BRIDGES
Mathematical Connections
in Art, Music, and Science

Bridges of Mathematics, Art, and Physics

Douglas D. Peden
Independent Artist
Essex, NY 12936-0003
E-mail: pedens@willex.com

Abstract

This paper is presented as an **introduction** to my art and its geometry in the union of three areas of our experience--mathematics, art, and physics.

1. Introduction

Artists invent worlds with their own laws and limitations, sometimes called style. Within this context, artists express themselves, their culture, indeed, a world we all share. Art as well as science and mathematics is a world to explore and discover that which defines us and expands the parameters of our experience. For example, in the visual arts we have various spatial approaches such as perspective, realist, cubist, atmospheric, etcetera. For the most part, they are all based on a Cartesian grid. Out of my art has grown a different mathematical structure based on a curvilinear coordinate system which I call **Wave Space Geometry.**

The two basic facets of **Wave Space Geometry** to be considered in this paper are what I call **Common Wave Space (CWS)** and **Interphase Wave Space (IWS)**. I will attempt to show how these geometries are used in my painting; and share some observations and speculations concerning my work and our own physical world.

2. Wave Space Geometry

Basic Definitions

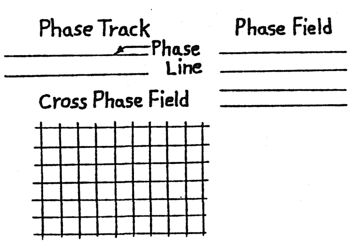

Figure 2.1

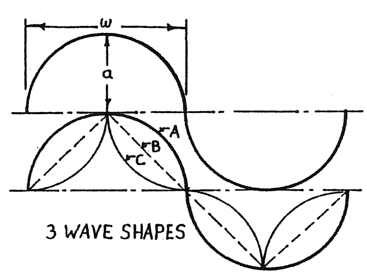

Figure 2.2

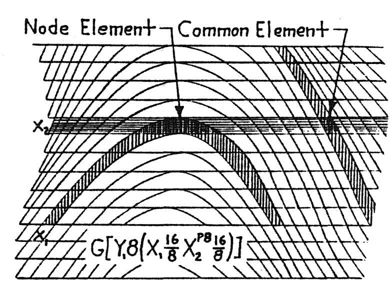

Figure 2.3

Refer to Fig.2.1 for the following definitions.
Phase Line: a continuous line division of space.
Phase Track: two parallel phase lines.
Phase Field: a series of phase tracks in one plane.
Cross-Phasing: crossing of two or more phase tracks.

Cross Phase Field: the crossing of two or more phase fields.
Wave dimensions and wave shapes are seen in Fig. 2.2 where **w** is ½ the basic wavelength, **W**, and **a** is its amplitude; the three basic **wave shapes** assumed are the **common** wave, A, the **line** wave, B, and the **concave** wave, C.

Common Wave Space(CWS) is defined by a curvilinear coordinate system. It is a wave grid pattern formed using an X or Y phase field orientation with one field a function of the other. This is what I call a functional geometric system and should not be confused with an overlay system where one phase field is randomly overlaid on another.

Interphase Wave Space (IWS) is a wave pattern formed by plotting a second generation wave field on a CWS grid.

Wave Space Equation
The working formula, or graphic equation, used here to draw the accompanying examples is:

$$S_w(WaveSpace) = G\left(Y_1 w_{y1}/a_{y1},\ X_1 w_{x1}/a_{x1},\ X_2^{pq} w_{x2}/a_{x2},\ \left(\begin{array}{c} X_m w_{xm}/a_{xm} \\ Y_m w_{ym}/a_{ym} \end{array}\right)\right)$$

where each term is drawn sequentially. Y_1 and X_1 indicate the primary wave field directions, the X_1 field being a function of the Y_1 field to form a CWS grid configuration. X_2 is the secondary wave field taken as a function of, and **interphasing** with the CWS coordinate grid. Interphasing occurs when one field track intercepts another, going in the same direction, and returns **without crossing**. The shape formed, I call a **nodal element**, or node, e_n. The other elemental shape which is formed by two or more phase tracks **crossing** each other, I call a **common element**, e_c, as seen in Fig.2.3. CWS is made of totally common elements; IWS is the summation of common and nodal elements in a proportion dependent on the interphasing geometries.

