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Mathematical Connections
in Art, Music, and Science

A Taxonomy of Ancient Geometry Based on the Hidden Pavements of Michelangelo's Laurentian Library

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Abstract

Based on Ben Nicholson's reconstruction of a set of hidden pavements within the Laurentian Library, the authors have hypothesized a system of geometry which may have led to many of the ancient masterpieces of architecture and design. The constituent parts of an ancient geometry, consisting of five major components are presented. Based on these principal geometries, a hypothetical taxonomny of ancient sacred geometry is presented. A proposal is made showing how this taxonomny relates to the pavements of the Laurentian Library.

Introduction

Architectural masterpieces from antiquity to the present have depended upon geometry for their creation. Scholars of the past 75 years have identified particular geometries that can be associated with individual buildings and building types. The well known constructions of the sacred cut based on $\sqrt{2}$, the eight pointed Brunes star, discovered by the Danish engineer Tons Brunes, the ad quadratum square-within-a-square, the golden mean, the Vesica Pisces, and Jay Hambridge's dynamic symmetry can be all exemplified by one or another building.

When considered together, the monuments from antiquity portray different elements of a singular body of knowledge sometimes referred to as sacred geometry. There is scant evidence that the component parts of this body of knowledge were ever considered as a whole, for few architectural drawings or masons' manuals have been handed down [1]. The standard way to come to terms with geometry is to organize it into parts, discrete 'chapters', which have a tentative relationship to one another. If all the parts are laid out together, it soon becomes clear that each construction can only be achieved if another generative construction has already been executed. Thus it is possible to see how each chapter links with another: for example the place where $\sqrt{2}$ and $\sqrt{3}$ constructions have a common origin is the Vesica Pisces.

In this context, the 15 pairs of geometric panels, lying hidden beneath the floorboards in Michelangelo's Laurentian Library, are of great interest because they demonstrate that the terra-cotta designs touch virtually every element of sacred geometry - except the pentagon. This paper proposes that the Laurentian pavement is a near complete taxonomy of ancient geometry. Following a 13 year analysis of the pavements, a taxonomy has been developed that commingles all the geometric systems handed down from antiquity into a single integrated whole.

The Laurentian Library

In 1774 a portentous accident occurred in the Reading Room of the Laurentian Library, part of the church complex of San Lorenzo, Florence.

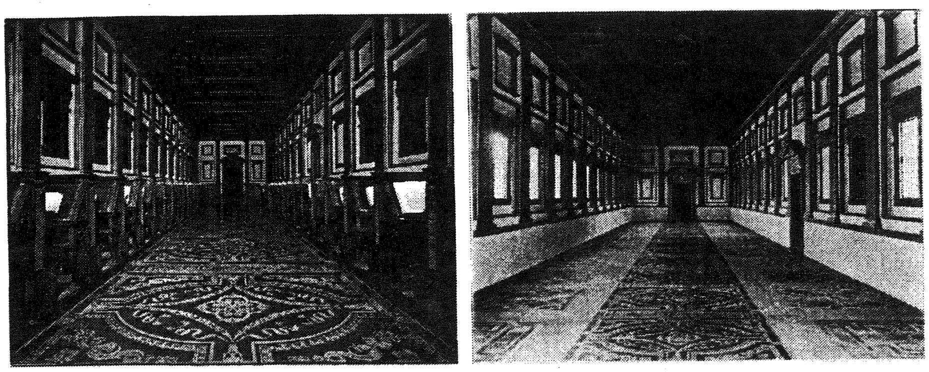


Figure 1. The Laurentian Library, Reading Room - with and without desks.

The shelf of desk 74, over laden with books, gave way and broke. During the course of its repair, workmen found a red and white terra-cotta pavement hidden for nearly 200 years beneath the floorboards. The librarian had trapdoors, still operable today, built into the floor so future generations could view these unusual pavements. Further details of the history and significance of the pavements can be found in Nicholson's Thinking the Unthinkable House [2], and in [3] and [4].

Overall, the pavement consists of two side aisles and a figurative center aisle. Each side aisle is composed of a series of fifteen panels and each panel is of a different design measuring about 8'6" x 8'6". The fifteen panels along one aisle mirror the ones on the other aisle, but differ by a very small degree and in subtle ways. When juxtaposed in a series, the fifteen pairs of panels appear to tell a story about the essentials of geometry and numbers. The nuances of the geometric structures of the Medici Panel (#2) and the Mask Panel (#13) are described in detail in [2] and [3] while the Sacred Cut Panel (#12) is described in [2] and [4].

Ben Nicholson has worked with students for thirteen years to reconstruct the system which the team of geometers and theologians, possibly including Michelangelo, may have used to create the original designs. He has recently collaborated with artist Blake Summers and architecture graduate student Saori Hisano to replicate all fifteen panels at full scale, working with straightedge and compass. As a result of this work, Nicholson and his team have discovered tenets of geometry which may have formed the basis of an organized system of ancient sacred geometry.

The Principal Geometries

We shall briefly describe the six principal geometries that form the constituent parts of ancient geometry. More detailed descriptions can be found in [4].

A. The Vesica Pisces and the Triangle Circle Grid

The *Vesica Pisces* is created by placing a point arbitrarily on the circumference of a circle and drawing a second circle of the same radius centered at that point (Figure 2a). In ancient sacred geometry the Vesica had spiritual significance, and engravings of Christ were often found within the central region (Figure 2b). A pair of axes placed in the Vesica results in a pair of equilateral triangles (Figure 2c). When each intersection point between circles is used as the center of a new circle, a triangular grid results. (Figure 2d) Four of these circles create the ten-pointed grid known as the *tectractys* of great significance in Platonic numerology. The tetractys can be shown to lead to the structure of the Pythagorean musical scale [4 and 5]. Figure 2e shows how the Vesica leads to a construction of an equilateral triangle in a square. Figure 2f shows a pair of Vesicas with axes at right angles to each other; the square and circle in the center have approximately the same perimeter giving an approximate *squaring of the circle* in length.

