoked once again. Shouldn't the *strange* quark have a $+{}^{2}{}_{3}$ partner that decays to an *up* quark? Just as in Maxwell's Equations, symmetry arguments were again used to predict yet undiscovered entities. The hypothesized new quark was given the name *charm* (a nice example of physicists not taking themselves too seriously) and not much time elapsed before the first "charmed" particle requiring this new building block was discovered.

The existence of the *charm* quark adds a third dimension to the plots of Fig. 2. Figure 3 shows the meson octet and the baryon decuplet extended in a third dimension showing the "charm" quantum number.

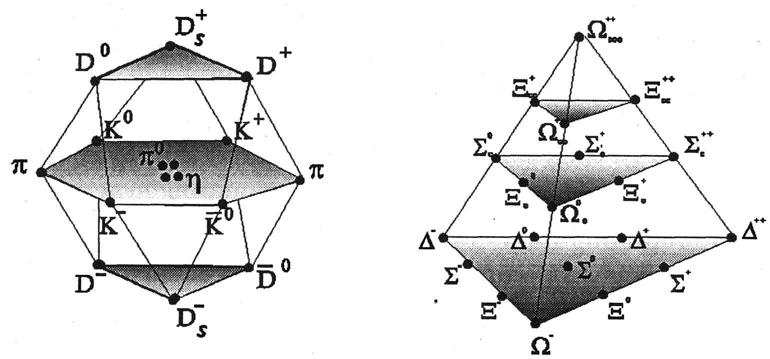

Figure 3: *The meson octet and baryon decuplet extended in a third dimension to include "charm"*
Note that the central plane of the meson plot and the base plane of the baryon plot are the groupings of Fig. 2

Subsequently, a third family of two quarks has been added to the collection. The plots shown in Fig. 3 above are then three-dimensional projections of five-dimensional groups of particles. Similar projections can be drawn for any subset of four of the six quarks.

In the current theory, each quark comes in three varieties (called colors). In all then, the model now has eighteen quarks and their corresponding eighteen anti-quarks plus six other "elementary particles" called "leptons" which are not governed by the strong nuclear force and finally six anti-leptons. Unfortunately that brings us up to 48 elementary particles again – the universe must be much simpler and more elegant than that!

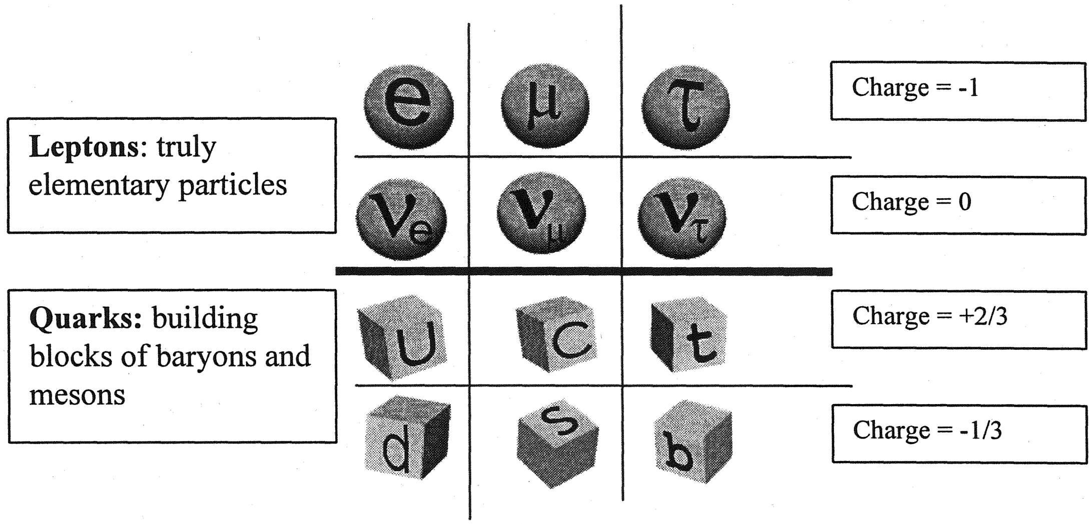

Figure 4: *The elementary particles of the "Standard Model"*
Three "families," each with two quarks and two leptons

Symmetry and Time

One of the more interesting symmetry questions in physics involves the direction of the flow of time. When one looks at a single atom, or at sub-atomic particles, any process that takes place can also happen in reverse, i.e. with time flowing in the opposite direction. If one could somehow photograph, for example, a collision between two atoms, or the absorption of a photon of light by an atom, or the capture of a neutron by an atomic nucleus, there would be no way for an observer of the resulting movie to tell if the movie were being run forward or backward. In other words, all interactions between individual atoms are symmetric with respect to reversal in time.

On a macroscopic scale, however, this symmetry disappears. While it is possible (and usually easy) to convert 100% of any quantity of any form of energy into heat energy, the Second Law Of Thermodynamics states that it is *not* possible to convert 100% of heat energy into *any* other form. This fact has major implications for our universe. On the one hand, if the law is simply true and absolute as stated then the universe will continue to expand forever and eventually all the stars will have burned themselves out and virtually all of the energy of the universe will be useless heat spread through all of space. On the other hand, if the expansion slows and eventually reverses into a contraction (as believed by Stephen Hawking and many other scientists) the Second Law of Thermodynamics cannot hold during the contraction phase. And, if the Second Law is somehow reversed, will the flow of time be reversed? Finally, what would it be like living in a universe where time runs backward?

In Conclusion

This paper has just scratched the surface of the role of symmetry ideas in the development of modern physics. Present day physicists like to believe in an ultimate symmetry of a universe where time undergoes an endless repeating cycle of forward and backward as the universe expands and contracts in an equally endless oscillation. (4)

At the ultimate extreme of contraction – the instant of the "big bang," all particles and all forces would be indistinguishable. Only as the universe cools and expands do particles separate into quarks then into protons and neutrons, and the primordial single force splits into distinct gravitational, electromagnetic and nuclear forces. This idea represents the ultimate dominance of Symmetry as the overriding principal of the structure of our universe. Modern physicists would like nothing better than to prove that the universe really does behave according to this model of "perfect symmetry."

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[1] Eugene Wigner, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences", in *Communications in Pure and Applied Mathematics,* Vol. 13, No. 1, February 1960; reprinted in *The World of Physics*, Jefferson Hane Weaver, Vol. III, p96, Simon and Schuster, New York, 1987

[2] E.P.Wigner, *Gruppentheorie und ihre Anwendung auf die Quantenmechanik der Atomspektren,* F. Vieweg und sohn, Braunschweig (1931). English translation by J.J. Griffin, Academic Press, New York (1959).
For a general discussion see: David J. Gross, *Symmetry in Physics: Wigner's Legacy,* Physics Today, December 1995, p46, American Institute of Physics

[3] In Gell-Mann's model, each *baryon* is made up of three quarks while each *meson* is composed of one quark and one anti-quark.

[4] . For a much more extensive discussion of these ideas, see Stephen W. Hawking, *A Brief History of Time,* Bantam, New York, 1988