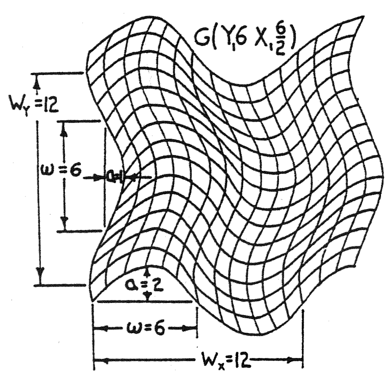

Figure 2.4 Common Wave Space(CWS)

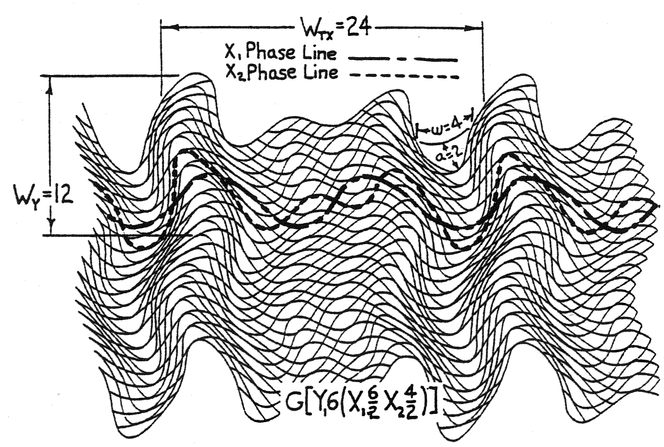

Figure 2.5 Interphase Wave Space(IWS)

An example of CWS is seen in Fig.2.4 where $w_{Y1}=6$, $a_{Y1}=1$, and $w_{X1}=6$, $a_{X1}=2$, or $S_W=G(Y_16/1X_16/2)$, written $G(Y_16X_16/2)$.

An example of IWS is seen in Fig.2.5 where the X_2 field of $w_{X2}=4$, $a_{X2}=2$ interphases with the CWS coordinate system of Fig.2.4 in the X_1 direction, the third term of S_W. The pq superscript in S_W indicates the phase position of the secondary wave on the primary wave and determines the rhythm of the interphase field as shown in Fig.2.6. There are W possible starting positions on wavelength W; in our example, 7 out of 12 positions are shown in a $G(X_16)$ wave field interacting with a secondary, X_26 wave field. In Fig.2.4 there are also 12 possible positions: $W=2w_{X1}=2x6=12$. In example, Fig.2.5, q=0 was chosen to establish the secondary, $X_24/2$ interphase wave field. **Also note** in the equation $G[Y_16(X_16/2X_24/2)]$ of Fig.2.5 that the parenthetic bracketing indicates what is shown, and will be so in subsequent equations.

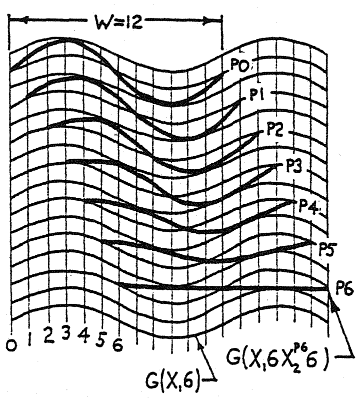

Figure 2.6 Phase Positions

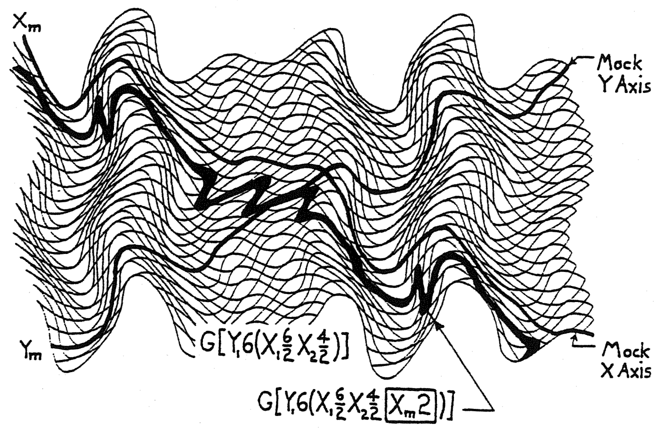

Figure 2.7 Mock Axes

The fourth term of S_W is what I call a **mock axes** coordinate system; in other words, a IWS configuration used as CWS grid. This is done by choosing an axes intersection point, i.e., an origin, with one coordinate starting in the primary field direction , the other starting in the secondary field direction and at each node alternating to the other field direction, giving the continuous, divergent x,y mock axes X_m and Y_m as exemplified in Fig.2.7. (Notice X_m is drawn from the upper left to lower right and Y_m from the lower left to upper right). The "layered" position of X_m and Y_m of S_W means that an X_m interphase field **or** Y_m interphase field **or** both simultaneously can be plotted. Fig.2.7 shows the single interphase track, X_m2.