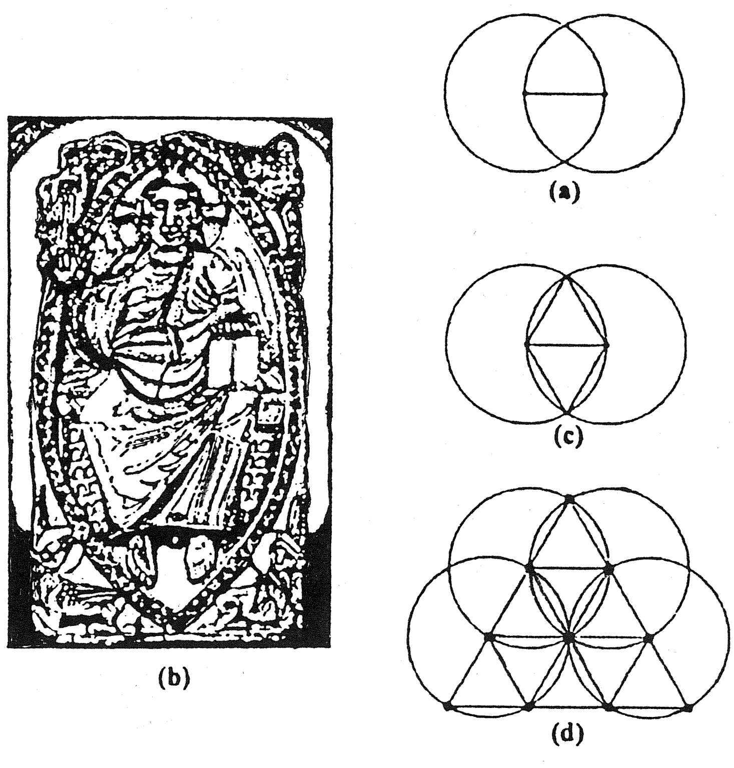


Figure 2. a) Vesica Pisces; b) Christ figure in Vesica; c) equilateral triangles fit into a Vesica; d) Vesica forms a triangular grid; e) construction of equilateral triangle in a square; f) pair of orthogonal Vesicas squaring the circle.

B. The Square Circle Grid and the Sacred Cut

The Vesica diagram is the initiating form for all two-dimensional ancient geometry as we shall see in the next section. Therefore it is not surprising that it leads in a natural manner to a fundamental construction of ancient geometry known as the *sacred cut*. Hisano's analysis follows:

1) Begin with a pair of Vesicas generated by three circles (Figure 3a). A pair of circles (light lines) are added to create two axes at right angles. Six additional circles are added to create a circle grid based on a square of nine circles (Figure 3b).

2) A square and its diagonals are highlighted within which the center circle of the nine circles is inscribed. This square is divided into four smaller squares, and a central square is highlighted in the upper left-hand square (Figure 3c). This highlighted square represents the central square of a subdivision of a square into three species of rectangles (Figure 3d). We shall refer to this as the sacred cut division of a square. The three rectangles are: a square (S) of proportion 1:1; a $\sqrt{2}$ rectangle (SR) of proportion $1:\sqrt{2}$; and a rectangle of proportion $1:\theta$ where $\theta=1+\sqrt{2}$, which we refer to as a Roman rectangle because of its prevalence in Roman architecture. The three proportions 1: 1, $1:\sqrt{2}$, and $1:\theta$ form the basis of a system of ancient Roman proportionality described by Theon of Smyrna, a second century AD Platonist philosopher and mathematician in his book The Mathematics Useful for Understanding Plato [6].

3) Within the central circle lies a pair of squares rotated at 45 deg. with respect to each other (Figure 3e). It is evident that this pair of squares also contains the outline of a regular octagon. Figure 3e shows that this pair of rotated squares leads again to the sacred cut subdivision of a square. In Figure 3f, square abcd is exploded outwards to square ABCD, and the sacred cut subdivision is replicated at a large scale. This process can be repeated at ever larger or smaller scales demonstrating the interdependence of parts to a whole, and visa versa (Figure 3g).

4) In Figure 3h we see yet another version of the sacred cut subdivision. This time four arcs of a circle are constructed with each arc drawn about a vertex of the square and passing through the center of the square. Each arc cuts the side of the square by a factor of $1/\sqrt{2}$. The eight points at which these arcs cut the side of the square are the vertices of a rectangular octagon. It is this construction from which Brunes first coined the term sacred cut. When the arcs are completed to circles (Figure 3i) square efgh explodes to square EFGH forming yet another sacred cut subdivision. As before, this process can be continued without limit.

5) The final display (Figure 3j), the Hisano diagram, shows how a star octagon is related to squares, circles, and triangles. Within the circle are numerous 45 deg. isosceles triangles with side to base in the ratio $1:\sqrt{2}$. The diagonals also cut each other in the ratio $1:\theta$. This star is a testament to the geometric integrity of the Roman system of proportions.

