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Mathematical Connections
in Art, Music, and Science

Mathematics and Poetry: Discrepancies within Similarities

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The usual way to talk about mathematics and poetry is to stress their discrepancies. Moreover, math and poetry are often considered as two poles of the human language, characterized by the strongest possible opposition. Several times we tried to deepen and order this way to look at the relation between math and poetry. Because we don't want to charge this presentation with bibliographical references, we will recall in this respect only our monograph "Mathematische Poetik", published in 1973 by Athenaeum Verlag (Frankfurt/Main); the initial version, in Romanian (Poetica Matematica) was published in 1970.

Step by step, we have realized that most oppositions between math and poetry take place within the framework of some similarities and give rise, in their turn, to new possible similarities; differences and similarities alternate in an endless succession. So, the right way to approach the math-poetry relation is to observe how some discrepancies emerge from some similarities and conversely.

1) A striking common feature of math and poetry: it is difficult, if not impossible, to define them. Math conceived as the science of numbers and spatial forms is for a longtime no longer acceptable. Math moved from quantity to quality and from numbers to structure; any attempt to capture in a single statement its great diversity fails. Similarly, poetry cannot be captured in a definition. Georges Braque said once that what counts in art is just what cannot be explained. Most people don't understand what the mathematicians are doing; they were usually confused with accountants, with engineers, with logicians and now with computer scientists. Poets, in their turn, are confused with people away from the contingent life, if not with crazy people. The ineffable nature of poetry is in contrast with its usually very specific external aspect, consisting of some prosodic features such as verses, meter, rhythm, rime. The difficulty in defining math is in sharp contrast with its very specific external aspect, consisting of artificial symbols, formulas, equations, etc. However, versification is neither necessary nor sufficient for poetry, while the use of mathematical symbols and formulas is neither sufficient nor (to some extent) necessary to guarantee that we are in the presence of a piece of mathematical thinking.

2) Both math and art are fields of research and creativity. As far as we know, no psychological difference was found between math and artistic creativity; great mathematicians such as Henri Poincaré and Jacques Hadamard and great artists such as Mozart suggested this.

3) Both math and poetry require a balance between invention and discovery. In math, definitions, axioms, postulates, theories and models are rather a matter of invention, while theorems are a

matter of discovery. Poetry too is an articulation of invention and discovery. The poetic work is invention, because it did not exist before the creative work of its author; it is also discovery, because the poet selects some combinations of words which potentially already exist in the language, waiting only to be actualized. Let us recall, in this respect, that Saint John Perse, who was both a poet and a scientist, observes (in "Discours de Stockholm") that the scientific itinerary observation-hypothesis-testing is analogue to the artistic itinerary emotion-creation-expression. Despite our above claim, the invention-discovery relation remains controversial. For Wittgenstein math is pure invention, while for Rene Thom (platonist) it is pure discovery. Music too is invention for some authors, discovery for other authors.

4) The importance of invention in both math and poetry follows to a large extent from the basic role of fiction in both these fields. The price math and poetry have to pay in order to reach a high level of rigor is to replace the real universe by a fictional one. Euclid's geometry starts with fictional entities: a point is that which has no part; a line is breadthless length; a straight line is a line which lies evenly with the points on itself. Similarly, Aeschylus tragedies start with fictional characters. For both, there is a scenario, corresponding to axioms, problems and theorems in the first case and to a specific action or conflict in the second case. However, the nature of mathematical rigor is different from that of artistic rigor; the former is predominantly logical, while the symptom of the latter is the feeling that nothing can be changed, added or eliminated.

5) Both math and poetry show a tendency towards higher and higher abstraction, stimulated by their fictional nature. In the case of math, we have to add, as a source of abstraction, the generalization process (see, for instance, the passage from arithmetics to algebra). In the case of poetry, an important source of abstraction is its mediated approach; poetry proceeds by suggestion, which is a mediated, indirect way, in contrast with math, where things are directly and explicitly told.