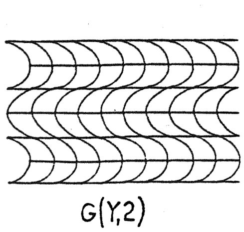

Figure 2.8 CWS

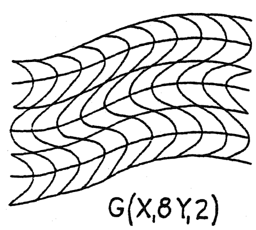

Figure 2.9 CWS

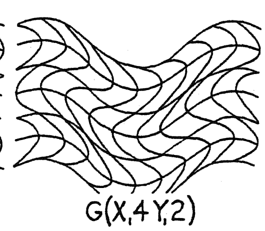

Figure 2.10 CWS

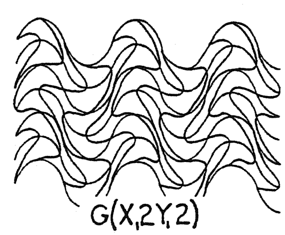

Figure 2.11 CWS

The above examples are given to further demonstrate how Wave Space works. CWS Figs.2.8, 2.9, 2.10, 2.11 show a progression in wave dynamics holding a_{X1} and Y_12 constant with w_{X1} going from infinity to 2:

Isolating the mock axis field X_m of Fig.2.5, we have Fig.2.12. The semi-colon in equations such as X_m:G(...) and Y_m:G(...) simply means X_m or Y_m **of** the specific geometry G(...). Fig.2.13 is an extension of Fig.2.5 space in the mock X axis direction by X_m2.

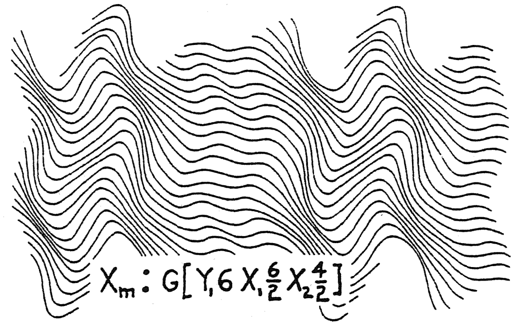

Figure 2.12 IWS

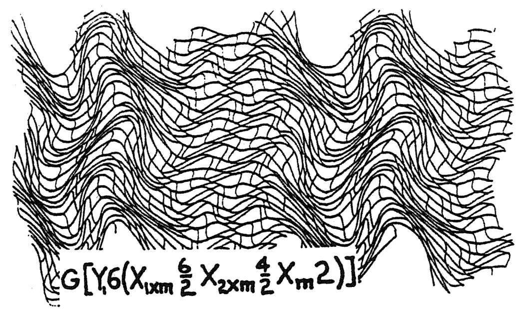

Figure 2.13 IWS

Figs.2.14 and 2.15 is Fig.2.13 separated into **its** X_m and Y_m phase fields. Fig. 2.16 is formed by extending Fig.2.5 in **both** mock axes directions simultaneously.

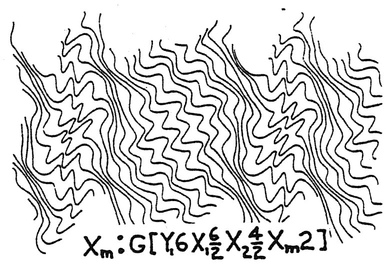

Figure 2.14 IWS

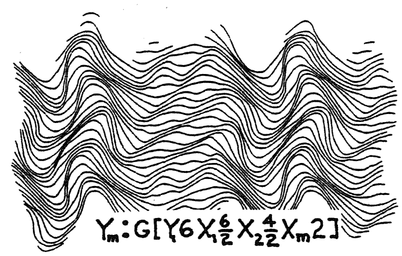

Figure 2.15 IWS

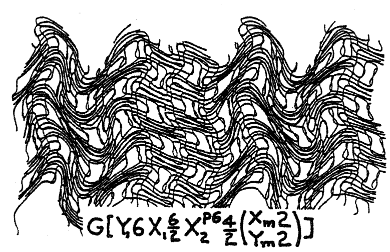

Figure 2.16 IWS

Variation is further increased if we consider the shape of the wave, w, or wave-train, W_T. Wave shape A was used in the previous examples. The other two basic shapes B and C, Fig.2.3, should be noted. Figs.2.17 and 2.18 show the B and C waves in the Fig.2.4 CWS geometry; Figs.2.19 and 2.20 show the B and C waves in the Fig.2.5 IWS geometry. The symbol above the wave dimensions indicates the wave type and its orientation: indicates a full wave; , indicates a half wave oriented apex up or down respectively.