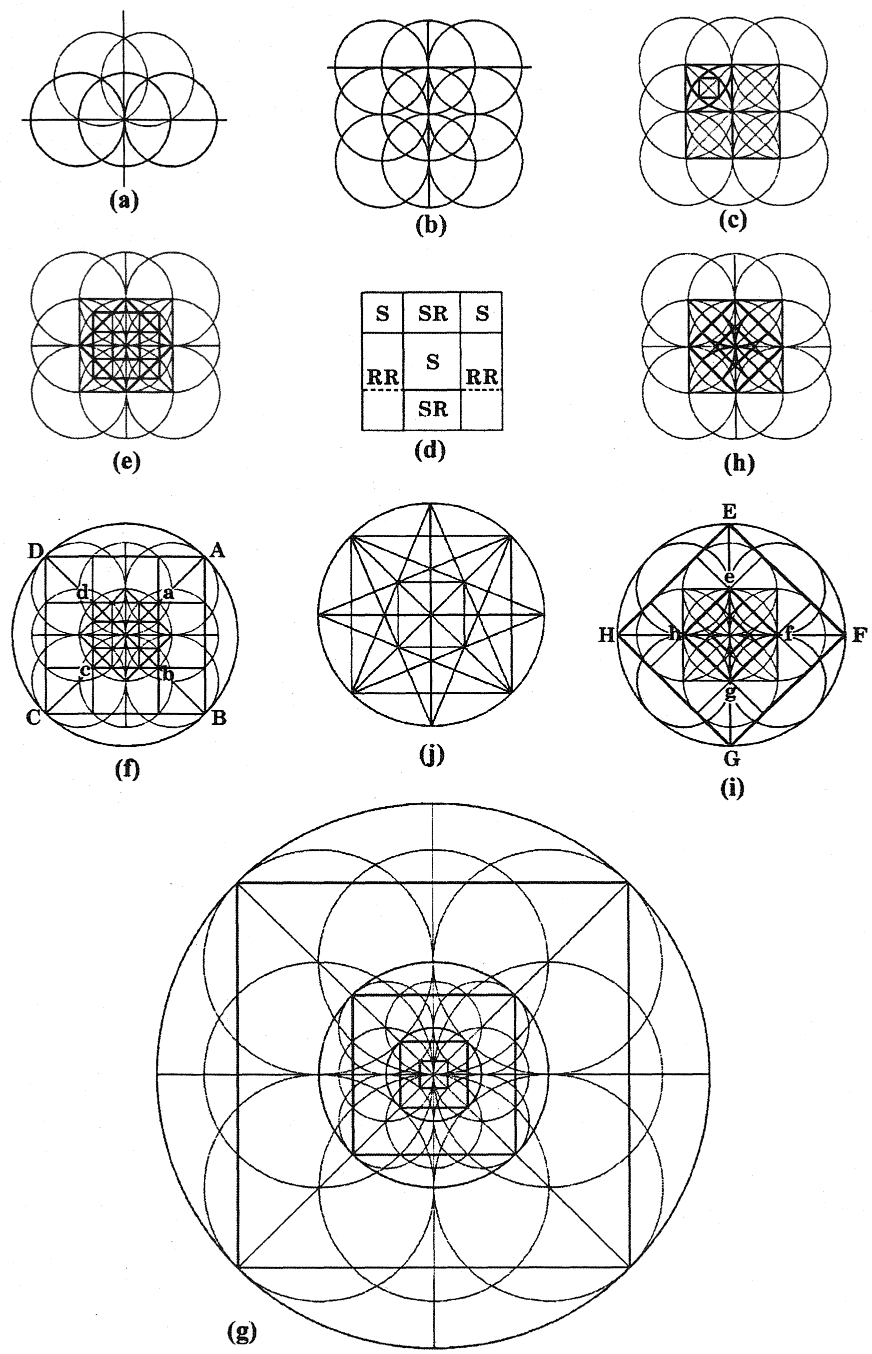


Figure 3. The sacred cut related to the square grid of circle

C. The Brunes Star

Brunes claimed to see an eight-pointed star of a new variety in an unspecified temple pavement in Pompeii (Figure 4a). He hypothesized that this star, along with the sacred cut, formed the basis of an ancient system of geometry important to temple construction [7]. The star is formed by dividing a half-square into four half-squares. When diagonals are placed within each square, the outline of the eight-pointed *Brunes star*, shown in Figure 4b, results. The Brunes star is remarkable since its interior is entirely subdivided into 3,4,5-triangles at four different scales as described in detail by Kappraff [4 and 8]. The Brunes star is a natural trisecting device. The intersections of the upward and downward pointing triangles with the diagonals of the outer square indicate the trisection points. However, the Brunes star is also a natural device with which to subdivide lengths into up to 10 parts without the use of a ruler as Figure 4c illustrates for 2,3,4,5,6, and 8 subdivisions. The case of subdivision into seven parts is shown in Figure 4d. All subdivisions are exact except for the case of seven which is in error by approximately 2%. The Brunes star also offers another means to approximately *square the circle* both in area and circumference (not shown) [4 and 8].

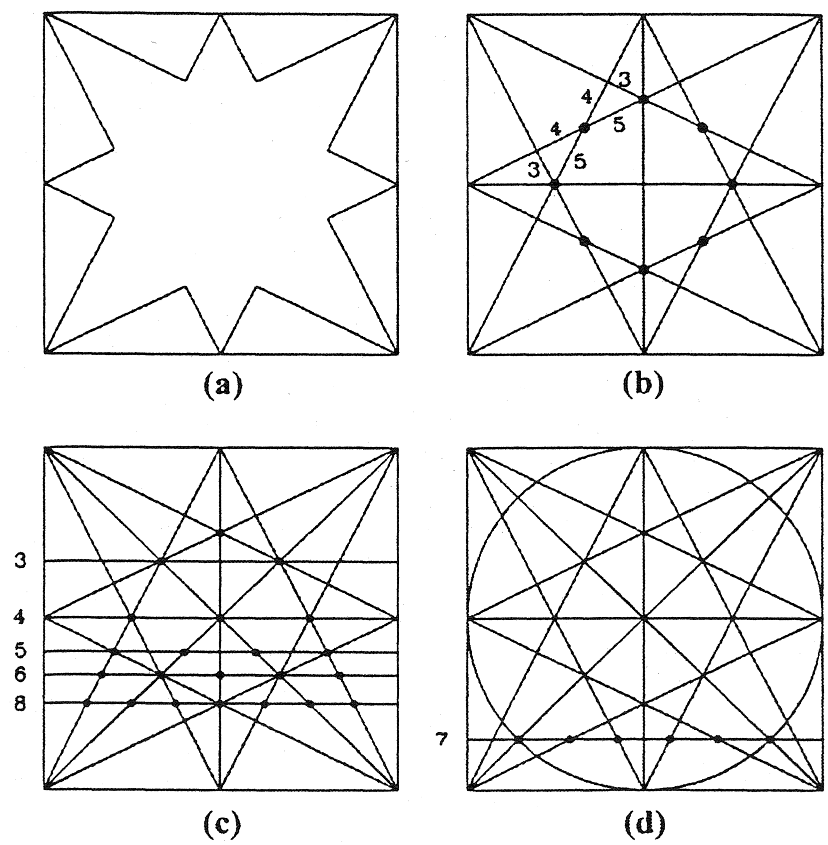


Figure 4. a) The Brunes star; b) inner structure of the Brunes star; c) equipartition; d) partition into 7 parts.

D. The Law of Repetition of Ratios

It has been conjectured by Jay Hambridge and others that the facades of buildings of antiquity were subdivided into self-similar proportions by using a method used during the Renaissance and known as the *law of repetition of ratios* [9], and referred by Hambridge as *dynamic symmetry* [10]. A diagonal is placed in a rectangle of any dimension and intersected by a second line segment at right angles to the diagonal (Figure 5a). This divides the rectangle referred to as the *unit* into another rectangle (unit) of the same proportions and a leftover rectangle referred to as a *gnomon* (Figure 5b). A series of such subdivisions results in set of whirling gnomons forming a logarithmic spiral (Figure 5c). Hambridge and his followers [10 and 11] have shown that an impressive repertoire of designs result from this construction. The gnomon of a Roman rectangle (RR) is a double square (not shown), while the gnomon of √2 rectangle (SR) is another SR. Figure 5d illustrates a subdivision of a SR by the law of repetition of ratios. Notice that the upward and downward pointed triangles of the Brunes star appear, and as a result this construction both bisects and trisects the SR. The √2 rectangle is particularly interesting since a square either added or subtracted from it results in a Roman rectangle, i.e., S+SR=RR and SR−S=RR (not shown). Also if a SR is either cut in two or two such rectangles combined, another SR results, i.e., SR+SR=SR. Kim Williams [12 and 13] has studied the proportion of the Medici Chapel in Florence and determined that its proportional system is based almost entirely on a √2 rectangle.