6) Another common source of increasing abstraction for math and poetry is their common interest in the hidden, invisible aspects of reality. This feature will be considered later, in connection with some other common features of math and poetry.

7) Both math and poetry realize, as a result and expression of their rigor and as a need of their creativity, a semiotic optimisation: maximal of meaning in minimum of expression. In other words, there is a trend towards the largest possible density of meaning. In math, the best example in this respect is the theorem, the fundamental brick of math. A theorem tells us, in a few lines a truth that which could require several pages in an attempt to translate it in ordinary language; practically, this is impossible, because the preciseness and the heuristic value of a theorem, of an equation, of a mathematical formula lies just in its shortness (in this respect, the use of some artificial means is essential for the math language).

Similarly, in a Shakespearean sonnet or in a Baudelaire poem we feel that no compression is possible. Consequently, like math, poetry cannot be translated in ordinary language.

However, math and poetry are different related to their length. For any mathematical work, we can make an abstract, because such a work is hierarchically organized, some parts of it being richer in information than some other parts. For instance, retaining the problem, the basic definitions and assumptions and the basic theorems, we get a good approximation of a mathematical work and we can, at least in principle, even reconstruct, starting from them, the other parts. No such abstract is possible for a piece of poetry, because there is no objective hierarchical organization of it (each personal reading may lead to a different hierarchy). It may be interesting to note that this situation corresponds to the status of randomness in the algorithmic-complexity approach (due to A. N. Kolmogorov and G. Chaitin) to information; a string over a given finite alphabet is random if it cannot be algorithmically compressed (no algorithm describing this string is shorter than it). In other words, poetry corresponds to the highest complexity the human language can acquire.

8) With the preceding aspect, we reach the linguistic level of human creativity and we can assert that both math and poetry try to exploit longer and longer, if possible, infinite contexts. In math, this is visible in its step-by-step approach, each step being explicitly and strongly based on the preceding steps. If you eliminate the first ten pages from a mathematical textbook, you risk understanding almost nothing from the remaining pages; the same operation in a textbook of history or geography may have no significant influence.

For poetry, the dependence on long distance contexts has another motivation: poetry is trying to diminish, if not to cancel, the conceptual dimension of language, by leaving the dictionary meaning of words and replacing it with a contextual, ad-hoc meaning the reader has to build and learn on his own. It was said once that reading poetry is like learning a foreign language; but in contrast with the proper foreign languages, for which a dictionary is usually available, poetry has no a priori dictionary and the reader has to compensate this absence by looking carefully at the whole poetic context. It may be interesting to note that a similar situation occurs in the cosmic language LINCOS proposed by Hans Freudenthal for a possible cosmic intercourse; no a priori dictionary is possible, in our attempt to communicate with intelligent beings from other celestial bodies.

However, this opposition between the conceptual nature of math and the contextual nature of poetry can be attenuated. Despite its explicit conceptual structure, math too is, to a large extent, contextual. Each mathematical entity has two parts: the conceptual part, given explicitly in a definition, and the contextual part, determined by its behavior in various particular situations. A very elementary example, showing the incapacity of a definition to capture the whole contextual behavior of an entity is the symbol 0 (zero). We have to distinguish between descriptive (gender-species) definitions, related to the conceptual part of an entity, and operational definitions, related to its contextual behavior. But it may happen, like in the case of zero, that even the operational definition cannot cover the whole contextual behavior, because the former should be finite and short, while the latter may be practically infinite. On the other hand, poetry, despite its anticonceptual tendency, cannot leave completely the conceptual structure of the ordinary language. A complete cancellation of the dictionary meanings and of the grammar of the language in which poetry is written would leave poetry unintelligible.