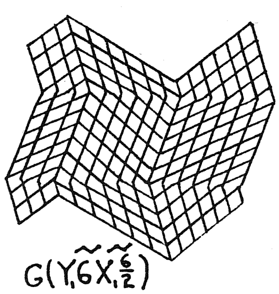

Figure 2.17 CWS B Wave

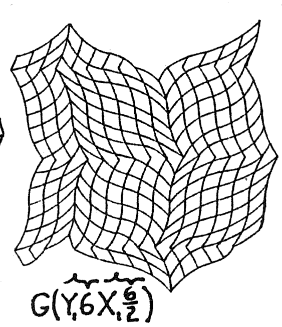

Figure 2.18 CWS C Wave

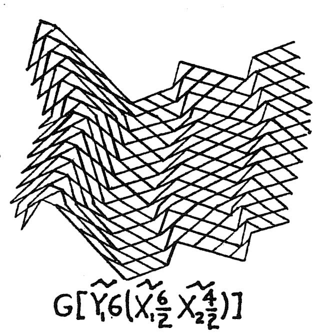

Figure 2.19 IWS B Wave

Figure 2.20 IWS C Wave

Another approach is the compounding of wavelengths and/or wave shapes as seen in Figs.2.21 and 2.22 in CWS; with a more complex CWS version in Fig.2.23 and its IWS extension, Fig.2.24. The various squiggles, as used above, show the specific make-up of a wave shape or wave train.

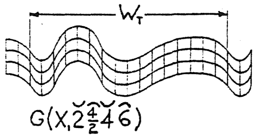

Figure 2.21 CWS Compound Length

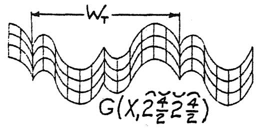

Figure 2.22 CWS Compound Shape

Figure 2.23 Compound CWS

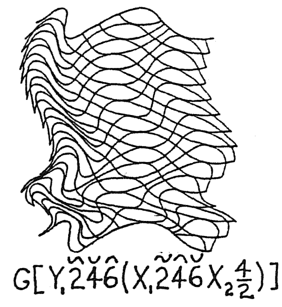

Figure 2.24 Compound IWS

3. Shapes in Wave Space

In Wave Space, the geometry defines the space. The elements, which are shapes in themselves, as squares in Cartesian space, are the increments of Wave Space. To get an idea of how shapes change from geometry to geometry, let us take a 2 x 2, 4 element square, and translate it in the following two geometries: Fig.3.1 shows the square in three regions of the CWS geometry $G(X_1 4/4 Y_1 4/2)$; here, the square appears as a rather amorphous blob, one shape being almost linear in its elongation. Fig.3.2 is an example of IWS geometry, $G(X_1 2X_2^{P2} 2/2)$, which converts the Cartesian square of our space into a 4 element circular shape in Wave Space.

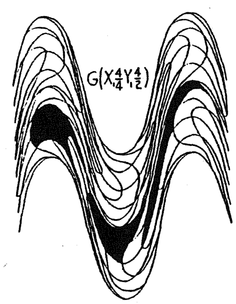

Figure 3.1 CWS

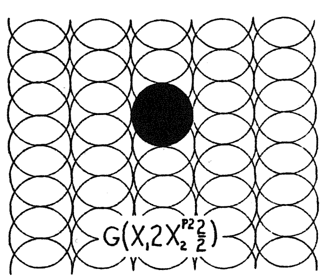

Figure 3.2 IWS

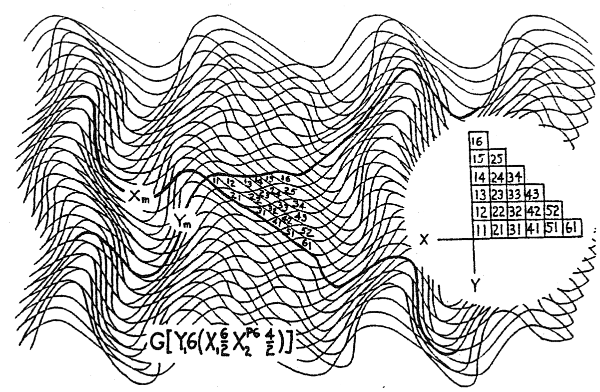

Figure 3.3 Translation

What we have done is take a square from our reality and put it in a different geometric reality. In a sense, it is still a square, and would be seen as such by the inhabitants of this new geometric space; however, from our frame of reference, it has become a circle. In this kind of physicality, it begs the question: What is "true" shape? The answer would seem to depend on your frame of reference, which is true; and, in this particular case, it depends on the geometric space the object occupies. Physically and philosophically this is interesting, considering the infinity of spatial configurations possible; that is, of course, if such spatial realities exist, a speculation we will consider later.