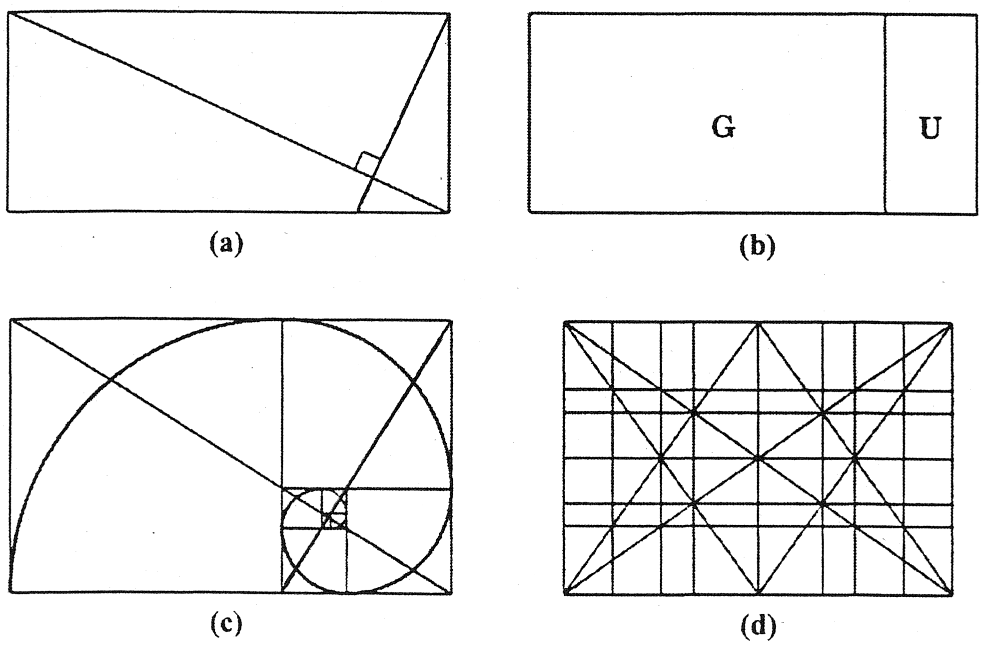


Figure 5. a) Law of repetition of ratios; b) unit and gnomon; c) series of whirling gnomonsform a log spiral; d) √2 rectangle subdivided the law of repetition of ratios.

E. The Golden Mean

A rectangle with proportions 1:ϕ is called a *golden rectangle*, where $\phi = (1 + \sqrt{5})/2$, is a number known as the *golden mean*. To construct golden rectangle ACFG, begin with a unit square ABCD. Add the semi-length AE of side AB to length EF by transposing length ED to EF (Figure 6a). Using the law of repetition of ratios, the gnomon of a golden rectangle is found to be a square. Thus, if a square is removed from a golden rectangle, another golden rectangle (KGBD) results. A series of 'whirling squares' forming a logarithmic spiral is the result of repeating this process (Figure 5c). Ann Macaulay, a Scottish researcher of the British Megalithic stone circles, has created the construction of a triangle with ratio of side to base of

ϕ:1 based on the double vesica (Figure 3a, 6b). If the four circles in Figure 6b have a diameter of 1 unit, it is easy to see that AD has golden mean length ϕ units. By transposing AD to AB and AC, an isosceles triangle ABC, referred to as a golden triangle, with ratio of side to base ϕ:1 is generated. The angle on the base line of a golden triangle is bisected to create a unit of the same proportion and a gnomon (Figure 6c) also with golden mean proportions at a smaller scale. This bisection can then be repeated. A regular pentagon can be subdivided into these two species of golden triangles (Figure 6d). The ratio of the diagonal to the side of a regular pentagon is ϕ:1, and the diagonals of a pentagon cut each other in the *golden section* (ratio of ϕ:1) as shown in Figure 6e. The golden mean occurs naturally in all areas of mathematics, art, and architecture, as well as in nature [5]. It is also a number that ties together an important class of polyhedra known as the *Platonic solids* [5].

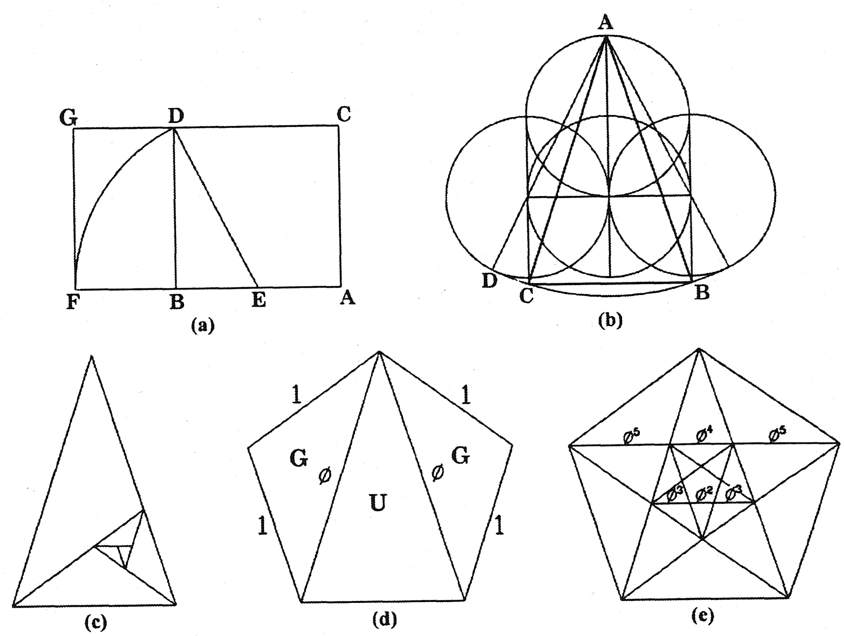
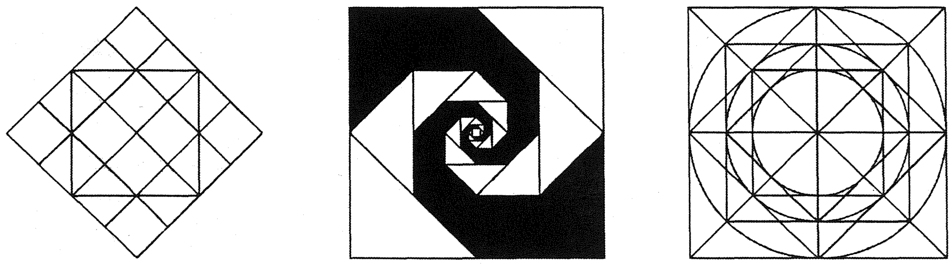


Figure 6. a) Construction of golden mean; b) Macaulay's construction of a golden triangle; c) self-similarity of the golden triangle; d) Golden triangles form a regular pentagon; e) diagonals of a pentagon cutting each other in the golden section.