9) A first consequence of the previous fact is the solidarity, in both math and poetry, between the local and the global aspects. A local error in a mathematical text may spread, contaminate and compromise it in its totality, while a single word in a piece of poetry may require, in order to be understood, to be related to the whole text. Moreover, some local aspects may have the capacity to account for the global ones. The statement of a central theorem in a mathematical text is responsible for the whole text; the instantaneous may account for the eternity and the blade of grass may account for the whole universe in a high quality piece of poetry (see William Blake's famous slogan: "To see the world in a grain of sand/and eternity in an hour"). This is the holographic principle, discovered in the field of photography by D. Gabor and illustrated by the mirror and by the fire; any part of a mirror or of a fire is equal in power with the whole mirror or fire. Analyticity in math is just of this type; the behavior of an

analytic function in the neighborhood of a point decides its global behavior. Any tree in a good poem accounts for all trees in the world.

10) Both math and poetry have a genuine relation with natural languages, despite the fact that they try to transgress, each of them in its way, the framework of natural languages. Already in the XVIIth century began the divorce between mathematical language and the ordinary language, by supplementing the latter with an artificial component in order to improve the former, by facing the increasing need of clarity and precision required by modern science. Ordinary language was born and evaluated in conditions of spontaneous communication humans developed in connection with their basic needs, so it was not prepared to face the requirement of rigor related to scientific research. Poetry too transgressed ordinary language, but in a different way; the elements of ordinary language got a fresh use, by replacing their denotative function with a conotative one, ruled by the poetic context.

However, if we go deeper in our analysis, this difference between math and poetry can be attenuated. Mainly in their modern form, math and poetry share more and more the use of what we could call "idiomatic expressions", understood as a type of integrative meaning; the meaning of a sequence s is different from the concatenation of the meanings of the terms of s. Take, for instance, the notation of the integral of a function f on the interval [a,b]. The expression f(x)dx under the integral sign, if we take it in the denotative way, is the product between f(x) and the differential dx of x. Nothing of this interpretation is valid when f(x)dx is placed under the integral sign.

11) Both math and poetry are faced with a semiotic crisis, which is a part of a more general crisis, defining the condition of human species today. This happens because both science and art shifted to a large extent their attention from the visible to the invisible aspects of the world. In science, attention moved beyond the macroscopic world, to the infinitely small (see quantum physics) to the infinitely large (see relativity theory); moreover, science tries to bridge these two extreme parts of the invisible (see quantum cosmology). Poetry in its turn is more and more directed to the hidden world, to a second reality, located beyond our senses. Poetry, art in general, becomes a metaphor of the invisible, built by means of the visible and this is equally valid for science.

12) Both math and poetry show a genuine presence of the element LUDUS (game involving practical difficulties) and of the element PAIDIA (freedom, vivacity), of both play and vitality. Both math and poetry are both a game and a play; a game, because they are possible only if the number of attempts is allowed to be larger than the number of successes, so the right to be wrong, to fail is a condition of creativity; a play, because they are based on a scenario, they are basically like a theatrical performance and they are possible only in a state of joy.

13) The genuine presence of paradox in both math and poetry follows from the distance they take in respect to the ordinary, everyday life.

14) The genuine presence of infinity can be observed in both math and poetry. Math is to a large extent the study of infinity by means of the finite, but, mainly in our century, also the study of the finite by means of the infinite, of the discrete by means of the continuous). The poetic work is finite in its appearance, but the reading process of poetry transforms it into an infinite structure.

15) Self-reference is essential in both math and poetry; see Goedel's incompleteness theorem (in its proof, arithmetics is the unavoidable metalanguage of arithmetics conceived as an object language);

on the other hand, self-reference is a basic feature of modern poetry, of modern art in general (poetry of poetry).

16) Imprecision too is genuine to both math and poetry. Most numbers are known only by their approximations. The history of math is to a large extent the history of approximation of what is of high complexity by what is of low complexity. Approximation, randomness, ambiguity, generality, fuzziness, roughness, fractalness, chaos, negligibility, indecidability are some of the main topics of mathematical research. Poetry, in its turn, cannot be conceived without imprecision. Moreover, poetry is favored by imprecision. The crepuscular is more poetic than the clear day. The hermetic, the fantasmagoric, the obscure, the strange, the mysterious, the vague are at home in poetry.

We stop here this analysis that requires further clarifications and investigations.