As an exercise, if we wish to translate from our Cartesian space to a Wave Space geometry, refer to Fig.3.3. Here, we have a geometry similar to Fig.2.5 with our mock axes arbitrarily chosen, as shown, and numbered to parallel our Cartesian grid(inset). It is now a matter of drawing a shape in Cartesian space and converting it to Wave Space, **or** do the reverse to see what shapes created in Wave Space look like in Cartesian space. For a demonstration, I show an imaginary character from Cartesian space, Fig.3.4, and show it in three other geometric configurations, Figs.3.4a,3.4b, and 3.4c.

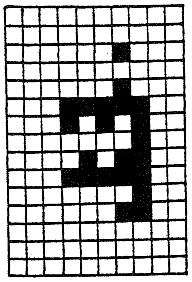

Figure 3.4 Character

Figure 3.4a

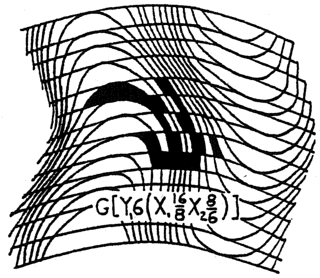

Figure 3.4b

Figure 3.4c

However, the idea of shapes changing outside our frame of reference is not new when we consider Einstein's illustration of a clock approaching the speed of light: From our frame of reference, we see the clock being flattened and time slowing; but, from the clock's frame of reference, everything has remained normal.

If we assume a shape to **move** in Wave Space, some curious things occur. A shape moving in our Newtonian(Cartesian grid) space will maintain its dimensional and geometric integrity. A shape moving in CWS will distort to a greater or lesser degree, depending on the geometry of the space it traverses; but the relationship of the elements will remain the same. However, a shape moving in IWS will not only distort, but it may fragment; that is, on an elemental level, the relationship of the elements will not remain intact. In IWS, the elements in a shape following one of the mock axes will keep a consistent relationship with one another regardless of the degree of distortion of the shape, as is shown in Fig.3.5a. Here, we have our 4-element square following the X_m axis through a number of successive incremental changes. If the shape should follow a specific phase track exclusively, as is the case in Fig.3.5b, some of the elements will separate(the cross-hatched figures) from their partners while passing through the nodal region; however, the square will be reformed as it passes through the next nodal region, thus following a cyclical process of fragmentation and reformation from region to region. For larger shapes comprising a vast number of elements, the individual incongruities would most likely be absorbed and seen as proportionally minor disturbances, fluctuations, or pulses.

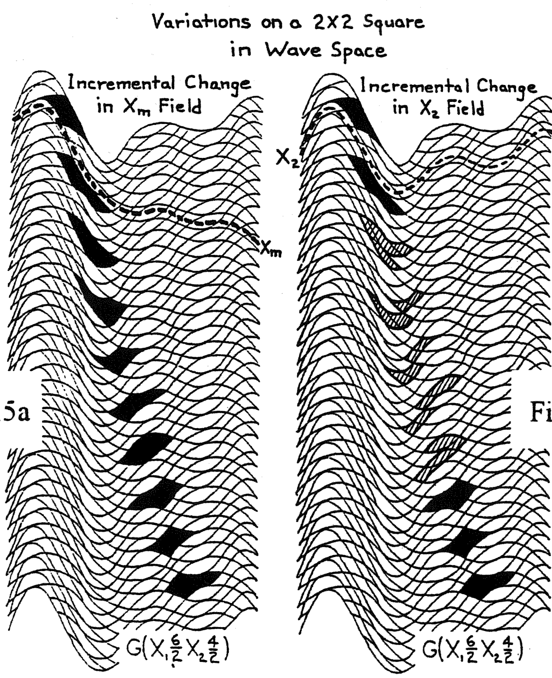

Figure 3.5a

Figure 3.5b

4. Wave Space Painting

Wave space art is holistic in that the shapes, themes, rhythms, and spatial textures are defined and influenced by the geometric configuration of the chosen space. The interweaving phase fields form the individual elements, the basic building blocks of larger shapes which in themselves can build to increasingly larger wholes.
The choice of the geometry to be used depends, of course, on the artist's intent. Some spatial configurations are more dynamic than others; in other words, they have a greater sense of action, or vitality, than those of a more gentle, "expressive feeling". For example, in Fig.3.4 our Cartesian character's attitude is somewhat static; it lacks a sense of motion that is expressed by the subsequent examples. This dynamic can then be emphasized or diminished by the use of color and tone in the rhythmic structure of the painting itself. For example, red in the character might suggest a hot, aggressive, passionate individual; while a blue might express a cool, thoughtful character; and green, a calm, comfortable personality. **Or** such colors could be used to characterize the background environment of the painting to enhance or diminish the drama.