F. The Ad Quadratum Square

A square placed within a square, so that the vertices of the inner square touch the midpoints of the outer square, is known as an *ad quadratum square*. Figure 7a shows that, when a pair of perpendicular axes are placed within the outer square, the axes and the inner square divide the outer square into eight 45 degree isosceles triangles. A second ad quad square results in a 4x4 square grid. In Figure 7b, a sequence of ad quad squares form a geometric series of areas and generate a logarithmic spiral known as a *Baravelle spiral*. The log spiral is the prototype of growth in nature [5] and forms the succession of tones of the musical scale [4]. The rooms of a Roman house were frequently proportioned using ad quadratum squares [14]. In sacred geometry, circles are often inscribed or circumscribed about squares. The square or rectangle, with its axes defining the directions of east, west, north, and south, represents the "earthly" realm while the circle, symbolizing the positions of the Zodiac, represents the "celestial" domain. In Figure 7c a geometric series of squares and inscribed circles are shown.



(a) (b) (c)

Figure 7. a) ad quadratum square; b) Baravelle spiral; c) geometric sequence of ad quad squares.

The Nicholson Taxonomy

The Laurentian Pavement unambiguously demonstrates that in 16th century Florence the different branches of geometry could be brought together as a singular entity - under one roof. As a result of the work of reconstructing the seemingly diverse set of designs found in the pavement panels, we realized that the designs were parts of a much larger picture which might constitute a taxonomy of ancient geometry when considered together. Whilst trying to visualize how the constituent parts of geometry mentioned above might fit together, we developed several prototype taxonomies whose form changed radically. A discussion of these changes may help to identify the shortfalls and value of making such a system.

A. Developing the Taxonomy

The first version was organized along a straight line (Figure 8a) off which hung the main branches of the system. The drawback to this linear system was that it lacked the flexibility to show how the constituent parts linked between each other. It created a false hierarchy that subordinated important constructions to the notion of the division of a line. The second version (Figure 8b) was organized around a central point, and the construction radiated out, ordered by concentric rings. The benefit to this system is that the constructions are visually presented in equal light, but it was still unable to show relationships between the parts that we knew to be there.

It was not until we abandoned linear and concentric principles of organization and opted for a rhizome system (Figure 8c) that the project began to make sense. It was possible to make many more juxtapositions of similar constructions formed by different systems. The taxonomy was animated by constant change as every new inclusion adjusted the relationship and composition of the whole taxonomy. Despite its rhizome form, the diagram had reached a point of instability characterized by hermetic loops that isolated the relationships we sought to exemplify.

The instability of the diagram was made firm with the decision to turn the whole taxonomy inside out (Figure 8d). Instead of dangling the taxonomy from a single point, the originating point became surrounded by the constructions that implied the genesis taking place within a void. The overall form was now capable of visually showing voids and complex relationships between the parts. A series of refinements then followed that inducted more and more new constructions considered necessary to make the taxonomy whole.

In the final version, the origin of the taxonomy appears within a void formed by the constructions of the whole taxonomy (Figure 9).