What we are doing here is creating a **world** of space, color, rhythm, and texture where the shapes become living things, experiencing the trials and tribulations of the environment they inhabit--plus their own episodic changes and transfigurations.

In the genre of art, my work would be considered abstract; however, the abstract components often imply literary content--that is, an **abstract narrative** or **metaphor**. In other words, I try to convey an idea or story exclusively through abstract forms, as suggested above. Indeed, our imaginary character of Fig.3.4 could be considered venturing through the three different spatial environments and experiencing various physical and/or emotional transfigurations as implied by the geometric dynamic. Another example can be seen in my IWS painting, "Come Together", Fig.4.1.

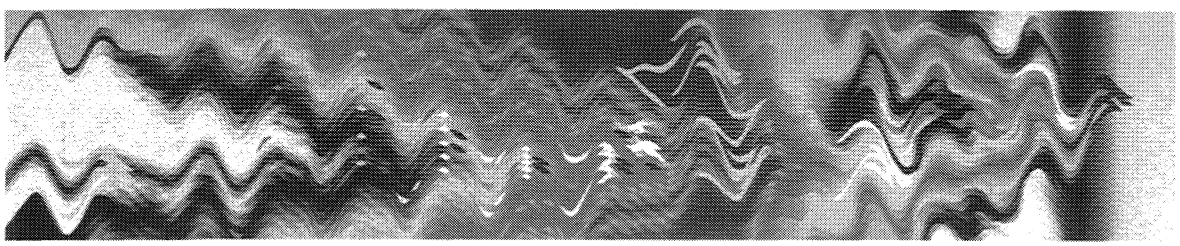

Figure 4.1 IWS "Come Together"

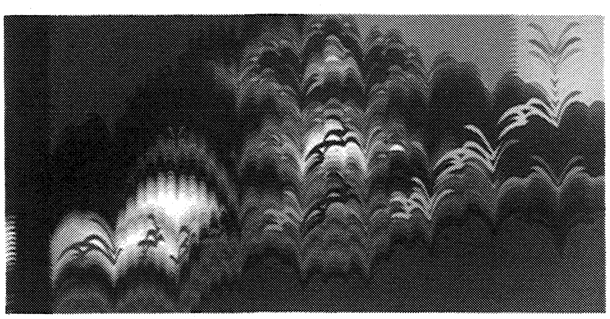

Figure 4.2 "Transfiguration"

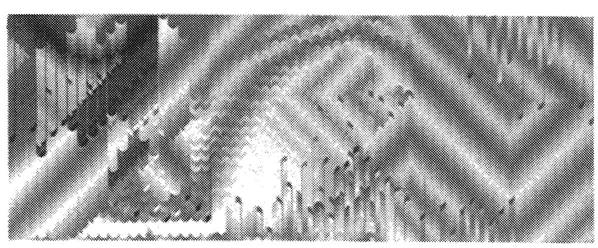

Figure 4.3 CWS "Rhapsody for 9 Notes"

This painting is episodic. Beginning at the left and progressing right, through a steady rhythmic space, separate elements, or parts, come together forming a complex whole--a shape theme, or leitmotif, at the mid-point of the painting. It is subsequently transformed into a linear, dancing combination of like figures, and proceeds further through a rhythmically agitated space to the end of the painting where it is seen penetrating a vertically static, dark rhythmic color column on the verge of entering a blank, monochromatic space. It could be the end of the story or the end of a chapter. It could be "read" from right to left for a different interpretation. In any event, how the painting is approached, experienced, and interpreted is entirely up to the individual observer.

Sometimes my art is perceived as visual music; and indeed, in many cases, it is my intent. In the context of the painting, my theme may be a single shape as in Fig.4.1 and 4.2; or as a group of separate elements, Fig.4.3, much in the same manner of music notation in a score. The theme, is subsequently developed through various color keys, color/tone harmonies, and thematic manipulations, i.e., turned, flipped, dissected, distorted, reversed, etcetera--all depending what I, the artist/composer, hope to convey.

5. Similar Worlds

Exploring the worlds of my work, patterns of our day to day experience appear, suggesting structures or methodologies in Wave Space geometry that parallels our own world. I have compared on many occasions, riding the local ferry, the rippling and cross-rippling of waves with the interacting wave fields of my own studies. Perhaps you have experienced the amusing distortion of shapes in a carnival mirror, and how they change from one region to another. Some other patterns observed are flame movements, as in my painting, Fig.5.1, "Aurora", and smoke in my study, Fig.5.2. In a detail of my painting "Edge of Chaos" and my painting "Garden of Middle Harmony",Figs.5.3 and 5.4, we have patterns reminiscent of gas and liquid currents, i.e., turbulence. Also in Fig.5.4, I am reminded of the spectral patterns seen in oil slicks; I have also seen this agitated schema as a colorful, vibrating pattern on the retinas of my eyes, which apparently is a migraine headache phenomenon.

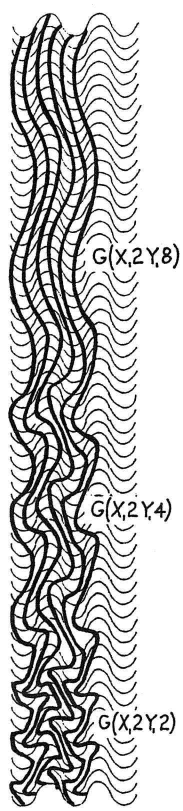

Figure 5.2 Heat Waves

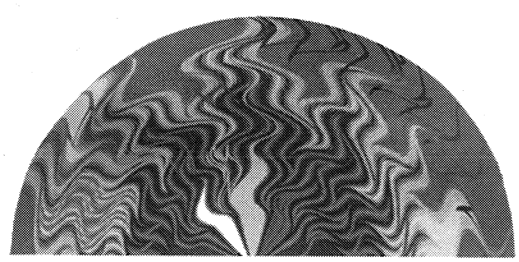

Figure 5.1 "Aurora"

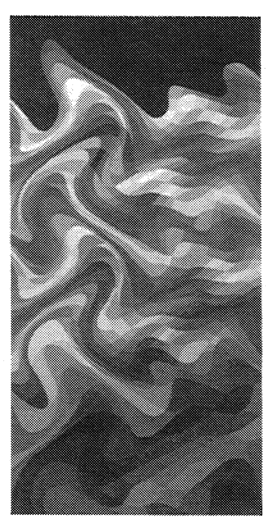

Figure 5.3 IWS Detail "Edge of Chaos"

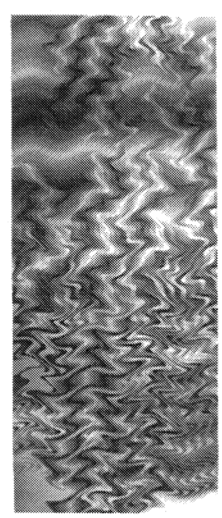

Figure 5.4 "Garden of Middle Harmony"

We find interphase fields where wave patterns are damped and reenergized as in Figure 5.5. Chaos and fractal geometries are suggested in my painting "Transfiguration", Figure 4.2, above. The wave track of Fig.2.7 suggests the meander of a river.

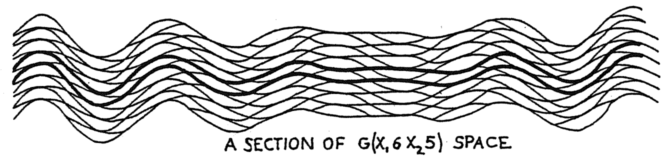

Figure 5.5 Phase Damping

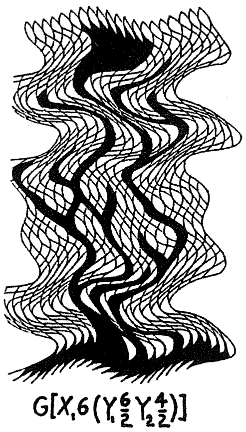

Figure 5.6

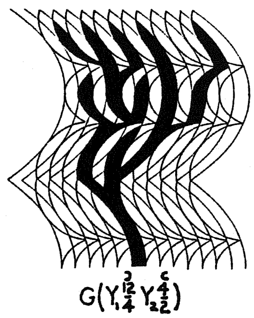

Figure 5.7

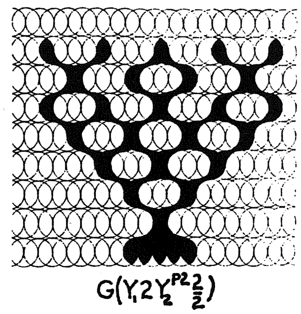

Figure 5.8

Plant Patterns

By utilizing the node as a bifurcation point, certain geometries suggest plant growth patterns, as in Figs.5.6, 5.7, and 5.8.