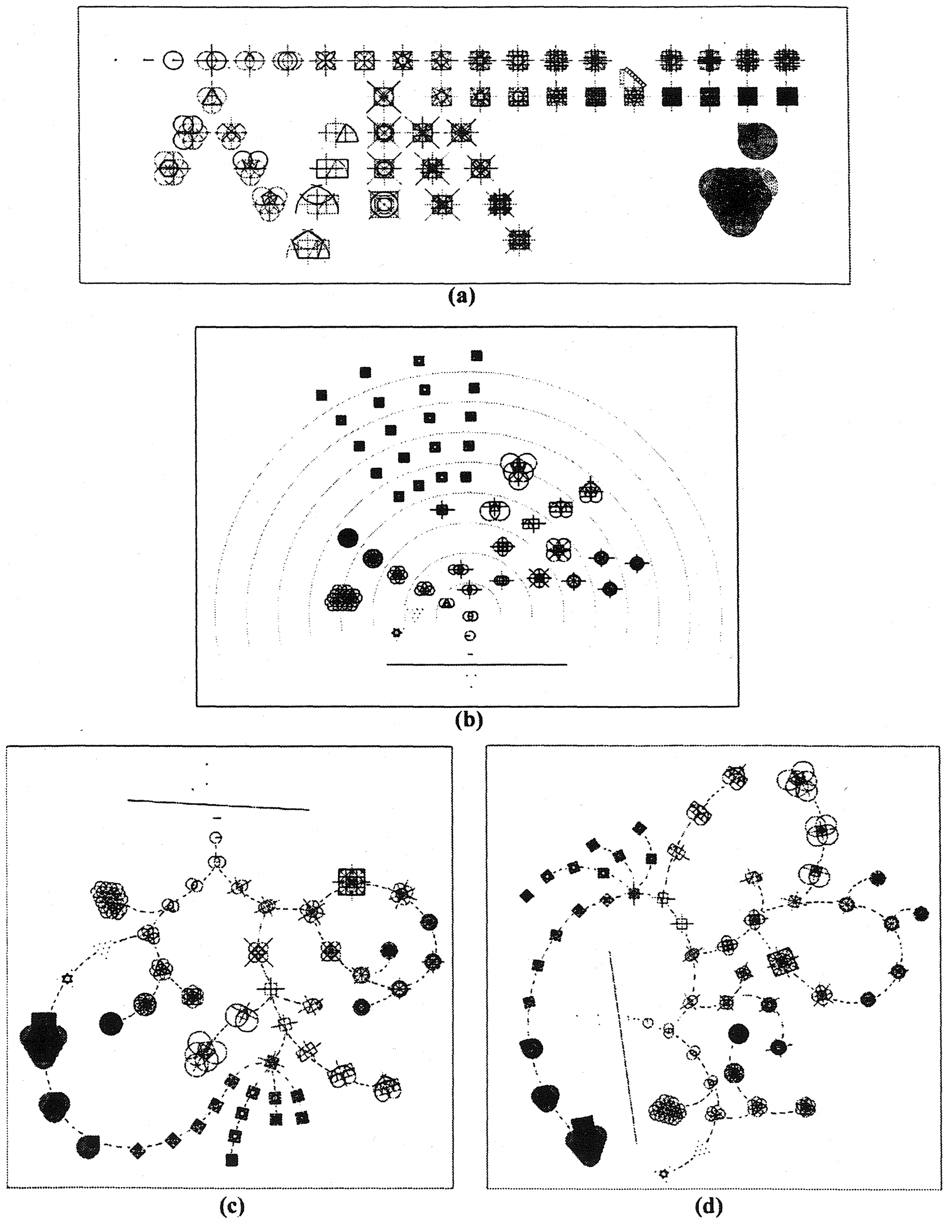


Figure 8. Development of the Taxonomy. a) linear pattern; b) concentric pattern: c) rhizome pattern; d) genesis within a 'void' - themes are linked by adjacency.

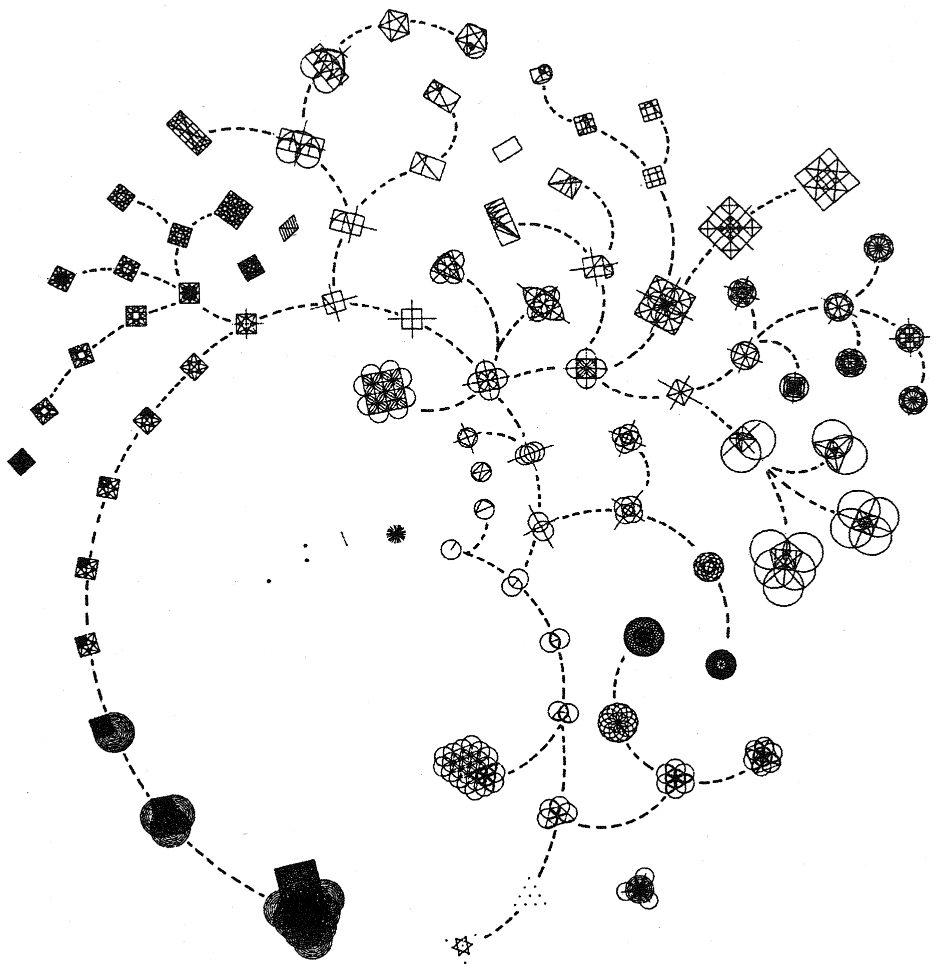


Figure 9. Nicholson's Taxonomy of Ancient Geometry

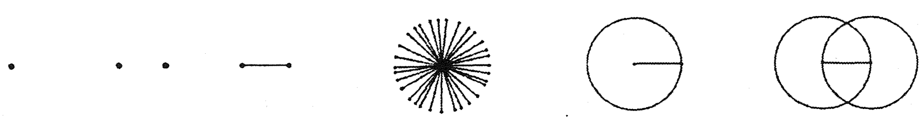


Figure 10. Point to Vesica.

The Nicholson taxonomy takes off from a point which splits by duality into a pair of points that are spanned by a line segment. The length of this line segment is of utmost importance as it is the only dimension required for the subsequent geometric construction, however complex they may be. A series of line segments are joined from a common point with points of the line segments defining a circle. A point placed on the circumference of the circle defines the center of a second circle, the Vesica Pisces. From the Vesica, the other figures of the taxonomy de-couple into roughly contiguous regions leading to the six main groups of constructions mentioned above, i.e. the triangle circle grid, the sacred cut etc..

It was now possible to rationalize each connection, break the hermetic loops, and organize the taxonomy so that all its constituent parts aligned correctly. From this developed system it was also possible to identify relationships between the principle Geometries. Some constructions, such as the sacred cut, which were generated by more than one principle geometry were placed in proximity to each other thereby underscoring the non-linearity of the taxonomy. Now the themes are linked through adjacency rather than being connected by a step by step process of construction.

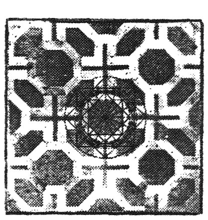
It only remained to add free-floating errant geometries: i.e., approximate structures that divide a circle into 9 parts or a line into 7 units to make the taxonomy complete.

B. Errancy in the Rational System: The Comma

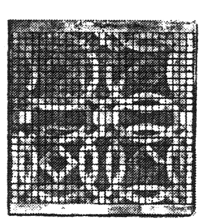
It has been demonstrated above that the root of the taxonomy begins with a point, which splits, forms a line segment, collects a group of points to form a circle, and is then developed into the Vesica Pisces. From this, all other figures of the taxonomy follow in a systematic manner. However, the apparent seamlessness of this system underscores the absence of vital connections to geometry, namely music and architecture. What happens when there is a misalignment of perfect systems of representation? The problem is especially apparent in contemporary architecture, but we will turn to early music theory to demonstrate a way to come to terms with the unavoidable errancy inherent in rational systems.