6. The Quantum Cosmic Connection

Exploring the many worlds of Wave Space geometry has led me to speculate on the two extremes of our own reality, i.e., micro or quantum space and cosmic space. The basic analogy between Wave Space and the "real" world is summed up in Fig.6.1. Here, the quantum world is viewed as a high frequency, densely packed nodular space compared to our essentially "linear", Newtonian world. This analogy seems to fit well when we consider Einstein positing a cosmic space which takes the form of a vast curvilinear geometric structure and the quantum scientists who view the microcosmos as a wave/particle paradigm of incredibly high frequencies.

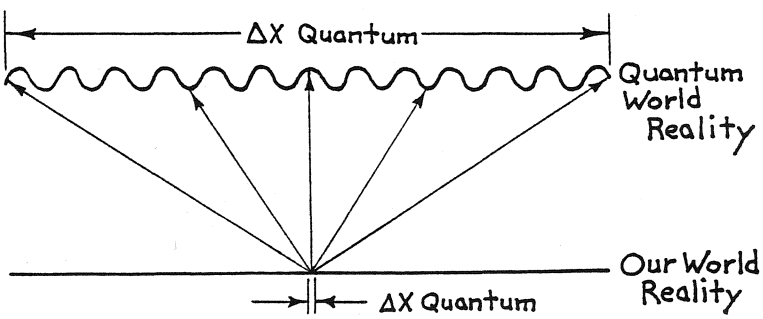

Figure 6.1 World Reality Scale

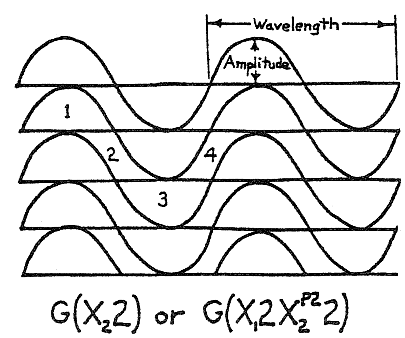

Figure 6.2 Minimum IWS Geometry

Let's consider the **limits** of Wave Space geometry. From the previous discussion, the macro limit would be a Cartesian grid, Newtonian reality, which is our basic frame of reference. Beyond our frame of reference, in the microworld, we assume the bottom limit of Wave Space geometry.

The minimum wave configuration is w/a=2, and its spatial configuration in IWS is given by the two equations, (1) $G(X_2 2)$ and (2) $G(X_1 2X_2^2 2)$, Fig.6.2. It should be noted, here, that the two equations give the same geometry but are physically different: The X_1 field line, the straight line portion of (1), is considered a wave with $w_{X1}/a_{X1} = \infty$, i.e., as a curve with an infinite radius of curvature. The straight line portion of (2) occurs with q=2, putting the secondary wave, X_2, 180° out of phase. This minimum configuration gives four elements per wave cycle, an equal distribution of common to nodal elements, or $e_c/e_n = 1$, and a maximum density of nodal elements. From this, e_c/e_n increases, as our reference scale increases, to our macroworld level and beyond, $e_c/e_n \rightarrow \infty$. Also, as e_c/e_n increases, the nodal density decreases and the space between the widening nodal regions takes on the appearance of a Cartesian grid; illustrated to some extent in Fig.2.3. In this region, we presume the predictable, Newtonian, world we are familiar with. Let me further suggest that in this geometry of widely separated nodal regions, that the nodal regions are analogous to the black hole regions of our own universe. This speculation may or may not be that far fetched if we consider Wave Space geometry in three or more dimensions rather than the two of this paper.

Another parallel to our world is, because of the density of nodes at the quantum level, and their bifurcating factor, a microshape or particle/wave packet would have more possible paths to follow in a given distance and therefore be harder to predict its position or velocity at a given time, unless we knew the precise geometry of the space and the laws that govern it. Also, a microshape moving in a densely nodular space, as in Fig.6.2, would be more difficult to identify due to its continual morphing from nodular to non-nodular space.

The above, of course, is pure science fiction; but even the physicists don't know what the quantum world is like, they have a strong feeling and good reason to believe it is quite different from our own experience. Many feel the quantum world to be a complex paradigm of wave fields; and, even on a cosmic scale, Einstein posits a curvilinear geometric structure. However, Einstein's conclusion, as I understand it, is that mass, or mass as energy, interacts with and distorts the surrounding space. In applying Wave Space to "our" space, I have to conclude that it is the space that creates and defines the mass, or to be more accurate, the illusion of mass. It is the interweaving and interdependence of complex geometries that create and contribute to the totality of reality itself.