The musical scale of Pythagoras was developed from a fundamental tone (line segment). All fundamental tones on the tone circle are equivalent (circle). A complete revolution about the tone circle gives rise to the same tone an octave higher or lower in pitch. All other tones of the 12 tone *chromatic* scale arise from the placement of one additional tone (the musical fifth) on the tone wheel (Vesica Pisces).

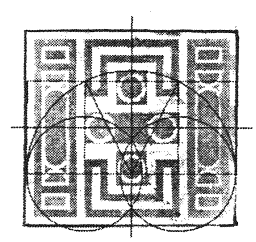
When the tones are represented, in terms of relative string length or inversely as frequencies, as the ratio of small whole numbers, the thirteenth tone of the scale deviates from the first tone by the length of about one-quarter of a whole tone known in musical parlance as a *comma*. Only 11 of the 12 tones can be represented by the ratio of small whole numbers, the twelfth tone must be represented by an awkward approximation to the $\sqrt{2}$. Fitting all 12 tones into the chromatic scale was therefore seen in ancient times as a struggle between the rational and irrational; the finite and the infinite. In modern times with the need to create a piano having only a finite number of keys, the comma was eliminated at the cost of giving up the expression of musical tones in terms of rational numbers. Tonal frequencies were now represented by irrational numbers.



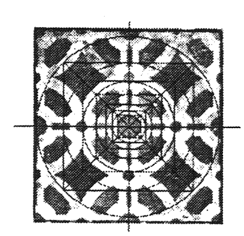
15 Index Panel



14 Medici Panel



13 Mask Panel



12 Dome Panel

In the works of Plato, the musical scale was used as a metaphor to express the tension between rational and irrational and the need to create a sense of limit, preferably self-limit, within the organization of society [4 and 15]. The errancy that is inherent in ancient musical scales was also recognized to exist in the incommensurability of the solar and lunar cycles. Musical theory was seen as an attempt to gain control over threatening infinity. It is interesting that Panel 14 east is laid out on a grid of 81 parts, while Panel 14 west is laid out on a grid of 80 parts. This errancy enshrines the comma of difference between the ancient Pythagorean and Just musical scales of 80:81. Also, while Panel (#2) fits into a 12 by 13 rectangle on one side of the library, it is framed by an 11 by 12 rectangle on the opposite side, another expression of the conundrums inherent within the musical scale.

The ancient musical scale can be shown to be based on a 3:4:5 relationship just as the Brunes Star is made up entirely of 3,4,5- right triangles (Figure 4b). The star octagon embodying the sacred cut (Figure 3j) supplies the $\sqrt{2}$ relationship required to complete the musical scale. Also there is an unavoidable errancy within Brunes's system into a subdivision of seven parts. The error is approximately the same as the comma of the musical scale.

C. Criteria for a Taxonomy

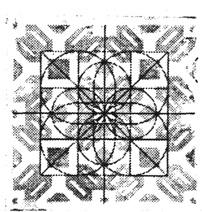
Following from the above remarks, we consider that a successful taxonomy of ancient geometry should have the following characteristics.

1. It should be complete in terms of accounting for the composition and assembly of a wide range of works of architecture and design.

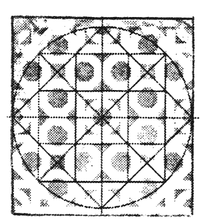
2. It should demonstrate how complex constructions follow inexorably from the primitive notions of a point and a line segment.

3. It should account for the principal geometries described above.

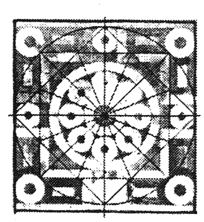
4. It should be easy to follow step by step.



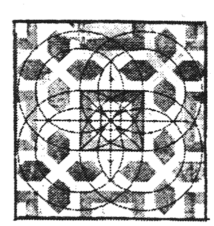
 # 11 Horoscope Panel



 # 10 St Peter Panel



 # 9 San Giovanni Panel



 # 8 Center Panel

An Order for the Library Collections

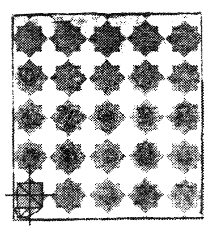
At its opening in 1571, the Laurentian Library housed 3000 books carefully chosen to touch upon virtually every endeavor of humanity. The books were organized into the branches of the traditional Tree of Knowledge, a concept envisioned since the time of the Stoics in antiquity [16], as well as by the standard method of ordering the Christian texts. The book categories can be seen today, written onto wooden boards that hang off each of the 88 desks in the Reading Room. Following the example of how the books are organized, the panel sequence is thought of as a journey. The books in the east range of desks commence with pagan tests (The Index) and lead through poetry, the quadrivium, the Christian philosophers, on to the Pentateuch. The books in the west range of desks begin with the trivium (grammar, rhetoric and oratory) and lead on up through the Tree of Knowledge to metaphysics.

A journey of a similar sort is also taken in Michelangelo's Sistine Chapel Ceiling frescoes, that has at one end a depiction of the baseness of Man and at the other end the majesty of God. Thus it would make sense that the geometric pavement underfoot could also be read as a 'book', an encyclopedic taxonomy that gathers the extant body of geometric knowledge.

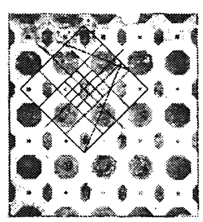
It might be asked what is the rationale behind the ordering of the fifteen panels. The problems we have had in making a sensible order with a linear system are no doubt the same that the Laurentian design team had with their chain of 15 designs. In making the taxonomy, we soon realized that a linear system of organization would not be able to accommodate the commingling of the various branches of the taxonomy. Deleuze's [17] concept for the integrated rhizome form is an appropriate structure to account for its interconnections.

An Order for the Fifteen Pavement Panels

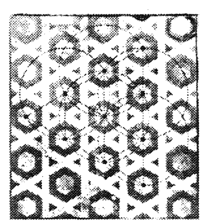
The sequential order of the 15 Laurentian panels is unlikely to have been predicated by geometric concerns alone for the pavement is far more than a taxonomy of geometry. The panels are drenched with symbolic numerology, references to the Church and the Medici Family, and any number of puns and quotations from literature. There are hints that the pavement may have formed a 'pictorial catalog' for the books adjacent to the panels, the geometry of the panels responding to the categories in which the books were arranged. Taking the above issues into consideration, a rationale is presented for an ordering of the 15 panels.



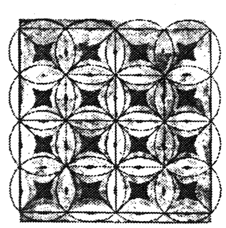
 # 7 Assisi Panel



 # 6 Forum Panel



 # 5 Tectractys Panel



 # 4 Star Panel

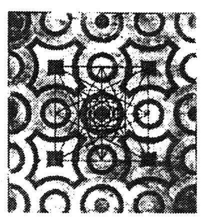
Next to the Library Reading Room entrance, Panel 15 (Index Panel) begins the sequence. The dominant geometry is the octagon, a traditional ideogram for the qualities of four-ness in creation [18]: the Islamic texts in the Library rest above this panel. Pane 14 (Medici Panel) revealing the Medici Coat of Arms is structured by the Platonic Lambda which was appropriated in the late Renaissance to proportion the body[2]. The subsequent Panel 13 (Mask Panel) continues the theme of corporeality by presenting the visages of comedy and tragedy alternatively laughing and grimacing, and regulated by the golden mean. Books of the Latin Poets rest upon this panel. This set of three panels suggests a progression from Oriental roots leading to the corporeal nature of mankind.

Next follows a series of five panels all of which require X and Y axes for their construction. Panel 12 (Dome Panel) introduces the squared circle, the geometric construction necessary to form the sacred cut present in the following four panels. Panel 11, 10, 9, & 8 all depend upon one or another version of the sacred cut, a $\sqrt{2}$ figure that provides an irrationally dimensioned square in the center of the design. The four panels tussle with the idea of the rational pitted against the irrational as well as exemplifying the complex rapport held between the notion of the center and the outside. Panel 7 (Assisi Panel) sums up the themes of the preceding 4 panels with an overt display of a √2 proportional system: this panel lies beneath the books on logic.

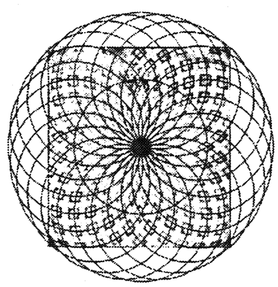
Panel 6 (Forum Panel) is closely related to Panel 7, and it is ordered by dynamic symmetry within the sacred cut. In association with Panel 7, it points to concepts of generative form. The next pair of panels presents themes of infinity by displaying universal and extended grids. Panel 5 (Tectractys Panel) presents the form of the Tectractys - the Decad - the mother of numerical systems. Panel 4 (Star Panel), an expansive √2 grid, sets up an infinite realm in readiness for the final three panels.

The first of the three, Panel 3 (Elements Panel), is regulated by the Brunes Star and follows the form of four intertwined circles favored in pavement design. It is the appropriate form with which to represent the four elements [19]. The penultimate panel, Panel 2 (Cosimo Panel), is chosen to reveal Duke Cosimo de Medici's personal empresa. The empresa is surrounded by a 96-leaf rosette whose formation coincidentally repeats Bode's numerical series that orders the relative distances of the known planets in the 16th century. Books on metaphysics are found adjacent to these two panels.

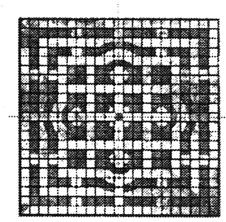
We are now left with one panel to close the series, perhaps charged to reveal the nature of God. Panel 1 (Cross Panel) sports a magnificent Cross that spreads across its center. The panel's geometry originates upon a central square surrounded by 10 ever increasing squares - a gnomon shot through with complex numerology that suggests the pliant nature of micro and macro [20].



 # 3 Elements Panel



 # 2 Cosimo Panel



 # 1 Cross Panel

That the books of the Hebrew Pentateuch are found here is an apt metaphor for the 10-ness that pervades the Hebrew Bible, i.e. the 10 Commandments, 10 generations to Abraham, and 10 more generations to the Flood. McClain has found that much of the numerology of the Hebrew Bible can be related to the ancient musical scale, a version of the scale based on the first 10 numbers [21].

It is worth mentioning here that the chosen number of panels, fifteen placed on each side of the Library, may also be related to ancient harmonic law. McClain suggests that the genesis of the ancient musical scale goes back to the third millennium BC, in Mesopotamia, where it was related to the number system base 60 used by that civilization. The base number 60 was taken to represent the fundamental tone of the scale [22]. This tone can be reduced by two octaves to 30 and 15 but no further using whole numbers. 60 (the unit 1) is the God number representing Anu "Father of the Gods" and head of the pantheon. He is the reference number or the fundamental tone; 30 (1/2) Sin, the moon, establishes the basic Sumerian octave matrix 1:2; while 15 (1/4), Ishtar, is the epitome of the feminine as virgin, wife, and everybody's mistress. McClain has found that the double octave is the key to understanding Hebrew and Greek musical theory [23].

Conclusion

Geometry and number have for centuries served as a metaphor for Sacred Knowledge. The extensive occurrence in the pavement panels of particular geometries and the numbers they spawn, suggest that the pavement designers were wholly cognizant of the bond between number and mythology that modern scholarship is once again making available for us. We have found no evidence to show how or when the pavements were designed despite extensive forays made by a colleague, Dr. Rolf Bagemihl, who has searched into archives in Florence, Pisa, and Rome [2] .

It is probable that the design team included specialists in the fields of library science, theology, geometry, music, and architecture, much like the interdisciplinary team that has undertaken this investigation. But even without knowing the authors and the logic behind the ordering of the 15 pairs of panels, the fact remains that the pavement includes so much of what constitutes two-dimensional geometry. At the very least, the pavements suggest that their designers were capable of conceiving the various branches of geometry as one unified system - a Body of Knowledge which for some reason was immediately concealed beneath the wooden dais of the library desks.

The difficulty of threading a unified story between the 15 panels points to the difficulty of arranging any body of knowledge, i.e. the Tree of Knowledge into a linear or paginated form as the 18th century French encyclopedists were soon to discover. We can count ourselves fortunate that we are active in an age when nonlinear, compound diagrams, made even more potent by cyberspace, make it possible to give new form to interconnective ideas which, of course has been the traditional responsibility of the architect since time began.

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